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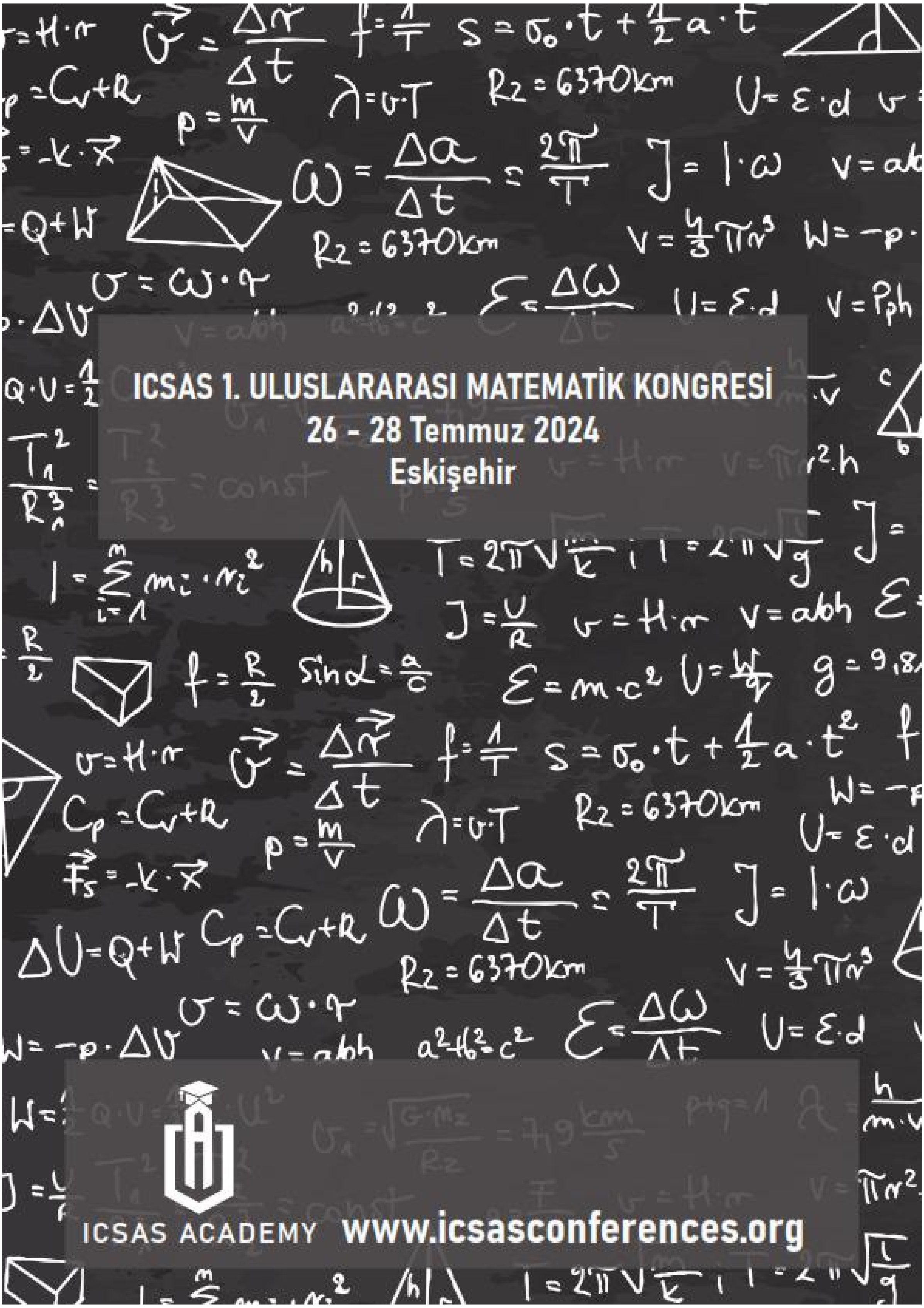

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JULY 26-28, 2024
ESKISEHIR

Edited By
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Sayı : E-16710634-03-903.07.02-2300384284 01.12.2023
Konu : Doç.Dr.Elif AKPINAR KÜLEKÇİ'nin Görevlendirilmesi

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İlgi : 29.11.2023 tarihli ve E-53120705-000-2300381989 sayılı belge.

Fakülteniz Peyzaj Mimarlığı Bölümü öğretim üyelerinden Doç.Dr.Elif AKPINAR KÜLEKÇİ'nin, Yükseköğretim Genel Kurulunun 15.06.2023 tarihli, 10 sayılı oturumunda alınan 2023.10.183 sayılı kararı gereğince Doçentlik Başvuru Şartlarında bulunan ve doçent olacak adaylardan istenen "Diğer uluslararası/ ulusal bilimsel toplantının düzenleme komitesinde resmi olarak görevlendirilmiş üniversite akademisyen temsilcisi bulunması zorunludur." maddesi gereğince, Academy Global Conference & Journals tarafından yapılan kongrelerin düzenleme kurullarında yolluksuz ve gündeliksiz olarak görevlendirilmesi Rektörlüğümüzce uygun görülmüştür.

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HALL / SALON 1	**Prof. Dr. Serkan TİMUR**	1	8 th GRADE SCIENCE COURSE TEACHING PROGRAM: A COMPARATIVE ANALYSIS OF CHANGES BETWEEN 2018 and 2024	Assist Prof. Dr., Ahmet UYAR
		2	INVESTIGATION OF INJURY ANXIETY LEVELS OF WOMEN BASKETBALL PLAYERS	Dilara TANDOĞAN Nursima ÜZÜMCÜ Berna ISPANAKÇI Prof.Dr. İlker ÖZMUTLU
		3	EXAMINATION OF PHYSICAL ACTIVITY LEVELS OF UNIVERSITY STAFF THROUGH VARIOUS VARIABLES	Hatice KILIÇ Nisanur İSLAM Prof.Dr. İlker ÖZMUTLU
		4	ÖĞRETMEN İŞ STRESİNİN NEDENLERİ	Dr. Öğr. Üyesi, Fadile AYDIN
		5	OKYANUSSAL DUYGULAR ÖLÇEĞİ’NİN TÜRKÇE’YE UYARLANMASI	Psk. Dan. Selin DEMİRBAŞ-YILMAZTÜRK Prof. Dr. HALİL EKŞİ
		6	MEB ANATOLIAN TALES SELECTED FOR PRESCHOOL CHILDREN EXAMINATION IN TERMS OF VALUES EDUCATION	Eğitim Bilimleri YL.Mezunu., ELÇİN GENİŞ
		7	AN EXAMINATION OF POSTGRADUATE THESES ON TRADITIONAL CHILDREN’S GAMES IN TURKEY	Prof. Dr., Adem BAYAR Yüksek Lisans Öğrencisi, Gökhan ÜNSAL
		8	SAHADAN SESLENİŞLER: ÖĞRETMENEVİ VE AKŞAM SANAT OKULU YÖNETİCİLERİNİN KARŞILAŞTIKLARI SORUNLAR VE ÇÖZÜM ÖNERİLERİNE YÖNELİK NİTEL BİR ÇALIŞMA	Prof. Dr., Adem BAYAR Yüksek Lisans Öğrencisi, Gökhan ÜNSAL
		9	ÖĞRETMEN ADAYLARININ EKOLOJİK ZEKÂLARI İLE BİYOÇEŞİTLİLİK OKURYAZARLIĞI ARASINDAKİ İLİŞKİNİN İNCELENMESİ	Emine Büşra GÜÇLÜOĞLU Prof. Dr. Serkan TİMUR

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HALL / SALON 2	**Assoc. Prof. Dr. Nazile Abdullazade**	1	TURKISH LEARNING CONCERNS OF REFUGEE STUDENTS STUDYING IN SECONDARY SCHOOL IN TURKEY	Mehmet KARDOĞAN Prof. Dr. Mehmet Nuri KARDAŞ
		2	TURKISH LEARNING STYLES OF REFUGEE STUDENTS STUDYING IN THE SECOND LEVEL OF PRIMARY EDUCATION	Mehmet KARDOĞAN Prof. Dr. Mehmet Nuri KARDAŞ
		3	OKUL DIŞI ÖĞRENME ORTAMLARINA İLİŞKİN SINIF ÖĞRETMENLERİNİN TUTUMLARININ VE UYGULAMALARININ İNCELENMESİ	MEB Sınıf Öğretmeni, Yasemin İŞGÖREN Doç.Dr.,Melike ÖZYURT
		4	AUGMENTED REALITY IN EFL LEARNING: A SYSTEMATIC REVIEW OF RESEARCH BETWEEN 2021-2023	Assist. Prof. Dr. Gülin ZEYBEK
		5	ORTAOKUL ÖĞRENCİLERİNİN ÇEVRESEL DEĞERLERE KARŞI TUTUMLARININ BELİRLENMESİ	Ferit AKÇAKAYA Dr. Öğr. Üyesi, Gonca ÇAKMAK
		6	İLKOKUL FEN BİLİMLERİ DERSİ ÖĞRETİM PROGRAMI KAZANIMLARININ VE ÇALIŞMA KİTABI SORULARININ YENİLENEN BLOOM TAKSONOMİSİNE GÖRE İNCELENMESİ	Öğretmen, Fatma ERGÜLEÇ Doç. Dr. Melike ÖZYURT
		7	COMMUNICATION STRATEGIES OF GENERATION X SCHOOL ADMINISTRATORS WITH GENERATION Z STUDENTS	SEDA EREN PROF. DR. SADEGÜL AKBABA ALTUN
		8	HUMAN RIGHTS' MOTIVES IN THE CREATION OF NAMIG KAMAL AND ALIAGHA VAHID	Assoc. Prof. Dr. Nazile Abdullazade
		9	YÖNETİCİLERDE İŞE TUTKUNLUĞUN DEMOGRAFİK DEĞİŞKENLER AÇISINDAN İNCELENMESİ	Öğretmen Kenan AKTÜRK Doç. Dr. Çiğdem BERBER ÇELİK

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HALL / SALON 3	**Prof. Dr. ALİ RIZA GÜL**	1	EVALUATION OF 3RD GRADE LIFE SCIENCE CURRICULUM ACQUISITIONS ACCORDING TO DETTMER TAXONOMY	Öğr. Gör. Dr. Akın KARAKUYU
		2	EXAMINATION OF STUDIES PREPARED RELATED TO CURRICULUM FIDELITY	Öğr. Gör. Dr. Akın KARAKUYU
		3	ORTAOKUL ÖĞRENCİLERİNİN IŞIK VE SESİN BENZERLİKLERİ VE FARKLILIKLARI İLE İLGİLİ SAHİP OLDUKLARI KAVRAMLARIN 2013 VE 2018 ÖĞRETİM PROGRAMINA GÖRE KARŞILAŞTIRILMASI	Uzman Öğretmen, FATMA BETÜL ÖZDEMİR Doç. Dr. MUALLA BOLAT
		4	YAPAY ZEKA DESTEKLİ EĞİTİMDE ÖĞRETMEN VARLIĞININ 6. SINIF FEN BİLİMLERİ DERSİ ELEŞTİREL DÜŞÜNME GÜCÜNE ETKİSİ	Doktor Öğretim Üyesi, Senem ÇOLAK YAZICI Nazlıcan YÜKSEL
		5	ALTINCI SINIF ÖĞRENCİLERİNİN MATEMATİKSEL MODELLEME YETERLİKLERİNİN İNCELENMESİ: ACİLE GELEN YÜKSEK TANSİYON HASTASI PROBLEMİ	Doç. Dr. Muhammet DORUK Öğr. Gör. Dr. Fikret CİHAN
		6	DEVELOPMENT OF A CURRICULUM FOR RAISING CLIMATE CHANGE AWARENESS IN PRIMARY SCHOOL STUDENTS	Prof. Dr. Eti Akyüz Levi Prof. Dr. Hale Sucuoğlu Doç. Dr. Ahmet Murat Ellez Tuğba Yılmaz
		7	6.SINIF TÜRKÇE DERSİ ÖĞRETİM PROGRAMININ IÇERİK VE DEĞERLENDİRME BOYUTUNDA HAMMOND DEĞERLENDİRME MODELİNE GÖRE DEĞERLENDİRİLMESİ	Yüksek Lisans Öğrencisi, AYŞEGÜL YILDIZ Prof. Dr., ŞENEL ELALDI
		8	2018 ve 2024 SINIF HAYAT BİLGİSİ DERSİ ÖĞRETİM PROGRAMLARININ KARŞILAŞTIRILMASI	Öğr. Gör. Dr. Burcu KARAFİL Öğr. Gör. Dr. Osman ÖZDEMİR
		9	İLAHİYAT FAKÜLTELERİNİN EĞİTİM PROGRAMLARININ VE YAPILARININ GÜNCELLENMESİ ÜZERİNE BAZI DÜŞÜNCELER	Prof. Dr. ALİ RIZA GÜL

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		2	A COMPARATIVE BIBLIOMETRIC ANALYSIS BASED ON THE ISTANBUL UNIVERSITY JOURNAL OF SOCIOLOGY AND THE AMERICAN SOCIOLOGICAL REVIEW	Assc. Prof. Nuriye ÇELİK
		3	SOCIAL MEDIA REGULATION: COMPARISON OF THE DIFFERENT MODELS	Associate Professor, TARANA MAHMUDOVA
		4	GÖÇÜN FELSEFESİ ÜZERİNE BİR TARTIŞMA	Dr. Erol AKYILDIRIM
		5	ALGI YÖNETİMİ	Dr. Erol AKYILDIRIM
		6	SOCIOLOGICAL ANALYSIS OF SHELTER AND HOUSING CRISIS: THE MOVIE ‘’THE TENANT’’	Dr. Arş. Grv. Özge Seda UĞRAŞ
		7	SOSYAL MEDYA KULLANICILARININ ALGORİTMALARA KARŞI TAKTİKLERİ	Dr. Öğr. Üyesi Hatice DURAN OKUR
		8	BAYBURT ÜNİVERSİTESİ ÖĞRENCİLERİNİN AKILLI TELEFON BAĞIMLILIĞI ÜZERİNE BİR ARAŞTIRMA	Doç. Dr. Emrah DOLGUNSÖZ Y. Lisans Öğr. Sema MELEKOĞLU
		9	SOCIAL INTEGRATION OF ROMA PEOPLE THROUGH MUSIC AGAINST SOCIAL EXCLUSION	Res. Asst. BURÇE ULUBİLGİN ÇUHADAR Prof. AYKUT B. ÇEREZCİOĞLU
		10	İLKOKULLARDA ÇALIŞAN SINIF ÖĞRETMENLERİ VE ORTAOKULLARDA ÇALIŞAN BRANŞ ÖĞRETMENLERİNİN MESLEKİ AÇIDAN MUTLULUK DÜZEYLERİ VE KARŞILAŞTIRILMASI	Doç.Dr. Süleyman YURTTAŞ Yüksek Lisans Öğrencisi Halil Ayvaz
		11	KONYA’YA BALKAN GÖÇÜ: BİR MUHACİR HİKAYESİ	Dr.Öğrt.Üy.Ayşegül SİLİ-KALEM

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		2	ENHANCING ENGINEERING ACCREDITATION: AN EXAMINATION OF INTERNAL QUALITY ASSURANCE AND SELF-EVALUATION METHODS	Mei-Ling Chen, Takumi Yoshida
		3	ENHANCING HISTORICAL LEARNING OUTCOMES THROUGH MULTIMEDIA INTEGRATION: A COMPARATIVE STUDY	Dr. Mei Lin Zhou, Prof. Akio Tanaka, Dr. Samuel Chike, Dr. Rosa Mwangi, Dr. Aisha N'Guessan
		4	ADVANCING SCIENCE EDUCATION: INNOVATIVE STRATEGIES FOR INTEGRATING NUCLEAR SCIENCE	Mei Lin Zhang, Tariq M. Abdel-Rahman
		5	ASSESSING PSYCHOTHERAPEUTIC SUPPORT FOR ENGINEERING STUDENTS: INFLUENCE OF THERAPIST ATTRIBUTES	Li Wei Zhang
		6	ADDRESSING LEARNING BARRIERS IN BUILDING MEASUREMENT COURSES	Lin Wei Zhang, Amina B. Mwangi
		7	Enhancing Education with Hybrid Learning Models: The Role of Digital Platforms	Akio Tanaka
		8	Learning and Behavior Modification through Casual Gaming: A Study on Environmental Awareness	Hanae Yamaguchi, Mwangi Ochieng
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HALL / SALON 2	**Dawei Zhang**	1	ADVANCING ROBOTICS EDUCATION THROUGH PROBLEM-BASED LEARNING: AN EVALUATION AT NANJING UNIVERSITY OF TECHNOLOGY, CHINA	Liu Wei, Zhang Hong, Chen Mei
		2	ANALYZING UNIVERSITY STUDENTS' COMPREHENSION OF NUMERICAL REPRESENTATION IN AVERAGE RATE OF CHANGE	Hiroshi Nakamura, Yuki Tanaka
		3	EXPLORING THE IMPACT OF MATHEMATICAL SELF-PERCEPTION, INTEREST, AND IDENTITY ON ACADEMIC PERFORMANCE	Lin Zhang, Musa Moyo
		4	ENHANCING EMBRYOLOGY EDUCATION THROUGH VIRTUAL REALITY: A NOVEL APPROACH	Hanae Tanaka, Li Wei Chen, Amina Khamis, Ryohei Nakamura, Kofi Boateng, Siti Rahmah
		5	A SURVEY OF CAREER ASPIRATIONS AMONG FINAL-YEAR STUDENTS AT THE FACULTY OF HEALTH SCIENCES, UNIVERSITY OF IBADAN, NIGERIA	R. Okafor, L. Adebayo, N. Chukwu, K. Bello, T. Ibrahim, A. Olaniyan, E. Ajayi, M. Adewale
		6	ENHANCING LEARNER INSIGHTS IN CORPORATE TRAINING USING XAPI: A STUDY ON BEHAVIOURAL PATTERNS AND PREDICTIVE ANALYTICS	Kenji Nakamura, Aisha Bello, Thabo Mokoena
		7	UNDERSTANDING ONLINE GRADUATE STUDENTS' ENGAGEMENT IN ACTIVE LEARNING: A CASE STUDY IN EAST ASIA	Hana K. Tanaka, Kofi B. Asante
		8	ENHANCING ONLINE GRADUATE STUDENT ENGAGEMENT THROUGH INSTRUCTOR STRATEGIES IN SOUTHEAST ASIA	Mei-Ling Chen, Dawei Zhang
		9	EVALUATING INTERACTIVE DYNAMICS IN HYBRID LEARNING ENVIRONMENTS: AN EXAMINATION OF DATA INTEGRATION AND ITERATIVE CONNECTIONS	Amina K. Al-Hassan, Kofi Mensah

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		2	EXPLORING THE ROLE OF PHYSICAL COMPUTING IN ENHANCING COMPUTATIONAL THINKING AND PROGRAMMING SELF-EFFICACY IN STEM EDUCATION	Li Wei Zhang
		3	ENHANCING ARTISTIC SKILLS IN EARLY CHILDHOOD EDUCATION THROUGH GRAPHIC ACTIVITIES: A STUDY IN ZAMBIA	Laila Nkhoma, Temba Moyo
		4	ENHANCING TEACHER DEVELOPMENT THROUGH A PRINCIPLE-CENTERED, TECHNOLOGY-INFUSED KNOWLEDGE BUILDING FRAMEWORK: INSIGHTS FROM A SECONDARY EDUCATION PROFESSIONAL LEARNING TEAM	Kaito Nakamura, Amina Sani
		5	EVALUATING COGNITIVE LOAD IN STUDENT PILOTS DURING TRAINING WITH MODERNIZED RECREATIONAL AIRCRAFT	Aiko Nakamura, Yuto Tanaka, Chikezie Okonkwo
		6	ADVANCING STEM EDUCATION THROUGH NEUROCOGNITIVE LEARNING FRAMEWORKS: AN INNOVATIVE APPROACH	Mei Ling Zhang, Aiko Nakamura
		7	LEVERAGING MOBILE LEARNING APPS TO ENHANCE METACOGNITIVE ABILITIES: A STUDY OF AN ENRICHMENT PROGRAM FOR HIGH-ACHIEVING STUDENTS	Haruto Tanaka, Li Na Wang
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HALL / SALON 4	**Dr. Clara Ruiz**	1	Embracing Education 4.0 Trends in Language Instruction	Dr. Clara Ruiz
		2	The Role of Etiquette and Public Speaking in Early Childhood Education: Implications for Academic and Professional Success	Maya Chen, Laila Nasser
		3	Exploring Intercultural Competence among Jewish and Arab Students in a Multicultural Academic Setting in Israel	Miriam Levy, Daniel Cohen
		4	Enhancing Mathematical Skills in Children with Autism through Project MIND: A Pilot Study	Dr. Alex Morton, Sarah Thompson, Laura Kim, Daniel Lee, Emily Park
		5	Innovations in Classroom Furniture: A Multicultural Workshop Experience with Chinese Design Students	L. J. Robertson, M. T. Elman, R. J. Gupta
		6	Evaluating the Efficacy of the VARK Learning Model in Undergraduate Education :	Eliza Turner, Thomas Bennett
		7	Enhancing Student Engagement and Learning Outcomes through Digital Polling Tools	Li Wei Zhang, Amina Kante
		8	The Influence of Educational Media on the Artistic Development of Young Learners: A Case Study	Yumi Tanaka, Chen Liwei
		9	Enhancing Writing Competence Through Precision Teaching: Insights from a Southeast Asian Educational Initiative	H. Tanaka, Y. Lee, N. Mwangi, J. Kim

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HALL / SALON 5	**Prof. Daniel Richards**	1	ENHANCING LANGUAGE PROFICIENCY AND INTERCULTURAL COMPETENCE: A PILOT PROGRAM FOR HIGHER EDUCATION STUDENTS FROM A NORMAL SCHOOL İN ATEQUIZA, MEXICO	Dr. Ana M. Rodríguez, Dr. Luis A. Morales, Dr. Elena T. Hernández
		2	ANALYZING THE INCIDENCE OF ACADEMIC ANXIETY AMONG DYSLEXIC UNIVERSITY STUDENTS	Dr. Emily Carter,
		3	ADVANCING MEDICAL EDUCATION İN BRAZIL THROUGH REALISTIC SIMULATION: INSIGHTS AND IMPLICATIONS	Dr. Maria J. Silva,
		4	TRANSFORMATIONS İN UNIVERSITY CURRICULUM POLICIES İN CHILE: AN IN-DEPTH ANALYSIS	Dr. Emilia R. Vargas,
		5	UTILIZING CHILDREN'S DRAWINGS TO UNDERSTAND THEIR EXPERIENCES İN EQUINE-ASSISTED THERAPY	Dr. Emily Johnson
		6	THE IMPACT OF COMMERCIALIZATION ON HIGHER EDUCATION: ANALYZING SHIFTS İN PEDAGOGICAL PRIORITIES	Dr. Emily Thompson, Prof. Daniel Richards
		7	EXPLORING THE IMPACT OF COLLABORATIVE CULTURES ON MIDDLE SCHOOL EDUCATORS	Emily Carter
		8	ASSESSING CHATBOT INTEGRATION İN UNIVERSITY LEARNING ENVIRONMENTS: INSIGHTS FROM A PRELIMINARY PILOT	Smith Johnson, M. Williams
		9	EXPLORING HYPERLEDGER IROHA FOR ENHANCING THE ISSUANCE AND VERIFICATION OF ACADEMIC CREDENTIALS	Eleni Papadopoulos, Dimitrios Georgiou, Maria Chatzidaki, Nikos Zervas, Sofia Karagiannis, Alexander Sotiropoulos

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HALL / SALON 1	**Prof. Dr. MURAT AKTAŞ**	1	A META-ANALYSIS: REVIEW OF ARTIFICIAL INTELLINGENCE RELATED POSTGRADUATE THESES WITH A FOCUS ON EDUCATION AND TRAINING	Dr. İlknur KAZAZ
		2	YAPAY ZEKÂ VE TOPLUMSAL CİNSİYET	Prof. Dr. MURAT AKTAŞ
		3	YAPAY ZEKÂ VE MESLEKLERİN DÖNÜŞÜMÜ	Prof. Dr. MURAT AKTAŞ
		4	ARTIFICIAL INTELLIGENCE AND THE SUSTAINABILITY OF URBAN MEMORY	Architect, TUĞBA ÖZDEN Assistant Professor, HANDE AKARCA
		5	ARTIFICIAL INTELLIGENCE SUPPORTED PARTICIPATORY DESIGN AND PLANNING STUDIES IN URBAN PUBLIC SPACES	Architect ELİF KÜBRA ÖZTÜRK Asst. Prof., PhD HANDE AKARCA
		6	ENVIRONMENTAL APPROACHES AND ARTIFICIAL INTELLIGENCE IN THE FASHION INDUSTRY	Betül Yazıcı Dr.Ögr.Üyesi Meral İSLER
		7	YAPAY ZEKA EKOSİSTEMİNİ HAREKETE GEÇİRMEK İÇİN KAMU KAYNAKLARININ ÖRNEK KULLANIMI	Dr, AHMET TÜMAY
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HALL / SALON 2	**Doç. Dr. , Funda KUTLU ONAY**	1	DETERMINATION OF MATURITY CLASSIFICATION FOR KHOLT DATE FRUIT USING VGGNET	Doktor Öğretim Üyesi, EBRU ERGÜN
		2	ANALYSIS OF THE NONLINEAR THERMAL RADIATION EFFECT OF A HYBRID NANOFLUID WITH CURVILINEAR FLOW ON A CURVED, OSCILLATING, AND STRETCHED SURFACE WITH A MACHINE LEARNING APPROACH	Assoc. Prof. Dr. Andaç Batur Çolak
		3	KAFES SİSTEMLERİNİN OPTİMİZASYONU İÇİN GÜNCEL METASEZGİSEL ALGORİTMALARIN KARŞILAŞTIRILMASI	Dr. Öğr. Üyesi Salih Berkan AYDEMİR
		4	ENTROPİ TABANLI HİBRİT AMAÇ FONKSİYONU TEMELLİ KAN EMİCİ SÜLÜK OPTİMİZE EDİCİ İLE ÇOK SEVİYELİ EŞİK SEÇİMİ	Doç. Dr. , Funda KUTLU ONAY
		5	USE OF GENERATIVE ARTIFICIAL INTELLIGENCE TOOLS IN DESIGN PROCESSES: AN EXPERIENCE ON ITALO CALVINO'S INVISIBLE CITIES	Mimar İREM YAŞAR Prof. Dr. SEMRA ARSLAN SELÇUK Doç. Dr. Sema ALAÇAM
		6	DİYABET TEŞHİSİ İÇİN YAPAY ZEKA VE AÇIKLANABİLİR YAPAY ZEKA UYGULAMALARI	Öğr.Gör. Emine Betül SÜRÜCÜ Dr.Öğr.Üyesi Kıyas KAYAALP
		7	REDESIGNING ROMAN FOOTWEAR USING ARTIFICIAL INTELLIGENCE	Dr. Öğr. Üyesi, ZEYNEP MEHLİKA ULUÇAM KIRBAĞ

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HALL / SALON 3	Doç. Dr. Mustafa TUNÇER	1	SCIENTIFIC OBJECTIVITY OR COLONIAL PERSPECTIVE? ORIENTALISTS' APPROACH TO THE ARABIC LANGUAGE AND ISLAMIC SCIENCES	Dr. Öğr. Üyesi Naci ÖZSOY
HALL / SALON 3	Doç. Dr. Mustafa TUNÇER	2	GENERAL CHARACTERISTICS OF THE OTTOMAN TAFSİR HERITAGE	Doç. Dr. Mustafa TUNÇER
HALL / SALON 3	Doç. Dr. Mustafa TUNÇER	3	KUR'ÂN'IN İKİ AŞAMADA İNDİRİLİŞİ ÜZERİNE BİR DEĞERLENDİRME	Doç. Dr. Mustafa TUNÇER
HALL / SALON 3	Doç. Dr. Mustafa TUNÇER	4	ORGANIZATIONAL CULTURE IN THE CONTEXT OF ORGANIZATIONAL COMMUNICATION: DIYANETSEN	Prof. Dr. Yusuf YURDİGÜL Nurullah ARDAHANLI
HALL / SALON 3	Doç. Dr. Mustafa TUNÇER	5	Hz. Peygamber Özelinde Pozitif Enerji	Doç. Dr. Fatih DEĞİRMENCİ Mustafa KARTAL
HALL / SALON 3	Doç. Dr. Mustafa TUNÇER	6	THE DEFEAT OF THE ARAB INTELLECTUAL THROUGH THE EYES OF A LITERARY CRITIC	Arş. Gör. YUNUS EMRE ÖZTÜRK Prof. Dr. MEHMET ŞİRİN ÇINAR

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HALL / SALON 4	**Dr. Öğr. Üyesi MAHİRE ASLAN**	1	A REVIEW ON THESES FOCUSED ON SCIENCE SCIENCES (2014–2024) IN THE FIELD OF CURRICULUM DEVELOPMENT AND INSTRUCTION	Yüksek Lisans Öğrencisi Ömer OKAY Prof. Dr. Melek ÇAKMAK
		2	INVESTIGATION OF TEACHERS' PERCEPTIONS OF ORGANIZATIONAL PEACE IN TERMS OF VARIOUS VARIABLES	Dr. Öğr. Üyesi MAHİRE ASLAN
		3	ORGANIZATIONAL IMAGE OF THE SCHOOL	Dr. Öğr. Üyesi MAHİRE ASLAN
		4	INVESTIGATING DIGITAL COMPETENCIES and 21 ST CENTURY SKILLS of SECONDARY SCHOOL STUDENTS	Dilek BOZDOĞAN Doç. Dr. Cenk AKAY
		5	DIGITAL TRANSFORMATION AWARENESS OF TEACHER CANDIDATES: A MIXED METHOD RESEARCH	Dilek BOZDOĞAN Doç. Dr. Cenk AKAY
		6	SCHOOL ADMINISTRATORS' AND TEACHERS' VIEWS ON THE OBSTACLES OF INSTRUCTIONAL LEADERSHIP BEHAVIOURS: EVIDENCE FROM PUBLIC HIGH SCHOOLS IN İZMİR	Prof. Dr. Osman Ferda BEYTEKİN Bedia KIRKULAK
		7	PECULIARITIES OF TEAM BUILDING IN VOLUNTEERING	Prof. Dr. Torybaeva Zhamilya Zakhanovna Zharkinbayeva Dinara Sabitovna
		8	PEDAGOG-PSİKOLOGLARIN MESLEKİ YETERLİLİKLERİNİ GELİŞTİRME YOLLARI	Prof. Dr. Torybaeva Zhamilya Zakhanovna Kuanyshbayeva Araylım Nurjanovna
		9	OKUL YÖNETİCİLERİNİN GERÇEK DOĞUM SIRALARININ YÖNETİCİLİĞE ETKİSİ	Dilan MENGÜÇ ÇAKICI Doç. Dr. Ramazan ATASOY Dr. Öğr. Üyesi Ramazan ÖZKUL

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HALL / SALON 1	**Dr. Kenji Nakamura**	1	Fostering Islamic Educational Values in Early Childhood through Narrative Techniques	Samuel Kofi Appiah, Amara Zahra Al-Hassan
		2	Gender Dynamics and Islamic Education in Contemporary Georgia: Insights from Kvemo Kartli	A. Omotoso, M. Zhang, K. Amari
		3	Exploring the Significance of Names Among Thai Muslim Students: An Examination of Values and Identity	Iman Al-Farouq, Mônica da Silva, Dr. Kenji Nakamura
		4	INTERACTİONS BETWEEN MALAY AND CHİNESE COMMUNİTİES: A CİVİLİZATİONAL ANALYSİS	Aisha Alimova, Liu Yanjun
		5	THE EMERGENCE OF ISLAMİC TOURİSM İN KAZAKHSTAN: A NEW TREND OR A RELİGİOUS REVİVAL?	M. T. Sharma, R. A. Patel, S. K. Zhou, L. M. K. Niyazov
		6	REVİSİTİNG APOSTASY LAWS: A CONTEMPORARY PERSPECTİVE	Sara Kofi, Ibrahim Ahmed
		7	Zamzam Water as Corrosion Inhibitor for Steel Rebar in Rainwater and Simulated Acid Rain	Ahmed A. Elshami, Stéphanie Bonnet, Abdelhafid Khelidj
		8	Islam, Gender and Education in Contemporary Georgia: The Example of Kvemo Kartli	N. Gelovani, D. Ismailov, S. Bochorishvili

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HALL / SALON 2	**Dr. Ana Rosa Costa**	1	COMPARATİVE ANALYSİS OF STATE AND RELİGİON RELATİONS İN CONSTİTUTİONS: A CROSS-NATİONAL STUDY	Akira Nakamura, Laila Juma, Roberto Souza
		2	CULTURAL AND RELİGİOUS IDENTİTY DYNAMİCS: COMPARATİVE INSİGHTS FROM CHRİSTİANİTY AND ISLAM İN AFRİCA AND ASİA	Dr. Ana Rosa Costa Dr. Hiroshi Tanaka,
		3	EXAMİNİNG THE INFLUENCE OF ISLAM ON DEVELOPMENT DYNAMİCS: INSİGHTS AND IMPLİCATİONS	Lian Zhang, University of Malaya, Malaysia and Ahmed Kone, University of Ouagadougou, Burkina Faso
		4	SİLENT BOUNDARİES: RELİGİON AND THE JUDİCİARY İN INDİA'S LOWER COURTS	Ananya Patel, Hiroshi Nakamura
		5	ISLAMİC PERSPECTİVES ON FERTİLİTY MANAGEMENT AND HEALTH CONSİDERATİONS	Dr. Amina Suleiman, Dr. Jun Park,
		6	Women with Disabilities: A Study of Contributions of Sexual and Reproductive Rights for Theology	Luciana Steffen
		7	The Suffering God and Its Relevance to the Understanding of Human Suffering in Jürgen Moltmann's Theology	Aldrin R. Logdat
		8	THE ROLE OF ISLAM İN THE POLİTİCAL LANDSCAPE OF CONTEMPORARY KAZAKHSTAN	Sofia Kim, Ahmed Youssef, Laila Hassan

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HALL / SALON 3	**Hiroshi Kinoshita**	1	The Role of Philosophical Hermeneutics in Enhancing Judicial Objectivity in Brazil	Lucas M. Silva, Ana P. Rodrigues
		2	Comparative Analysis of Spiritual Influences on Architecture: Islamic and Gothic Styles	J. Ribeiro, A. Liu
		3	Analyzing the Interplay between Religion and Development with a Focus on Islam	Lina Marais Hiroshi Kinoshita
		4	Exploring Ancient Wisdom for Contemporary Social Harmony: Insights from Sufi and Islamic Thought Maria S. Oliveira, Hiroshi Takeda	Maria S. Oliveira, Hiroshi Takeda
		5	Balancing Efficiency and Empathy in the Context of Open Knowledge: A Pedagogical Perspective	Amina Bakare, Minh-Hoang Nguyen, Siti Zaharah, Kwame Kofi Asante, Jianyu
		6	The Evolution of Democratic Ideals in Pakistan: An Examination Through the Perspectives of Islamic Thought and Comparative Political Theory	Dr. Farhan Malik
		7	The Role of Religious and Ethical Values in National Security: A Kazakh Perspective	A. N. Zhanibekov, B. K. Serikbayev, C. M. Tulegenova, D. A. Askarov, E. K. Kenzhebekova
		8	Analyzing Servant Leadership: A Critical Literature Review	Amina Diallo, Momo Kone, Kenji Tanaka
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HALL / SALON 4	**Li Wei Chen**	1	Unveiling Symbolism in Hindu Temple Architecture: A Philosophical Perspective	M. Hwang, E. J. Mbeke
		2	Integrating Ethical Frameworks: A Comparative Study of Asian and African Perspectives on Business Ethics	Amina Idris, Yohan Park
		3	Innovations in Open Science: Transforming Research Paradigms	A. Liu, M. Okafor
		4	Reevaluating Constructivist Paradigms: An Existential and Phenomenological Perspective	Amara N'Diaye, Li Wei
		5	Reconstructing Self Through Temporal Dynamics: Analyzing Zhao Tao's Role in Jia Zhangke's Cinematic Universe	Liang Wei
		6	Enhancing Construction Efficiency: A Study on the Adoption of Lean Practices	S. K. Ngugi, A. W. Chien
		7	Analyzing Aesthetic Dimensions in Museum Architecture	R. K. Dlamini, S. Zhou
		8	Integrating Philosophical Perspectives into Interdisciplinary Physical Education Programs	Amina Khamis, Jibril Adamu
		9	Integrating Buddhist Principles in Addressing Mental Health Challenges	Li Wei Chen

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HALL / SALON 5	**Carlos Silva**	1	The Influence of Cultural Philosophy on Individual Identity in Turkic Traditions	Dr. K. Shunji, Prof. M. Adebayo, Dr. L. Fong
		2	Traditional Eastern Practices in Contemporary Sustainable Architecture	L. Tanaka, N. Ndungu
		3	Exploring Postmodern Tragi-Comedy: An Analysis of Tom Stoppard's 'Rosencrantz and Guildenstern Are Dead'"	Mei-Ling Chen, Carlos Silva
		4	The Role of Islam in Shaping Cultural Values in Kazakhstan	Li Xue, Kofi Agyeman, Amina El-Omari, Hiroshi Takeda, Fatoumata Diallo
		5	Artistic Responses to Climate Crisis: Exploring Innovative Approaches to Sustainable Futures through Interdisciplinary Art Practice	Amina Bello, Jianyu Zhang, Mikhail Ndumba
		6	Unveiling Darkness: Exploring Existential Themes and Musical Narratives in "True Detective	Aiko Tanaka, Liu Wei
		7	Rethinking Absence: The Role of Silence and Pause in Samuel Beckett's Waiting for Godot	Jun-Ho Kim, Meilin Xu
		8	Advancements in Constraint Management Theory: A Comprehensive Review	Mei-Ling Chen, Hiroshi Takahashi, Samuel Nkrumah
		9	The Influence of Work Ethic on Economic Growth: Comparative Analysis of Emerging Asian and African Economies	Hanako Takeda, Aisha Kone, Jiro Nakamura, Farah Njeri, Mei Lin Zhang

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HALL / SALON 6	**Dr. Ana Oliveira,**	1	Robust Variogram Fitting Using the Modified Huber Norm	Mariana Costa, Zhen Li, Amina Njeri
		2	Evaluating the Effectiveness of Stratified Double Median Ranked Set Sampling for Population Mean Estimation	Laura N. Delgado Aiko Tanaka
		3	Advancing the Extended Trapezoidal Technique for Numerical Resolution of Volterra Integro-Differential Equations	Amina Zuberi, Rajesh Kumar Patel
		4	A Novel Computational Approach for Hyper-Elastic Structural Analysis Using Lagrangian-Hamiltonian Framework	Dr. Ana Oliveira, Prof. Yassir Malek, Dr. Lin Zhang
		5	INNOVATIVE DESIGN OF FRACTIONAL ORDER CONTROLLERS FOR VIBRATION REDUCTION IN AIRCRAFT WING STRUCTURES	Leila Martins, Yassir Bouaziz, Elena Kovač, Nikoleta Petrovic
		6	Performance Analysis and Modeling of Loading Factors in Centrifugal Compressor Impellers	Dr. Maria de Souza, Prof. Wei Liu
		7	Advanced Discrete Evolutionary Splines for Modeling Occlusion in Temporomandibular Disorders	Sofia Mendes Hiroshi Tanaka

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HALL / SALON 7	**Dr. Alejandro Silva ,**	1	ENHANCING FORECAST ACCURACY THROUGH NORMALIZATION OF REALIZED VOLATILITY IN LONG-MEMORY MODELS	M. A. Silva, L. Zhang, and T. O. Nkosi
		2	ENHANCING LOUDSPEAKER DESIGN PARAMETERS THROUGH AIR VISCOSITY DAMPING OPTIMIZATION	Julia Martin, Chen Wei, Ahmed El-Sayed, Sofia Ivankova, Paulo Silva
		3	PROPERTIES OF QUASI-CONFORMALLY FLAT LP-SASAKIAN MANIFOLDS WITH CONSTANT COEFFICIENT	Elena Rodrigues Aiden Schmidt
		4	A NOVEL APPROACH TO NUMERICAL SOLUTIONS FOR REACTION-DIFFUSION SYSTEMS ON CLOSED SURFACES	Dr. Niazi Barakat, Dr. Anna Zhen, Dr. Hassan Ghanem
		5	AN ADVANCED NUMERICAL TECHNIQUE FOR DIFFUSION AND CAHN-HILLIARD EQUATIONS ON DYNAMIC SPHERICAL GEOMETRIES	Li Xianjun, Maria Orellana
		6	ANALYSIS OF DYNAMIC STABILITY IN AN EXTENDED MODEL OF THE ENDOCRINE FEEDBACK SYSTEM	Dr. Alejandro Silva , Dr. Mei Ling Tan
		7	CHARACTERIZATION OF (Λ, M)-FUZZY SUBGROUP STRUCTURES IN OPERATOR-GROUPS	Mei Lin, Akira Tanaka, and João Oliveira
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HALL / SALON 8	**Lina Oliveira**	1	Stability and Dynamics of a Human-Mosquito Malaria Model with Infected Immigrants	Tariq Patel, Mei Ling Zhang
		2	Reliability Assessment of Data Centers at Kigali Institute of Science and Technology Using LRU Algorithm	A. M. Hassan, Nadia Faye, Kofi Mensah
		3	Integrating Python Programming with Analytic Geometry Concepts	Maya L. Tanaka Emmanuel Akinola,
		4	Advanced Implicit Eulerian Approach for Modeling Highly Deformable Elastic Membranes in Newtonian Fluids	Lina Oliveira, Haruto Tanaka, Anwar Ahsan
		5	Advanced Analytical Techniques for Corotational Maxwell Fluids in Wire Coating Processes	Li Wei Zhang, Olufemi Adewale, Elsa Müller
		6	An Analysis of Stochastic Integrals in Catastrophic Event Models	Dr. Leila Martins, Dr. Huan Zhao, Dr. Amina Osei
		7	Optimal Block Design Strategies for Main Effects in Experimental Studies	Lian Chen, Fatima Ahmed

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HALL / SALON 1	**Dr. Öğretim Üyesi ÖZGÜR SOYSAL**	1	ALBERT CAMUS ve JEAN-PAUL SARTRE'ın VAROLUŞ FELSEFESİ HAKKINDA	Azra GİRAY
		2	NİETZSCHE VE TRANSHÜMANİZM: TRANSHUMAN ASLINDA BİR ÜSTİNSAN MI?	FATMA BUSE CİVAN
		3	THE POSITION AND IMPORTANCE OF HERMES IN PHILOSOPHY OF ISHRAQ	Doç. Dr. Zeynep KANTARCI BİNGÖL Doktora Öğrencisi Rıdvan YILDIZ
		4	THE EFFECTS OF PLATO'S PHILOSOPHY ON SEHÂBEDDİN SUHRAWARDI	Doç. Dr. Zeynep KANTARCI BİNGÖL Doktora Öğrencisi Rıdvan YILDIZ
		5	SOME OBJECTIONS TO VIRTUE ETHICS	Res. Asst. Dr. Enes DAĞ
		6	RETHINKING PHILOSOPHICAL ANTHROPOLOGY	Dr. Öğretim Üyesi ÖZGÜR SOYSAL
		7	ÇOCUKLAR İÇİN FELSEFENİN ÇOCUKLARDA ELEŞTİREL DÜŞÜNME BECERİSİNE ETKİSİNİ İNCELEYEN ÇALIŞMALARIN SİSTEMATİK GÖZDEN GEÇİRİLMESİ	Bilim Uzmanı, ZEYNEP KORKMAZ
		8	BEDENLENMİŞ ZİHİN DÜŞÜNCESİ BAĞLAMINDA ENAKTİVİZM DÜŞÜNCESİNE GENEL BİR BAKIŞ	Dr. Zafer AKDAĞ
		9	FENOMENOLOJİ GELENEĞİNİN ZİHİN FELSEFESİNE ETKİSİ	Dr. Zafer AKDAĞ
		10	KRİZLER ÇAĞINDA "CÜRETKAR" BİR TAVIR ALIŞ OLARAK FELSEFE	Dr. ZEYNEP BERKE

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HALL / SALON 2	**Prof. Dr., Lale KABADAYI**	1	A DISCUSSION ON THE CONCEPT OF PAIN AND ITS EXISTENTIAL IMPORTANCE	Doç. Dr., Ferdi SELİM
		2	BİLGİ, İLGİLİ ALTERNATİFLER VE GERÇEKLİK VARSAYIMI	Arş. Gör. Dr. Nusret Erdi ELMACI
		3	AD HOC HYPOTHESES IN KARL POPPER’S PHILOSOPHY OF SCIENCE	Assoc. Prof. Dr. Alper Bilgehan YARDIMCI Selin GÜLEROĞLU
		4	THE CHANGE OF SCIENTIFIC KNOWLEDGE IN THOMAS KUHN AND KARL POPPER: SIMILARITIES-DIFFERENCES	Assoc. Prof. Dr. Alper Bilgehan YARDIMCI Anıl ÇELİK
		5	İKLİM KURGUSU TÜRÜ AÇISINDAN ÇEVRESEL HETEROTOPYALARI OKUMAK: KÖRFEZ (2017) VE SELYATAĞI (2018) FİLMLERİ	Öğretim Görevlisi, Damla PİRLİ Doktora Öğrencisi, Atakan PİRLİ
		6	SCHİLLER’İN SANAT FELSEFESİNDE ETİK PROBLEMİ	Yüksek Lisans Öğrencisi, BETÜL CANSU DİŞÇİOĞLU
		7	METODOLOJİK YANLIŞLAMACILIĞA KARŞI İNCELTİLMİŞ YANLIŞLAMACILIK: KARL POPPER VE IMRE LAKATOS	HAZARCAN İDİL TUFANTOZ
		8	MÜKEMMEL GÜNLER FİLMİNDE “PARANTEZE ALMA”	Prof. Dr., Lale KABADAYI
		9	YAPAY SÜPER ZEKÂ PROBLEMATİĞİ: HOMO SAPİENS’İN ÜSTEL TEKNOLOJİK İLERLEME KARŞISINDAKİ DRAMI ÜZERİNE FELSEFİ BİR İNCELEME	Dr. Öğr. Üyesi, MUAMMER AKTAY

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HALL / SALON 3	**Prof. Dr Şebnem YÜCEL**	1	TAŞIYICI BİTKİLERİN TMS 16: MADDİ DURAN VARLIKLAR STANDARDI KAPSAMINDA MUHASEBELEŞTİRİLMESİ: ÜZÜM ÜRETİM İŞLETMESİNDE BİR UYGULAMA	Dr. Öğr. Üyesi MEHMET MURAT GUTNU
		2	DEMATEL METHOD ONE OF MULTI-CRITERIA DECISION MAKING TECHNIQUES: BIBLIOMETRIC ANALYSIS	Yüksek Lisans Öğrencisi Havana ÇETİNKAYA Prof. Dr Şebnem YÜCEL
		3	VIKOR METHOD ONE OF MULTI-CRITERIA DECISION MAKING TECHNIQUES: BIBLIOMETRIC ANALYSIS	Yüksek Lisans Öğrencisi Gamze KESİCİ Prof. Dr Şebnem YÜCEL
		4	GLASS CEILING SYNDROME: AN ASSESSMENT IN THE CONTEXT OF THE WORLD, AMERICA, EUROPEAN UNION AND JAPAN	Yüksek Lisans Öğrencisi, YUSUF OKTAY ÜNLÜER Doç. Dr., BURCU DOĞANALP
		5	EVALUATION OF GREEN BOND ISSUANCES IN TURKEY: 2016-2024	Dr. Öğr. Üyesi Çağatay MİRGEN Doç. Dr. Yusuf TEPELİ
		6	EXAMINING THE EFFECTS OF THE USE OF ARTIFICIAL INTELLIGENCE IN THE FIELD OF ORGANIZATIONAL BEHAVIOR	Öğr. Gör. Dr., MUKADDES GÜLER
		7	CORPORATE COMMUNICATION IN HEALTHCARE INSTITUTIONS: AN ANALYSIS ON STATE HOSPITAL EMPLOYEES	Prof.Dr. MİKAİL BATU

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HALL / SALON 4	**Dr. Gülşen KIRPIK**	1	TÜRKİYE'DE FAALİYET GÖSTEREN BANKALARIN DEPREM BÖLGESİ İÇİN SAĞLADIĞI KREDİ PAKETLERİ VE FİNANSMAN OLANAKLARI	Arş. Tubay YİNAÇ Dr.Öğr.Üyesi İsmet BOLAT
		2	MÜLTECİ VE GÖÇMENLERİN TÜRKİYE EKONOMİSİNE ETKİLERİ ÜZERİNE BİR ARAŞTIRMA	Dr.Öğr.Üyesi İsmet BOLAT Arş. Tubay YİNAÇ
		3	EVALUATION OF THE GLASS CEILING SYNDROME FROM THE PERSPECTIVE OF FEMALE EMPLOYEES: THE CASE OF SAFRANBOLU	Assoc. Prof. HALİME GÖKTAŞ KULUALP ERAY EKİNCİ
		4	THE RELATIONSHIP OF BEHAVIOR MODELS WITH GENERAL PERFORMANCE: A CASE STUDY ON FREE PHARMACY PERSONNEL	Pharm. Ebru UZUN Dr. Gülşen KIRPIK
		5	INVESTIGATION OF THE RELATIONSHIP BETWEEN TRUST IN THE MANAGER AND GENERAL PERFORMANCE: THE CASE OF ŞANLIURFA COMMUNITY PHARMACIES	Pharm. Muhammed Ali ŞEKER Dr. Gülşen KIRPIK
		6	WAYS TO EVALUATE AND DEVELOP THE EXPORT POTENTIAL OF AZERBAIJAN'S KARABAKH AND EASTERN ZENGEZUR REGIONS	Assoc. Prof. Amiraslanova Dilara Amiraslan
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HALL / SALON 1	**Ana Carvalho,**	1	Enhancing Public Relations Strategies in Nonprofit Organizations: An Albanian Case Study	Leila Asante, Tariq Nabil, Youssef El-Masri
		2	Comparative Analysis of Financial Market Integration: Insights from European and Global Bond Markets	Dr. Amara Kone, Dr. Liang Wei
		3	The Dynamics of Financial Integration in Emerging Bond Markets	Ana Carvalho, Kofi Mensah, Li Wei
		4	Evolution and Efficacy of International Criminal Jurisprudence in Addressing Major Atrocities	Amara Bello, Hiroshi Tanaka
		5	Navigating Challenges in SME Relationships: Evidence from Southeast Asia	Aisha Kone, Abdul Rahman Abebe, and Hiroshi Tanaka
		6	Strategic Approaches and Emerging Trends in Public Relations for Media Outlets in Thailand	Mei-Ling Chen John Doe
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HALL / SALON 2	**Ana Clara Silva**	1	Regional Diplomacy and Global Engagement: An Examination of Sub-National Governments' Foreign Relations in Mexic	Ana Maria Ribeiro Liu Wei Carlos Hernández,
		2	Examining the Impact of Air Passenger Transport on Economic Growth in Kenya	Samuel Njeri, Grace Adesina, Daniel Mwangi, Aisha Khamis
		3	Economic Diplomacy Between South Korea and Japan: A WTO Dispute Analysis	Lia M. Torres, Yuki K. Hwang
		4	Ethical and Legal Implications of Artificial Intelligence in Military Applications: A Comparative Analysis	Ana Clara Silva Yu Wei
		5	Assessing Self-Perceived Employability of International Relations Students at the University of Warmia and Mazury in Poland	Adama Sow, Felicia Kwabena, and Ahmad Chukwu
		6	The Influence of Eastern Powers and European Nations on Religious Conflicts in Central Asia	Haruki Nishida, Jovana Marković, Adama N'Diaye
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HALL / SALON 3	**Prof. James Anderson,**	1	THE ROLE OF SOCIAL WORK AGENCIES IN PROMOTING REFLECTIVE PRACTICES IN HONG KONG'S WELFARE SYSTEM	Alex Thompson, University of Hong Kong
		2	ASSESSING THE IMPACT OF SOCIAL ENTERPRISES ON COMMUNITY DEVELOPMENT	Dr. Emily J. Harrison, Dr. Robert K. Stevens, Dr. Priya S. Kumar, Dr. Maya L. Patel
		3	ENHANCING WORKER WELL-BEING THROUGH SUSTAINABLE PRACTICES: THE ROLE OF ERGONOMICS AND CORPORATE SOCIAL RESPONSIBILITY	Dr. Emily Thompson, Prof. James Anderson, Dr. Sarah Lopez
		4	INFLUENCE OF GENDER, WORK EXPERIENCE, AND SOCIAL ENGAGEMENT ON ENTREPRENEURIAL SKILLS DEVELOPMENT IN ENTREPRENEURSHIP EDUCATION: A STUDY AT UNIVERSITY OF PARIS-SACLAY	J. Dubois M. Martin L. Lefevre
		5	ENHANCING PROFESSIONAL COMPETENCY AND ADDRESSING CHALLENGES IN CHILD CASE MANAGEMENT: INSIGHTS FROM MALAYSIA'S DISTRICT SOCIAL WELFARE SERVICES	Dr. Emily Carter, Prof. James Lee, Dr. Hana Patel, Dr. Michael Roberts
		6	A COMPREHENSIVE MODEL FOR FIELD WORK PRACTICUM IN LABOUR WELFARE: INSIGHTS AND INNOVATIONS	Aisha Idris Kimiko Sato
		7	The Impact of Work Values, Work-Value Alignment, and Work Centrality on Organizational Citizenship Behavior	Dr. Jane Smith, Dr. Mark Johnson
		8	GENDER DISPARITIES AND EMPOWERMENT INITIATIVES IN HARYANA: AN ANALYTICAL STUDY	Dr. Priya Sharma, Dr. Rajeev Mehta, Dr. Ananya Sinha

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HALL / SALON 4	**Dr. Ayesha Rahman**	1	EVALUATING THE EFFECTIVENESS OF BRIEF THERAPEUTIC INTERVENTIONS ON THE MENTAL HEALTH OF WOMEN FAMILY SUPERVISORS	Jane Doe, John Smith, Alice Johnson, Robert Brown
		2	A COMPARATIVE STUDY OF FAMILY DYNAMICS IN RURAL BANGLADESHI COMMUNITIES	Dr. Ayesha Rahman
		3	TRANSGENERATIONAL SYNERGY IN CHINESE FAMILY ENTERPRISES: A MODEL PROPOSAL	Emily Zhang, Michael Chen, David Wong
		4	THE POLITICAL JOURNEYS OF SOCIAL WORKERS: A QUALITATIVE EXAMINATION OF THEIR ENGAGEMENTS	Dr. Alex Johnson,
		5	ENHANCING SOCIAL WORK EDUCATION FOR CHILDREN AND YOUTH: ADDRESSING THE GAPS IN THE POLISH CURRICULUM	Marta Czechowska-Bieluga,
		6	RETHINKING DEVELOPMENTAL SOCIAL WORK: POST-APARTHEID CHALLENGES AND PROSPECTS IN SOUTH AFRICA	Dr. John Smith,
			ADVANCED MODULAR SYSTEM FOR ASSESSING AND MONITORING FACTORS CONTRIBUTING TO WORK-RELATED HEALTH ISSUES	Dr. Emily S. Carter, Dr. James T. Harrison, Dr. Olivia M. Brooks, Dr. Alexander L. Thompson
		7	EXPLORING SOCIAL SUSTAINABILITY PRACTICES: INSIGHTS FROM THE RETAIL SECTOR IN EGYPT	Sarah Ahmed, David Smith

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HALL / SALON 5	**Dr. Amina Al-Mohamed**	1	Reimagining Intelligence: Insights from Information Theory	Eduardo Silva, Amina Al-Sayed, Akira Nakano
		2	Leveraging Artificial Intelligence in Systems Engineering: Insights from a Remote Sensing Application	Amina Z. N'Guessan, Hiroshi T. Nakamura
		3	Enhancing Speech Recognition Through Advanced Statistical Models	Dr. Amina Al-Mohamed, Dr. Li Wei,
		4	Strategic Decision-Making Through Advanced Data Analytics	Amina Nkosi, Ryo Tanaka, and Kofi Asante
		5	Integrative Framework for Intelligent Enterprise Systems	Maria Silva, Jun-Ho Lee
		6	Forecasting Telemarketing Success in Banking Using Deep Learning Techniques	Javier Morales, Liu Wei, and Amara Ndiaye
		7	Enhancing Software Reliability through Advanced Computational Techniques	Aisha Nkosi, Hiroshi Tanaka, Pedro Lima, Eleni Papadopoulos
		8	Advanced Approaches for Precipitation Forecasting Using Machine Learning Techniques: A Comparative Analysis	Léa Roussillon, Mikhail Ivanov, Amina Jalloh, Hiroshi Nakamura, Sofia Silva
		9	Advancements in Artificial Intelligence Approaches for Dissolved	Gas Analysis in Transformers: A Comprehensive Review
		10	Exploring Proactive Strategies in Innovation Management	Dr. Liang Wei, Dr. Emil Kato

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HALL / SALON 6	**Dr. Ana Oliveira**	1	Exploring Proactive Strategies in Innovation Management	Dr. Liang Wei, Dr. Emil Kato
		2	Advanced Approaches for Precipitation Forecasting Using Machine Learning Techniques: A Comparative Analysis	Léa Roussillon, Mikhail Ivanov, Amina Jalloh, Hiroshi Nakamura, Sofia Silva
		3	Advancements in Artificial Intelligence Approaches for Dissolved Gas Analysis in Transformers: A Comprehensive Review	Carlos Silva, Mei-Ling Zhang, Amina Jallow
		4	Optimization of Cost in Parallel Job Shop Scheduling using Hybrid Swarm Intelligence	Li Mei Chen, Andre Oliveira
		5	Rethinking Higher Education in the Age of Emerging Technologies: AI's Transformative Influence	Dr. Elena Souza, Prof. Wei Zhang
		6	Advanced Computational Networks for Knowledge Representation in Educational Systems	Mariana Silva Hao Chen Oluwaseun Adeyemi
		7	Enhancing Strategic Insights with Geo-Intelligence: A Comprehensive Overview	Dr. Ana Oliveira Dr. Wei Chen,
		8	Innovative Architectures for Enhanced Stability in Artificial Neural Networks	Dr. Mei Ling Zhao João Pereira,
		9	Enhancing Diagnostic Accuracy in Diabetes Management Using Machine Learning Techniques	João Silva, Mei Ling Chen
		10	Exploring the Evolution and Impact of Artificial Intelligence: A Systematic Review of Emerging Applications and Technologies	Dr. Amina Bakri, Dr. Hiroshi Tanaka,
		11	Evaluating Bias and Transparency in AI Systems Using Statistical Methods from Metrology	Dr. Ananya Gupta, Prof. Omar Diallo

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HALL / SALON 1	**Asst. Prof. Damla BARLAK**	1	-RİCCİ BOURGUİGNON SOLİTONLARI KABUL EDEN PARA-SASAKIAN MANİFOLDLAR	Prof. Dr. Mehmet ATÇEKEN Doç. Dr. Tuğba MERT
		2	APPROXIMATION OF CONE SECTIONS WITH A CLASS OF LINEAR POSITIVE OPERATORS	PHD Student EMİNE GÜVEN Assoc. Prof. Dr., Nazmiye GÖNÜL BİLGİN
		3	STATISTICAL CONVERGENCE OF ORDER FOR DOUBLE SEQUENCES OF FUZZY NUMBERS	Asst. Prof. Damla BARLAK
		4	IMAGES OF CYCLIC CODES OVER FINITE RING R	Dr. Öğr. Üyesi Tülay YILDIRIM TURAN
		5	SOME APPLICATIONS OF MULTIVARIATE PADÉ APPROXIMANT METHOD	Graduate Student, Abdulkadir CEYLAN Assis. Prof. Dr. Muammer AYATA
		6	PROPERTIES OF PERMUTING TRI DERIVATION ON LATTICES	Hilal ÜNAL Hasret YAZARLI
		7	YARI EKSENDE SÜREKSİZLİK KOŞULLARINA SAHİP STURM-LIOUVILLE DENKLEMİ ÜZERİNE	Öğrenci, Yusuf Ilgın EMİROĞLU Prof.Dr.Selma GÜLYAZ ÖZYURT
		8	CHATTERJEA TYPE CONTRACTION MAPPINGS FOR RECTANGULAR SOFT B-METRİC SPACES	Assoc. Prof. Dr., Simge ÖZTUNÇ Phd., Sedat ASLAN

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HALL / SALON 2	**Prof. Dr. Ali DEMIR**	1	Evistence Result for p- Laplacian Fractional Boundary Value Problems	GÖZDE ADALI PROF. DR., FATMA SERAP TOPAL
		2	AĞLARDA ETKİN UZAKLIK ÇEKİM MERKEZLİĞİ MODELİ İÇİN ALGORİTMA TASARIMI	Yüksek Lisans Öğrencisi, YAKUP DENİZ YAMAÇ Doç. Dr., ZEYNEP NİHAN BERBERLER
		3	EULER METHOD FOR INITIAL VALUE PROBLEMS OF LINEAR FRACTIONAL DIFFERENTIAL SYSTEM	Asst. Prof. Dr. Suleyman CETINKAYA Prof. Dr. Ali DEMIR
		4	ON THE SOLUTION OF TIME FRACTIONAL WAVE PROBLEM WITH NEUMANN BOUNDARY CONDITIONS	Asst. Prof. Dr. Suleyman CETINKAYA Prof. Dr. Ali DEMIR
		5	KESİRLİ TÜREV İÇEREN HADAMARD TİPİ ÇOK NOKTALI SINIR DEĞER PROBLEMİNİN ÇÖZÜMLERİNİN VARLIĞI	ESRA NUR SOYLU DOÇ. DR. ERBİL ÇETİN
		6	ÖKLİD UZAYINDA MANNHEİM EĞRİLERİ VE MANNHEİM EĞRİ ÇİFTLERİ	Eda YILDIRIM Nural YÜKSEL

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HALL / SALON 3	**Dr. Bedri Şahin**	1	GÜNEYU KORE'DE KOOKMİN'İN ÖNEMİ VE MİLLİ KİMLİK İNŞASI	Yüksek Lisans Öğrencisi, HATİCE ÇİFTÇİOĞLU
		2	DEVELOPMENT OF INTELLIGENCE UNITS IN TURKEY AND DEEP STATE ANALYSIS	Konsept Uzmanı Aysuhan Gürel Dr. Öğretim Üyesi Adem Ali İren
		3	EVALUATION OF THE RELATIONS BETWEEN THE RED-GREEN COALITION GOVERNMENT OF THE FEDERAL REPUBLIC OF GERMANY (GERMANY) (1998-2005) AND THE NORTH ATLANTIC TREATY ORGANIZATION (NATO)	Dr. Bedri Şahin
		4	GLOBAL CLİMATE CHANGE REFLECTİONS ON THE WORLD AND TURKEY THROUGH CAUSES AND CONSEQUENCESES	SELMA ŞAHİN
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HALL / SALON 4	**Prof. Dr. Ali Fuat ERSOY**	1	ADLİ DESTEK GÖREVLİLERİNİN ETİK İKİLEM VE ETİK KARAR VERME SÜREÇLERİ ÜZERİNE NİTEL BİR ARAŞTIRMA	Doktora öğrencisi Ganime YEĞİN Prof. Dr. Ali Fuat ERSOY
		2	VAROLUŞÇU YAKLAŞIM PERSFEKTİFİNDEN “GÜNÜMÜZ MODERN TOPLUMUNDA KADIN VEYA ERKEK OLMAK” ÜZERİNE NİTEL BİR ARAŞTIRMA	Doktora öğrencisi Ganime YEĞİN Prof. Dr. Ali Fuat ERSOY
		3	ÖTEKİ OĞUL: BİR FİLM ANALİZİ	SHU Dr. MERVE ERCAN
		4	SAĞLIK TEDBİRİ UYGULAMA SÜRECİNİN SOSYAL HİZMET BAKIŞ AÇISIYLA DEĞERLENDİRİLMESİ	Sosyal Çalışmacı, Bülent MACİT Prof. Dr. , Gülsüm ÇAMUR
		5	TÜRKİYE’DE BEKAR ANNE OLMAK: SOSYAL HİZMET PERSPEKTİFİNDEN BAKIŞ	Nazlı Deniz SARAL Prof. Dr. Gülsüm ÇAMUR
		6	SOSYAL HİZMETTE MÜDAHALE ARAŞTIRMALARININ YERİ VE ÖNEMİ	Prof. Dr. OĞUZHAN ZENGİN Arş. Gör. MUSTAFA EMİN AKÇİN
		7	SOSYAL HİZMET UZMANLARININ PSİKOTERAPİ ALANINDAKİ KATKILARI: AİLE TERAPİSİ, ÇÖZÜM ODAKLI KISA TERAPİ VE ANLATI TERAPİSİ ÜZERİNE BİR İNCELEME	Arş. Gör. MUSTAFA EMİN AKÇİN Prof. Dr. OĞUZHAN ZENGİN

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HALL / SALON 1	**Dr. Rina Nakashima,**	1	Enhancing Business Decision-Making through Geospatial Analytics: The Emergence of Location Intelligence	Sofia Ribeiro, Chen Wei, and Ahmed El-Sayed
		2	Strategic Frameworks in the Evolving Electric Mobility Market	Amina Fadli, Kenji Yamamoto
		3	Strategic Insights into CRM Systems in Modern Enterprises: Navigating Implementation Challenges and Emerging Opportunities	Dr. Rina Nakashima, Dr. Ahmed Bello
		4	Developing Interdisciplinary Skills for Business Interpreters: Addressing Market Needs	Akira Tanaka, Janelle Ndlovu
		5	The Critical Role of Optimal Business Models in the Success of High-Tech Firms: A Nanotechnology Perspective	Linh Nguyen Amara Diallo, Petra Novak,
		6	Advancing Context-Awareness in Business Process Management Systems	Tadeusz Kowalski, Maria Lungu
		7	Adaptive and Scalable Framework for Business Rules Management in Data Validation	Linh Nguyen Amina Dlamini
		8	Evaluation of Technological Proficiency in Enhancing Business Outcomes of Professional Service Firms	Dr. Adama Juma Dr. Mei Ling Dr. Miguel Rodriguez,

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HALL / SALON 2	**Aleksandar Chonevski**	1	HARMONIZING BUSINESS PROCESS MODELING STANDARDS: A COMPARATIVE ANALYSIS	Dr. Akinwale Adebowale Dr. Mei-Lin Wong,
		2	ENHANCING BUSINESS PROCESS MODELS THROUGH TARGETED REFACTORING	Nguyen Thi Lan Amina Diouf Lukas Müller Carlos Silva
		3	ENTREPRENEURSHIP AND THE DISCOVERY AND EXPLOITATION OF BUSINESS OPPORTUNITIES: EMPIRICAL EVIDENCE FROM THE MALAWIAN TOURISM SECTOR	Aravind Mohan Krishnan
		4	THE RELATIONSHIP BETWEEN BUSINESS-MODEL INNOVATION AND FIRM VALUE: A DYNAMIC PERSPECTIVE	Yung C. Ho, Hui C. Fang, Ming J. Hsieh
		5	IMPLICATIONS ABOUT THE IMPACT OF COVID-19 ON INTERNATIONAL TRADE IN SAUDI ARABIA	Anwar Kashgari
		6	HOW INTERNATIONAL COLLEGE STUDENTS UNDERSTAND ENTREPRENEURIAL READINESS AND BUSINESS-RELATED SKILLS: A QUALITATIVE STUDY	Aleksandar Chonevski
		7	ADOPTION OF E-BUSINESS BY THAI SMES	Pisit Chanvarasuth
		8	Optimizing Supply Chain Strategies for SMEs in Dynamic Markets	Ravi Chandran Lerato Mbeki, Katarina Kovács Aisha Yusuf, Yuki Takahashi

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HALL / SALON 3	**Viktor Radulescu,**	1	The Impact of Investment on Economic Development: A Comparative Study	Kwame Mensah Li Wei Maria Silva
		2	Optimizing Supply Chain Strategies for SMEs in Evolving Markets	Lu Wen, Tatenda Chikosi Ewa Nowak, Fatou Diarra Hyun-Jin Lee,
		3	Investigating Visitor Experiences in Serengeti Wildlife Park	Zhang Wei, Amina Nkosi
		4	Optimizing Supply Chain Strategies for SMEs in Dynamic Markets	Ravi Chandran, Lerato Mbeki, Katarina Kovács Aisha Yusuf, Yuki Takahashi,
		5	Social and Economic Impacts of Mining Sector Reforms: A Case Study from Romania	Viktor Radulescu, Ana Ionescu
		6	Evaluating the Influence of Government Expenditures on Private Consumption and Economic Growth: Evidence from Indonesia	Lila Handayani
		7	Sustainable Value Metrics: A Comparative Study of Uganda and East Africa	Maria A. Nkosi, David K. Amara
		8	Exploring the Influence of Uncertainty on Investment Decisions: A Monte Carlo Simulation Approach	Elena Silva, Kofi Osei,

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HALL / SALON 4	**Dr. Mei Ling Chen**	1	ANALYZİNG THE IMPACT OF MEDİA PROFİLİNG ON SOCİAL ISSUES: PERSPECTİVES FROM GHANA AND VİETNAM	Dr. Kwame Mensah Dr. Linh Tran,
		2	CULTİVATİNG YOUTH ENGAGEMENT İN ACADEMİC DİSCİPLİNES: A NEW PARADİGM FOR UNİVERSİTY CULTURE	A. L. Zhao, B. S. Khamis, M. T. Wanjiku, E. A. Nogueira
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η-RİCCİ BOURGUİGNON SOLİTONLARI KABUL EDEN PARA-SASAKIAN MANİFOLDLAR

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ÖZET

Ricci-Bourguignon akışı kavramı, Ricci akışının bir uzantısı olarak, Lichnerowicz'in yayımlanmamış bir makalesi ve Aubin'in bir makalesine dayanarak J.P. Bourguignon tarafından ortaya çıkartılmıştır. Ricci-Bourguignon akışı, sabit noktalarının solitonlar olduğu Riemannian manifoldlar üzerindeki içsel bir geometrik akıştır. Bu nedenle Ricci Bourguignon solitonları

$\frac{\partial g}{\partial t} = -2(Ric - \rho r g), g(0) = g_0,$

şeklinde tanımlı Ricci-Bourguignon akışına özdeş benzer çözümler üretir, burada Ric; Ricci eğrilik tensörü, r; g yarı Riemannian metriğine göre skaler eğrilik, ve ρ; sıfırdan farklı bir reel sabittir.

(M, g) bir semi-Riemann manifold olmak üzere

$$\frac{1}{2}L_\xi g + S + 2(\lambda - \rho r)g + 2\mu\eta \otimes \eta = 0,$$

denklemini sağalayan M üzerindeki (g, ξ, λ, μ) dörtlüsüne η-Ricci Bourguignon soliton denir, burada λ ve μ reel sabitler, η; ξ vektör alanının duali ve S; M manifoldunun Ricci eğrilik tensörüdür.

Ricci solitonlar özellikle son yıllarda büyük ilgi görmüş ve birçok matematikçi tarafında incelenmiştir. Öteyandan para-Sasakian manifoldlar geometri, matematik ve fizik için oldukça önemli bir manifold çeşididir. Bu çalışmada η-Ricci Bourguignon solitonları para-Sasakian manifoldlar üzerinde ele aldık. Para-Sasakian manifold için concircular eğrilik tensörü, projektif eğrilik tensörü, W-eğrilik tensörleri gibi bazı özel eğrilik tensörleri üzerinde η-Ricci Bourguignon solitonları detaylı bir şekilde inceledik. Özellikle bu eğrilik tensörleri ve η-Ricci

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Bourguignon solitonları kullanarak Ricci pseudosimetrik ve Ricci semisimetrik para-Sasaskian manifoldların önemli karakterizasyonlarını verdik.

Anahtar Kelimeler : η-Ricci Bourguignon soliton, Hiperbolik Sasakian Manifold, Ricci Pseudoparalel Manifold.

1 PARA SASAKIAN MANIFOLDS ADMITTING η -RICI BOURGUIGNON SOLITONS

Hamilton revealed a new concept, the Ricci flow, in 1982. He obtained the canonical metric of a smooth manifold with the concept of Ricci flow. In the following years, Ricci flow played a very active role in the study of Riemann manifolds. It has become very useful especially for Riemannian manifolds with positive curvature. Poincare conjecture proven by Perelman using Ricci flow in [1],[2]. Ricci flow, which is an evolution equation for metrics in Riemannian manifolds, is defined as

$$\frac{\partial}{\partial t} g(t) = -2S\big(g(t)\big).$$

The limit of solutions of Ricci flow is called Ricci soliton. If the Ricci flow moves with the only one set of parameters, it is called a solution of the Ricci flow.

Especially for 20 years, Ricci solitons occupied a very important place for geometry and managed to attract the attention of many authors. In addition, the Ricci soliton was solved by Perelman in 1904 with the help of Ricci soliton, and the long-standing Poincare conjecture turned Ricci solitons into a very important one. In [3], Sharma studied the Ricci solitons in contact geometry. Thereafter Ricci solitons in contact metric manifolds have been studied by various authors such as Bagewadi et al. in [4],[5],[6],[7], Bejan and Crasmareanu in [8], Blaga in [9], Chandra et al. in [10], Chen and Deshmukh in [11], Deshmukh et al. in [12], He and Zhu [13], Atçeken et al. in [14], Nagaraja and Premalatta in [15], Tripathi in [16] and many others.

The concept of Ricci Bourguignon flow as an extension of Ricci flow has been introduced by J.P. Bourguignon. Ricci Bourguignon flow is an intrinsic geometric flow on Riemannian manifolds, whose fixed points are solitons. Therefore, the Ricci Bourguignon solitons generate self-similar solution to the Ricci Bourguignon flow

$$\frac{\partial g}{\partial t} = -2(Ric - \rho r g), g(0) = g_0,$$

where Ric is the Ricci curvature tensor, r is scalar curvature with respect to the semi-Riemannian metric g and ρ is a non-zero real constant.

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On the other hand, quasi-Einstein metrics or Ricci solitons serve as solitons to Ricci flow equation. Aubin has given the notion of Ricci Bourguignon flow in a complete Riemannian manifold. A semi Riemannian manifold of dimension $n \geq 3$ is said to be Ricci Bourguignon soliton of

$$\frac{1}{2}L_V g + Ric + (\lambda - \rho r)g = 0,$$

where L_V denotes the Lie derivative operator along the vector field V, ρ is a non-zero constant and λ is real constant. Similar to the Ricci soliton, a Ricci Bourguignon soliton (M, g, V, λ, ρ) is called expanding if $\lambda > 0$, steady if $\lambda = 0$, and shrinking if $\lambda < 0$.

Perturbing the equation that define Ricci Bourguignon solitons by multiple of a certain (0,2) tensor field $\eta \otimes \eta$, we obtain a slightly more general notion, namely η-Ricci Bourguignon solitons such as

$$L_V g + 2Ric + 2(\lambda - \rho r)g + 2w\eta \otimes \eta = 0,$$

where w is a real constant and η is 1-form. Particularly, if we choose $\rho = 0$ in this equation, the η-Ricci Bourguignon soliton reduces to the η-Ricci soliton.

In this paper, we considered Ricci-pseudosymmetric para-Sasakian manifolds admitting almost η −Ricci Bourguignon solitons in some curvature tensors. Ricci pseudosymmetry concepts of para Sasakian manifolds admitting η −Ricci Bourguignon soliton have introduced according to the choice of the curvature tensor. Then, again according to these curvature tensor, necessary and sufficient conditions are given for para-sasakian manifold admitting η −Ricci Bourguignon soliton to be Ricci semi-symmetric. Then some characterizations are obtained and some classifications have made.

2 PRELIMINARIES

A $(2n+1)$-dimensional smooth manifold M has an almost paracontact structure (ϕ, ξ, η) if it admits a tensor field ϕ of type (1,1), a vector field ξ and a 1-form η satisfying the following conditions;

$$\phi^2 q_1 = q_1 - \eta(q_1)\xi, \quad \eta(\xi) = 1, \quad \phi\xi = 0, \quad \eta \circ \phi = 0. \tag{1}$$

If an almost paracontact manifold is endowed with a semi-Riemannian metric tensor g such that

$$g(\phi q_1, \phi q_2) = -g(q_1, q_2) + \eta(q_1)\eta(q_2), \tag{2}$$

for all vector fields q_1, q_2 on M, then $M(\phi, \xi, \eta, g)$ is said to be almost paracontact metric manifold. It is clear that

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$$g(\xi, q_1) = \eta(q_1). \tag{3}$$

The fundamental 2-form Φ of an almost paracontact metric manifold $M(\phi, \xi, \eta, g)$ is defined by

$$\Phi(q_1, q_2) = g(q_1, \phi q_2). \tag{4}$$

If $d\eta = \Phi$, then almost paracontact metric manifold $M(\phi, \xi, \eta, g)$ is called paracontact metric manifold. If a paracontact metric structure is normal, this structure is called para-Sasakian. That means

$$d\eta(q_1, q_2) = \frac{1}{2}[\eta(q_2)q_1 - \eta(q_1)q_2 - \eta([q_1, q_2])] = 0. \tag{5}$$

It well known that the necessary and sufficient condition for an almost paracontact manifold to be para-Sasakian is that

$$(\nabla_{q_1}\phi)q_2 = -g(q_1, q_2)\xi - \eta(q_2)q_1 + 2\eta(q_1)\eta(q_2)\xi. \tag{6}$$

For all q_1, q_2 in (6), it is clear that

$$\nabla_{q_1}\xi = -\phi q_1. \tag{7}$$

The manifold M^{2n+1} is called para-Sasakian manifold or P-Sasakian manifold**,** where ∇ denote the Levi-Civita connection on M^{2n+1}. If the relation

$$(\nabla_{q_1}\eta)q_2 = -g(q_1, q_2) + \eta(q_1)\eta(q_2)$$

is satisfied specifically, the para-Sasakian manifold is called the special para-Sasakian manifold or the Sp-Sasakian manifold.

Lemma 1 *A $(2n+1)-$dimensional para-Sasakian manifold M^{2n+1} provides the following relations.*

$$R(q_1, q_2)\xi = \eta(q_1)q_2 - \eta(q_2)q_1, \tag{8}$$

$$R(\xi, q_1)q_2 = \eta(q_2)q_1 - g(q_1, q_2)\xi, \tag{9}$$

$$\eta(R(q_1, q_2)q_3) = g(\eta(q_2)q_1 - \eta(q_1)q_2, q_3), \tag{10}$$

$$S(q_1, \xi) = -2n\eta(q_1), \tag{11}$$

$$Q\xi = -2n\xi, \tag{12}$$

$$(\nabla_{q_1}\eta)q_2 = g(q_1, \phi q_2) \tag{13}$$

for any vector fields q_1, q_2 on M^{2n+1}, where ∇ is the Levi-Civita connection, R and S denote the Riemannian curvature tensor and Ricci tensor of M^{2n+1}, respectively.

Precisely, a Ricci Bourguignon soliton on a (semi) Riemannian manifold (M, g) is defined as a triple (g, ξ, λ) on M satisfying

$$L_\xi g + 2S + 2(\lambda - \rho r)g = 0,$$

where L_ξ is the Lie derivative operator along the vector field ξ and λ is a real constant. Futhermore, generalization is the notion of η −Ricci Bourguignon soliton defined by Aubin as a quadruple (g, ξ, λ, μ) satisfying

$$L_\xi g + 2S + 2(\lambda - \rho r)g + 2\mu\eta \otimes \eta = 0, \tag{14}$$

where λ and μ are real constants, ρ is a nonzero constant, r is the scalar curvature of M, η is the dual of ξ and S denotes the Ricci tensor of M.

3 η −RICCI BOURGUIGNON SOLITONS ON RICCI PSEUDOSYMMETRIC PARA-SASAKIAN MANIFOLDS

Now let (g, ξ, λ, μ) be almost η −Ricci Bourguignon soliton on para-Sasakian manifold. Then we have

$$(L_\xi g)(q_1, q_2) = L_\xi g(q_1, q_2) - g(L_\xi q_1, q_2) - g(q_1, L_\xi q_2)$$

$$= \xi g(q_1, q_2) - g([\xi, q_1], q_2) - g(q_1, [\xi, q_2])$$

$$= g(\nabla_\xi q_1, q_2) - g(q_1, \nabla_\xi q_2) - g(\nabla_\xi q_1, q_2)$$

$$+ g(\nabla_{q_1}\xi, q_2) - g(\nabla_\xi q_2, q_1) + g(q_1, \nabla_{q_2}\xi),$$

for all $q_1, q_2 \in \Gamma(TM)$. By using ϕ is anti-symmetric and in view of (7), we have

$$(L_\xi g)(q_1, q_2) = 0. \tag{15}$$

Thus, in a para-Sasakian manifolds, from (14) and (15), we have

$$S(q_1, q_2) = (\rho r - \lambda)g(q_1, q_2) - \mu\eta(q_1)\eta(q_2). \tag{16}$$

It is clear from (16) that the $(2n+1)$ −dimensional para-Sasakian admitting η −Ricci Bourguignon soliton $(M^{2n+1}, g, \xi, \lambda, \mu)$ is an η −Einstein manifold provided $\lambda \neq \rho r$ and $\mu \neq 0$.

For $q_2 = \xi$ in (16), this implies that

$$S(\xi, q_1) = (\rho r - \lambda - \mu)\eta(q_1). \tag{17}$$

Taking into account of (11), we conclude that

$$\lambda + \mu = 2n + \rho r. \tag{18}$$

Again it is clear from (16)

$$Qq_2 = (\rho r - \lambda)q_2 - \mu\eta(q_2)\xi. \tag{19}$$

Definition 1 *Let M^{2n+1} be an $(2n+1)$ −dimensional para-Sasakian manifold. If $R \cdot S$ and $Q(g,S)$ are linearly dependent, then the manifold is said to be Ricci pseudosymmetric.*

In this case, there exists a function F_R on M such that

$$R \cdot S = F_R Q(g,S).$$

In particular, if $F_R = 0$, the manifold M^{2n+1} is said to be Ricci semi-symmetric.

Let us now investigate the Ricci pseudosymmetry case of the $(2n+1)$ −dimensional para-Sasakian manifold.

Theorem 1 *Let M^{2n+1} be para-Sasakian manifold and (g,ξ,λ,μ) be almost η −Ricci Bourguignon soliton on M^{2n+1}. If M^{2n+1} is a Ricci pseudosymmetric, then either the η-Ricci Bourguignon soliton reduces to the Ricci Bourguignon soliton or $F_R = -1$.*

Proof. Let's assume that para-Sasakian manifold M^{2n+1} be Ricci pseudosymmetric and (g,ξ,λ,μ) be almost η −Ricci Bourguignon soliton on para-Sasakian manifold M^{2n+1}. That's mean

$$(R(q_1,q_2)\cdot S)(q_3,q_4) = F_R Q(g,S)(q_3,q_4;q_1,q_2),$$

for all $q_1,q_2,q_3,q_4 \in \Gamma(TM^{2n+1})$. From the last equation, we can easily write

$$\begin{aligned} &S(R(q_1,q_2)q_3,q_4) + S(q_3,R(q_1,q_2)q_4) \\ &= F_R\left\{S\left((q_1 \wedge_g q_2)q_3,q_4\right) + S(q_3,(q_1 \wedge_g q_2)q_4)\right\}. \end{aligned} \tag{20}$$

If we choose $q_4 = \xi$ in (20) and make use of (9), (11), we have

$$\begin{aligned} &-2n\eta(R(q_1,q_2)q_3) + S(q_3,\eta(q_1)q_2 - \eta(q_2)q_1) \\ &= F_R\{-2ng(\eta(q_1)q_2 - \eta(q_2)q_1,q_3) + S(q_3,\eta(q_2)q_1 - \eta(q_1)q_2)\}. \end{aligned} \tag{21}$$

If we use (16) in (21), we get

$$[(\lambda - \rho r - 2n)(1 + F_R)]g(\eta(q_2)q_1 - \eta(q_1)q_2,q_3) = 0. \tag{22}$$

This completes the proof.

We can give the results obtained from this theorem as follows.

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Corollary 1 *Let M^{2n+1} be para-Sasakian manifold and (g, ξ, λ, μ) be almost η −Ricci Bourguignon soliton on M^{2n+1}. If M^{2n+1} is a Ricci semisymmetric, then M^{2n+1} is an Einstein manifold.*

Corollary 2 *Let M^{2n+1} be para-Sasakian manifold and (g, ξ, λ, μ) be almost η −Ricci Bourguignon soliton on M^{2n+1}. If M^{2n+1} is a Ricci semisymmetric, then Aat least one of the following is satisfied:*

i. If $\rho r > -2n$, then M^{2n+1} is an expanding.

ii. If $\rho r = -2n$, then M^{2n+1} is an steady.

iii. If $\rho r < -2n$, then M^{2n+1} is an shrinking.

For an $(2n + 1)$ −dimensional semi-Riemann manifold M, the projective curvature tensor is defined as

$$P(q_1, q_2)q_3 = R(q_1, q_2)q_3 - \frac{1}{2n}[S(q_2, q_3)q_1 - S(q_1, q_3)q_2]. \tag{23}$$

For an $(2n + 1)$ −dimensional para-Sasakian manifold, if we choose $q_3 = \xi$ in (23), we can write

$$P(q_1, q_2)\xi = 0, \tag{24}$$

so, we have

$$\eta(P(q_1, q_2)q_3) = 0. \tag{25}$$

Definition 2 *Let M^{2n+1} be an $(2n + 1)$ −dimensional para-Sasakian manifold. If $P \cdot S$ and $Q(g, S)$ are linearly dependent, then the manifold is said to be projective-Ricci pseudosymmetric.*

In this case, there exists a function F_P on M^{2n+1} such that

$$P \cdot S = F_P Q(g, S).$$

In particular, if $F_P = 0$, the manifold M^{2n+1} is said to be projective-Ricci semisymmetric.

Let us now investigate the projective-Ricci pseudosymmetry case of the $(2n + 1)$ −dimensional para-Sasakian manifold under condition the almost η −Ricci Bourguignon soliton.

Theorem 2 *Let M^{2n+1} be para-Sasakian manifold and (g, ξ, λ, μ) be almost η −Ricci Bourguignon soliton on M^{2n+1}. If M^{2n+1} is a projective-Ricci pseudosymmetric, then M^{2n+1} is either projective-Ricci semisymmetric or η −Ricci Bourguignon soliton (g, ξ, λ, μ) reduces Ricci Bourguignon soliton (g, ξ, λ).*

Proof. Let's assume that para-Sasakian manifold M^{2n+1} be projective Ricci pseudosymmetric and (g,ξ,λ,μ) be almost η −Ricci Bourguignon soliton on para-Sasakian manifold M^{2n+1}. That's mean

$$(P(q_1,q_2)\cdot S)(q_3,q_4) = F_P Q(g,S)(q_3,q_4;q_1,q_2),$$

for all $q_1,q_2,q_3,q_4 \in \Gamma(TM)$. From the last equation, we can easily write

$$S(P(q_1,q_2)q_3,q_4) + S(q_3,P(q_1,q_2)q_4)$$
$$= F_P\left\{S\left((q_1\wedge_g q_2)q_3,q_4\right) + S\left(q_3,(q_1\wedge_g q_2)q_4\right)\right\}. \quad (26)$$

If we putting $q_4=\xi$ in (26),and make use of (11), (24) and (25), we have

$$F_p[-2ng(\eta(q_1)q_2-\eta(q_2)q_1,q_3) + S(q_3,\eta(q_2)q_1-\eta(q_1)q_2)] = 0 \quad (27)$$

If we use (16) in (27), we get

$$F_P(\lambda-\rho r-2n)g(\eta(q_1)q_2-\eta(q_2)q_1,q_3) = 0.$$

This completes the proof.

We can give the result obtained from this theorem as follows.

Corollary 3 *Let M^{2n+1} be para-Sasakian manifold and (g,ξ,λ,μ) be almost η −Ricci Bourguignon soliton on M^{2n+1}. If M^{2n+1} is a projective-Ricci pseudosymmetric, then at least one of the following is satisfied:*

i. M^{2n+1} is a projective Ricci semisymmetric.

ii. If $\rho r > -2n$, then M^{2n+1} is an expanding.

iii. If $\rho r = -2n$, then M^{2n+1} is an steady.

iv. If $\rho r < -2n$, then M^{2n+1} is an shrinking.

For an $(2n+1)$ −dimensional M semi-Riemann manifold, the $\mathcal{M}$ −projective curvature tensor is defined as

$$\mathcal{M}(q_1,q_2)q_3 = R(q_1,q_2)q_3 - \frac{1}{4n}[S(q_2,q_3)q_1 - S(q_1,q_3)q_2 + g(q_2,q_3)Qq_1 - g(q_1,q_3)Qq_2]. \quad (28)$$

For an $(2n+1)$ −dimensional para-Sasakian manifold, if we choose $q_3=\xi$ in (28), we can write

$$\mathcal{M}(q_1,q_2)\xi = \frac{1}{2}[\eta(q_1)q_2-\eta(q_2)q_1]$$
$$-\frac{1}{4n}[\eta(q_2)Qq_1-\eta(q_1)Qq_2], \quad (29)$$

by means of (29), we get

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$$\eta(\mathcal{M}(q_1,q_2)q_3)=\frac{1}{2}g(\eta(q_2)q_1-\eta(q_1)q_2,q_3)$$
$$+\frac{1}{4n}S(\eta(q_2)q_1-\eta(q_1)q_2,q_3). \quad (30)$$

Definition 3 *Let M be an $(2n+1)$ −dimensional para-Sasakian manifold. If $\mathcal{M}\cdot S$ and $Q(g,S)$ are linearly dependent, then the manifold is said to be $\mathcal{M}$ −projective Ricci pseudosymmetric.*

In this case, there exists a function $F_{\mathcal{M}}$ on M such that

$$\mathcal{M}\cdot S=F_{\mathcal{M}}Q(g,S).$$

In particular, if $F_{\mathcal{M}}=0$, the manifold M^{2n+1} is called $\mathcal{M}$ −projective Ricci semi-symmetric.

Next, let us now investigate the $\mathcal{M}$ −projective Ricci pseudosymmetry case of the $(2n+1)$ −dimensional para-Sasakian manifold.

Theorem 3 *Let M^{2n+1} be para-Sasakian manifold and (g,ξ,λ,μ) be almost η −Ricci Bourguignon soliton on M^{2n+1}. If M^{2n+1} is a $\mathcal{M}$ −projective Ricci pseudosymmetric, then*

$$F_{\mathcal{M}}=\frac{(\rho r-\lambda)^2+4n(\rho r-\lambda+n)}{4n(\lambda-\rho r-2n)},$$

provided $\lambda\neq\rho r+2n$.

Proof. Let's assume that para-Sasakian manifold be $\mathcal{M}$ −projective Ricci pseudosymmetric and (g,ξ,λ,μ) be almost η −Ricci Bourguignon soliton on para-Sasakian manifold. That's mean

$$(\mathcal{M}(q_1,q_2)\cdot S)(q_3,q_4)=F_{\mathcal{M}}Q(g,S)(q_3,q_4;q_1,q_2),$$

for all $q_1,q_2,q_3,q_4\in\Gamma(TM^{2n+1})$. From the last equation, we can easily write

$$S(\mathcal{M}(q_1,q_2)q_3,q_4)+S(q_3,\mathcal{M}(q_1,q_2)q_4)$$
$$=F_{\mathcal{M}}\left\{S\left((q_1\wedge_g q_2)q_3,q_4\right)+S\left(q_3,(q_1\wedge_g q_2)q_4\right)\right\}. \quad (31)$$

If we choose $q_4=\xi$ in (31) and make use of (11), (29) and (30) in (31), we have

$$F_{\mathcal{M}}\{-2ng(\eta(q_1)q_2-\eta(q_2)q_1,q_3)+S(q_3,\eta(q_2)q_1-\eta(q_1)q_2)\}$$
$$=-ng(\eta(q_2)q_1-\eta(q_1)q_2,q_3)-S(\eta(q_2)q_1-\eta(q_1)q_2,q_3)$$
$$-\frac{1}{4n}S(q_3,\eta(q_2)Qq_1-\eta(q_1)Qq_2). \quad (32)$$

If we use (16) in (32), we have

$$(\rho r-\lambda+n)g(\eta(q_1)q_2-\eta(q_2)q_1,q_3)-\frac{\rho r-\lambda}{4n}S(\eta(q_2)q_1-\eta(q_1)q_2,q_3)$$
$$=F_{\mathcal{M}}[-2ng(\eta(q_1)q_2-\eta(q_2)q_1,q_3)+(\rho r-\lambda)g(\eta(q_2)q_1-\eta(q_1)q_2,q_3)]. \quad (33)$$

If we use (16) in (33), we have

$$\left[\frac{1}{4n}(\rho r-\lambda)^2+(\rho r-\lambda+n)+(2n-\lambda+\rho r)F_{\mathcal{M}}\right]g(\eta(q_1)q_2-\eta(q_2)q_1,q_3)=0.$$

This completes the proof.

We can give the results obtained from this theorem as follows.

Corollary 4 *Let M^{2n+1} be para-Sasakian manifold and (g,ξ,λ,μ) be almost η −Ricci Bourguignon soliton on M^{2n+1}. If M^{2n+1} is a $\mathcal{M}$-projective Ricci semisymmetric, then at least one of the following is satisfied:*

i. η −Ricci Bourguignon soliton (g,ξ,λ,μ) reduces Ricci Bourguignon soliton (g,ξ,λ).

ii. If $\rho r>-2n$, then M^{2n+1} is an expanding.

iii. If $\rho r=-2n$, then M^{2n+1} is an steady.

iv. If $\rho r<-2n$, then M^{2n+1} is an shrinking.

Example 1 *We consider a* 3 −*dimensional manifold* $M=\mathbb{R}^3$ *with standard cartesian coordinates. We choose the vector fields*

$$e_1=e^x\frac{\partial}{\partial y},e_2=e^x\left(\frac{\partial}{\partial y}-\frac{\partial}{\partial z}\right),e_3=-\frac{\partial}{\partial x},$$

which are linearly independent at each point of M. Let g be the Riemannian metric defined by

$$g(e_i,e_j)=\delta_{ij}.$$

Let η be the 1 −form defined by

$$g(q_3,e_3)=\eta(q_3),$$

for any vector field q_3 on M. We define the (1,1)-type tensor field ϕ as

$$\phi(e_1)=e_1,\phi(e_2)=e_2\text{and}\phi(e_3)=0.$$

One can verify easily that with $\xi=e_3$, (ϕ,ξ,η,g) defines an almost paracontact structure on M [18]. Let ∇ be the Levi-Civita connection with respect to g. Then we have

$$[e_1,e_2]=0,[e_1,e_3]=e_1,[e_2,e_3]=e_2.$$

Taking $e_3=\xi$ and using Koszul's formula, we obtain

$$\nabla_{e_1}e_2=0,\qquad \nabla_{e_1}e_3=e_1,\qquad \nabla_{e_1}e_1=-e_3,$$
$$\nabla_{e_2}e_3=e_2,\qquad \nabla_{e_2}e_2=-e_3,\qquad \nabla_{e_2}e_1=0,$$
$$\nabla_{e_3}e_3=0,\qquad \nabla_{e_3}e_2=0,\qquad \nabla_{e_3}e_1=0.$$

From the above it can be easily seen that (ϕ, ξ, η, g) is a para-Sasakian structure on M. Hence $M(\phi, \xi, \eta, g)$ is a $3-$dimensional para-Sasakian manifold [18]. By using above result, we can easily obtain the following:

$$R(e_1, e_2)e_2 = -e_1, \qquad R(e_1, e_3)e_3 = -e_1, \qquad R(e_2, e_1)e_1 = -e_2,$$

$$R(e_2, e_3)e_3 = -e_2, \qquad R(e_3, e_1)e_1 = -e_3, \qquad R(e_3, e_2)e_2 = -e_3,$$

$$R(e_1, e_2)e_3 = 0, \qquad R(e_3, e_2)e_3 = e_2, \qquad R(e_3, e_1)e_2 = 0.$$

Tracing the Riemann curvature tensor, the components of the Ricci tensor is given by:

$$S(e_1, e_1) = -2, \qquad S(e_2, e_2) = -2, \qquad S(e_3, e_3) = -2,$$

$$S(e_1, e_2) = 0, \qquad S(e_1, e_3) = 0, \qquad S(e_2, e_3) = 0.$$

That is

$$S(q_1, q_2) = -2g(q_1, q_2).$$

Thus we have

$$\lambda = 2.$$

This manifold in the example is always expanding.

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APPROXIMATION OF CONE SECTIONS WITH A CLASS OF LINEAR POSITIVE OPERATORS

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ABSTRACT

The emergence of the concepts is based on Apollonius' book Conica. "conic sections" are very important geometric objects that we encounter in many stages of daily life, especially in locating ships in maritime. On the other hand, approximation theory is one of the important fields of study used not only in mathematics but also in solving problems in many scientific fields. In this study, it is aimed to approximation to conic sections with linear positive operators, which are important tools of approximation theory. By calculating test functions under the operator, it has been shown with the help of these equations that the operator satisfies the Volkov type theorem used in approximation to two-variable functions. Additionally, the conic sections that will be obtained after applying the defined operator to these functions are examined. Research findings were supported with visual elements. The results obtained show that the defined operator generally behaves similarly to the function to be approximated. Therefore, the defined operator is a suitable operator for approximating conic sections.

Key Words: Cone Sections, Volkov Theorem, Bernstein Type Operators.

1. INTRODUCTION

In the field of mathematical analysis, the approximation of continuous functions by a sequence of simple functions, such as algebraic polynomials, has been an important research topic for the past two centuries. After the proof by Weierstrass that continuous functions on a closed interval can be approximated by polynomials, this difficult and complex problem has been simplified by many mathematicians (Weierstraas, 1885). One of the first such mathematicians was Bernstein, who provided the first application of Weierstrass' theorem in the interval [0,1] with a sequence of polynomials (Bernstein, 1912). There are many generalizations of this operator in the literature. Subsequently, important general theories have appeared in the literature without choosing special polynomials. Korovkin and Volkov's theorems (Korovkin, 1953), (Volkov, 1957) are important theorems that accelerated the work in the field of

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approximation theory. According to these theorems, approximation is provided for each function of the space with the help of the equations provided for the test functions.

Many different generalizations of the Bernstein polynomial have been investigated by many different authors for both univariate and bivariate functions (Izgi, 2012), (Büyükyazıcı, 1999), (Bilgin and Eren, 2021), (Aral et al., 2018), (Aral et al., 2012). One of these generalizations is the polynomials defined by Izgi and his students, given by equation (2) below. An important work that brings a different perspective to approximation theory by using Bernstein polynomials is the application of classical Bernstein operators to conic equations by Alhazzori under the supervision of Tunç (Alhazzori, 2023).

In this study, the (2) operator given in (Cilo et al., 2012) is generalized as two variables and double indexed and its approximation properties are given. Inspired by the studies of (Cilo et al., 2012) and (Alhazzori, 2023), conic equations were applied and graphs for these approaches were drawn.

Definition 1.1

Let $x \in [-1,1]$ and $f \in C[-1,1]$

$$C_n(f;x) = \frac{1}{2^n}\sum_{k=0}^{n}\binom{n}{k}(1+x)^k(1-x)^{n-k}f\left(2\frac{k}{n}-1\right) \qquad (1)$$

is called $C_n(f;x)$ operatör (Cilo et al, 2012).

Definition 1.2

A general conic equation for $A,B,C,D,E,F \in \mathbb{R}$ and $A^2+B^2+C^2 \neq 0$ is as follows:

$x^2 + Bxy + Cy^2 + Dx + Ey + F = 0.$

This equation is a quadratic equation in two unknowns. By choosing the coefficients appropriately, the solution set of the equation represents a circle, ellipse, hyperbola or parabola. The expression $B^2 - 4AC$ is called a discriminant. Except in some degenerate cases, the following is true for the equation

if $B^2 - 4AC > 0$, then it is a hyperbola

if $B^2 - 4AC < 0$ then it is an ellipse,

if $B^2 - 4AC = 0$ then it denotes a parabola (Thomas & Finney, 1984).

If $A = C \neq 0$ and $B = 0$ in the equation

$x^2 + y^2 + D'x + E'y + F' = 0,$

Here

$D' = \frac{D}{A}, E' = \frac{E}{A}, F' = \frac{F}{A}$

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is and equation

$i)$ $D'^2 + E'^2 - 4F' > 0$, it is a circle of radius $\sqrt{\frac{D'^2+E'^2-4F'}{4}}$ centered at $\left(-\frac{D'}{2}, -\frac{E'}{2}\right)$,

$ii)$ if $D'^2 + E'^2 - 4F' = 0$, then it is the point $\left(-\frac{D'}{2}, -\frac{E'}{2}\right)$

$iii)$ denotes the empty set if $D'^2 + E'^2 - 4F' < 0$ (Sabuncuoğlu, 2003).

2. A DOUBLE INDEX TWO VARIABLE BERNSTEIN TYPE OPERATOR

Definition 2.1

Double-index Bernstein operators in $[-1,1] \times [-1,1]$, n −th order with respect to the x variable and m −th degree with respect to the y variable can be defined as:

$$C_{n,m}(f; x, y) = \frac{1}{2^{n+m}} \sum_{k=0}^{n} \sum_{j=0}^{m} f\left(\frac{2k}{n} - 1, \frac{2j}{m} - 1\right) P_{n,k}(x) Q_{m,j}(y) \qquad (2).$$

Here

$P_{n,k}(x) = \binom{n}{k} (1 + x)^k (1 - x)^{n-k}$

$Q_{m,j}(y) = \binom{m}{j} (1 + y)^j (1 - y)^{m-j}$.

Lemma 2.1

For any $n, m \in \mathbb{N}$ and $\lambda, \mu \in \{0,1,2\}$, $e_{\lambda,\mu}(x, y) = x^\lambda y^\mu$, on $[-1,1] \times [-1,1]$ for the two-variable Bernstein type operator defined with (2)

(i) $C_{n,m}(e_{0,0}; x, y) = 1$,

(ii) $C_{n,m}(e_{1,0}; x, y) = x$,

(iii) $C_{n,m}(e_{0,1}; x, y) = y$,

(iv) $C_{n,m}(e_{1,1}; x, y) = xy$,

(v) $C_{n,m}(e_{2,0}; x, y) = \frac{n-1}{n} x^2 + \frac{1}{n}$,

(vi) $C_{n,m}(e_{0,2}; x, y) = \frac{m-1}{m} y^2 + \frac{1}{m}$,

(vii) $C_{n,m}(e_{2,2}; x, y) = \frac{n-1}{n} x^2 + \frac{m-1}{m} y^2 + \frac{1}{n} + \frac{1}{m}$

equality is ensured.

Theorem 2.1

For every continuous function f on $[-1,1] \times [-1,1]$

$$\lim_{n.m \to \infty} \left\| C_{n,m} f - f \right\|_{C([-1,1] \times [-1,1])} = 0$$

equality is achieved.

The application of double-index, two-variable these Bernstein operators defined on the quadratic region to conic equations will be discussed.

Definition 2.2

For $A, B, C, D, E, F \in \mathbb{R}$, the image of

$$f(t,u) = At^2 + Btu + Cu^2 + Dt + Eu + F = 0$$

under the $C_{n,m}(f; x, y)$ operator is in the form as

$$C_{n,m}(f; x, y) = A'x^2 + B'xy + C'y^2 + D'x + E'y + F' = 0.$$

The above coefficients are easily obtained from the equations in Lemma 2.1 and are as follows.

$$A' = \frac{n-1}{n} A, \qquad B' = B, \qquad C' = C\frac{m-1}{m}, \qquad D' = D, \qquad E' = E,$$

$$F' = F + \frac{1}{n} A + \frac{1}{m} C. \tag{3}$$

Remark 2.1

Since uniform convergence is valid for every function f in Theorem 2.1, uniform convergence is valid for $f(t,u)$ given above.

Examination of the Circle

Let $A, B, C, D, E, F \in \mathbb{R}$ and

$$f(x,y) = Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \tag{4}$$

equation denotes a circle in Cartesian plane if $A = C = 1, B = 0$ and $D^2 + E^2 - 4F > 0$.

Remark 2.2

In the following sections, Δ_1, Δ_2 expressions will be defined as:

$$\Delta_1 = D^2 + E^2 - 4F$$

and

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$\Delta_2 = F - 2.$

Theorem 2.2

Let the equation $f(x, y) = 0$ given by (4) denote a circle and let $n \in \{2,3,4 \dots\}$. In this case, a necessary and sufficient condition for $C_{n,n}(f; x, y)$to denote a circle is

$[n^2\Delta_1 + 4n\Delta_2 + 8] > 0.$

Remark 2.3

1) In Theorem 2.2, if $n = 1$, the equation $C_{n,n}(f; x, y) = 0$ represents a line. The equation of this line is $Dx + Ey + F + 2 = 0$.

2) In Theorem 2.2, the equation $C_{n,n}(f; x, y) = 0$ if $n > 1$,

$\left(D\frac{n}{1-n}, E\frac{n}{1-n}\right)$ with centered on $\frac{\sqrt{n^2\Delta_1+4n\Delta_2+8}}{2n-2}$ at radius

$$x^2 + y^2 + D\frac{n}{n-1}x + E\frac{n}{n-1}y + F\frac{n}{n-1} + \frac{2}{n-1} = 0$$

refers to the circle.

3) When $\Delta_1 > 0$, the condition $n^2\Delta_1 + 4n\Delta_2 + 8 > 0$ in Theorem 2.2 will hold for every sufficiently large natural number n. Therefore, if the equation $f(x, y) = 0$ denotes a circle, then $C_{n,n}(f; x, y) = 0$ denotes a circle except for a finite number n.

Example 2.1

$f(x, y) = x^2 + y^2 + 8x - 6y - 10 = 0$

Considering the circle specified by the equation $\Delta_1 = 140$ and $\Delta_2 = 12$.

For every $n \in \{2,3,4 \dots\}$

$n^2\Delta_1 + 4n\Delta_2 + 8 > 0$

$140n^2 - 48n + 8 > 0$

Since, according to the theorem, the equation $C_{n,n}(f; x, y) = 0$ indicates a circle as:

$$x^2 + y^2 + \frac{Dn}{n-1}x + \frac{En}{n-1}y + \frac{nF + 2}{n-1} = 0$$

is in the form.

When $n \in \{2, 3, \dots, 10\}$ is taken in the $C_{n,n}(f; x, y)$ operator, the approach to the $f(x, y)$ function is given in Figure 2.1.

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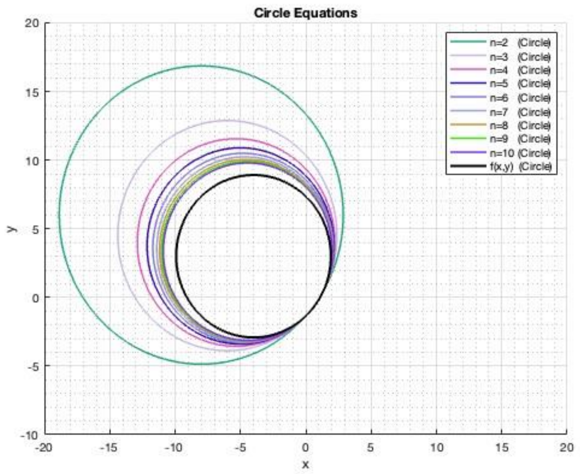

Figure 2.1

Theorem 2.3
Let the equation $f(x, y) = 0$ given in (4) denote a circle and let $n, m \in \{2,3,4 \ldots\}$. Then, when $n \neq m$, the equation $C_{n,m}(f; x, y) = 0$ denotes an ellipse.
Example 2.2

$$f(x, y) = x^2 + y^2 + 8x - 6y - 10 = 0$$

If the circle specified by the equation is considered

$$C_{n,m}(f; x, y) = \frac{n-1}{n}x^2 + \frac{m-1}{n}y^2 + 8x - 6y + \frac{m+n-10nm}{nm} = 0$$

equations denote an ellipse under the condition $n \neq m$.

In $C_{n,m}(f; x, y)$ operator, for $m = 5,6,7$ when $n = 2$, for $m = 5,6,7$ when $n = 3$ and for n=4 when m=5,6,7 the approach to the $f(x, y)$ function is given in Figure 2.2.

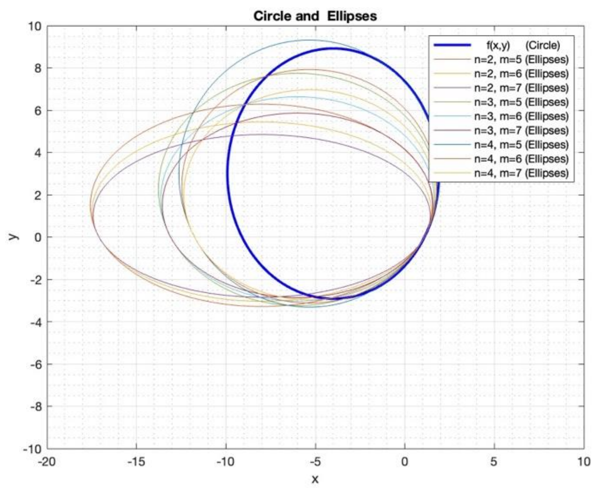

Figure 2.2

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Theorem 2.4

Let the equation $f(x,y)=0$ given in (4) specify a circle. In this case, if $n \neq m$ and $min(n,m)=1$, the equation $C_{n,m}(f;x,y)=0$ indicates a parabola.

Example 2.3

$f(x,y)=x^2+y^2+8x-6y-10=0$

considering the circle for $n=1$ specified by the equation,

$$C_{1,m}(f;x,y)=\frac{m-1}{m}y^2+8x-6y+\frac{1-10m}{m}=0$$

the equations each specify a parabola under the condition $m>1$.

If we take $m=2,3,4$ for $C_{1,m}(f;x,y)$, the approximation to the function $f(x,y)$ is given in Figure 2.3.

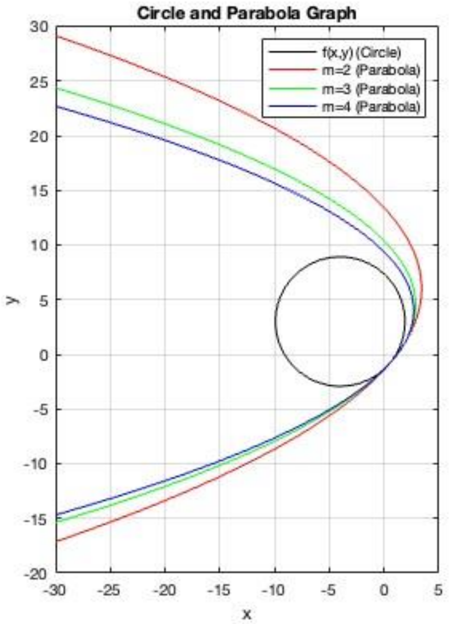

Figure 2.3

Investigation of the ellipse

Theorem 2.5

Let the equation $f(x,y)=0$ given by (4) specify an ellipse. In this situation

$i)$ For every $n,m \in \mathbb{N}$ if $C_{n,m}(f;x,y)=0$ satisfies the inequality $\frac{B^2}{4AC}<\left(\frac{n-1}{n}\right)\left(\frac{m-1}{m}\right)$ then the equation specifies an ellipse.

$ii)$ For every $n,m \in \{2,3,4\ldots\}$ if $C_{n,m}(f;x,y)=0$ satisfies the inequality

$\frac{B^2}{4AC} > \left(\frac{n-1}{n}\right)\left(\frac{m-1}{m}\right)$ then the equation specifies a hyperbola.

$iii)$ For every $n, m \in \mathbb{N}$ if $C_{n,m}(f; x, y) = 0$ satisfies the inequality $\frac{B^2}{4AC} = \left(\frac{n-1}{n}\right)\left(\frac{m-1}{m}\right)$ in this case the equation specifies a parabola.

Example 2.4

Consider the ellipse

$f(x, y) = 16x^2 + 25y^2 - 96x + 100y - 156 = 0.$

In this case,

$$\frac{0^2}{4.16.25} = 0$$

obtained. According to this

$$C_{n,m}(f; x, y) = 16\left(\frac{n-1}{n}\right)x^2 + 25\left(\frac{m-1}{m}\right)y^2 - 96x + 100y - 156 + \frac{16}{n} + \frac{25}{m} = 0.$$

If $n = 4, m = 5$ is taken in equation, the equation specifies an ellipse

$C_{4,5}(f; x, y) = 12x^2 + 20y^2 - 96x + 100y + 9 = 0$

If $n = 1, m = 5$ is taken in equation, the equation specifies the parabola

$C_{1,5}(f; x, y) = 20y^2 - 96x + 100y - 156 + 21 = 0$

For the $C_{n,m}(f; x, y)$ operator, the approach of the equations $C_{4,5}(f; x, y)$ and $C_{1,5}(f; x, y)$ to the $f(x, y)$ function is given in Figure 2.4 .

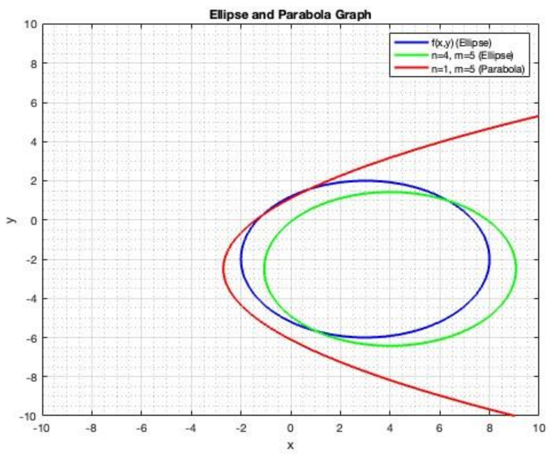

Figure 2.4

Theorem 2.6

Let the equation $f(x, y) = 0$ given by (4) specify an ellipse. In this case $n \neq m$ and

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if $min(n,m) = 1$, the equation $C_{n,m}(f;x,y)$

i) Specifies a parabola for $B = 0$.

ii) Specifies a hyperbola for $B \neq 0$.

Example 2.5

Considering the ellipse specified by the equation

$f(x,y) = 16x^2 + 25y^2 - 96x + 100y - 156 = 0,$

then the equation

$$C_{1,m}(f;x,y) = 25\left(\frac{m-1}{m}\right)y^2 - 96x + 100y - 156 + \frac{25}{m} = 0$$

specifies a parabola for every $m \geq 2$.

When $m \in \{2,3,\dots,10\}$ is taken in the $C_{1,m}(f;x,y)$ operator, the approach to the $f(x,y)$ function is given in Figure 2.5.

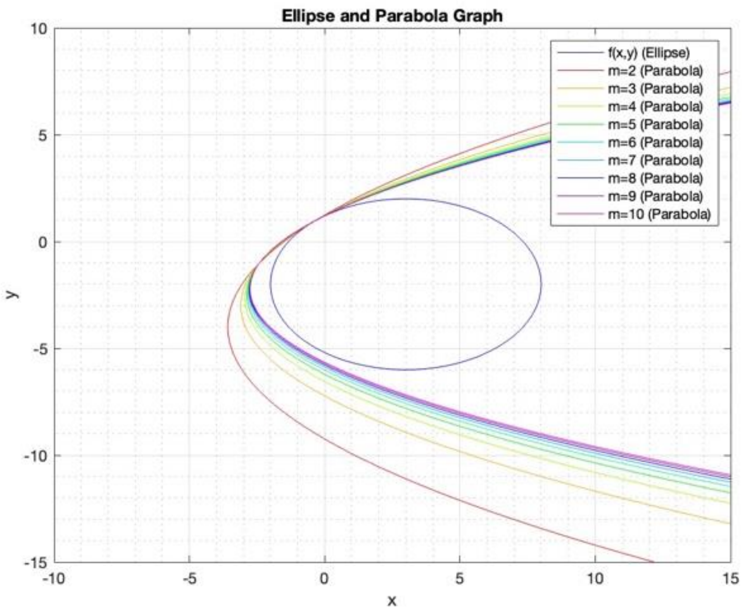

Figure 2.5

Examining the hyperbola

Theorem 2.7

Let the equation given by (4) specify a hyperbola. In this case, for each $n,m \in \{2,3,4 \dots\}$, $C_{n,m}(f;x,y) = 0$ also indicates a hyperbola.

Remark 2.4

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1. Let the equation given in (4) specify a hyperbola and let $AC < 0$. In this case, the possibility of $B = 0$ arises. If this situation occurs, the equation $C_{n,m}(f; x, y) = 0$ for every $n, m \in \mathbb{N}$ with $min(n, m) = 1$ and $n \neq m$ indicates a parabola. Because under these conditions $(B')^2 - 4A'C' = 0$.

2. If $B \neq 0$, Theorem 2.2 is valid for every $n, m \in \mathbb{N}$ that is not simultaneously 1.

Example 2.6

a) Considering the hyperbola specified by the equation

$x^2 + 2xy - y^2 + 2x + 5y - 7 = 0,$

the equations

$$C_{n,m}(f; x, y) = \frac{n-1}{n}x^2 + 2xy - \frac{m-1}{m}y^2 + 2x + 5y - 7 + \frac{1}{n} = 0$$

specify a hyperbola for every $n, m \in \mathbb{N}$.

The approximation to the function $f(x, y)$ with the operator $C_{n,m}(f; x, y)$ when $n = 2$ and $m = 2$; $n = 5$ and $m = 5$; $n = 8$ and $m = 8$ is given in Figure 2.6.

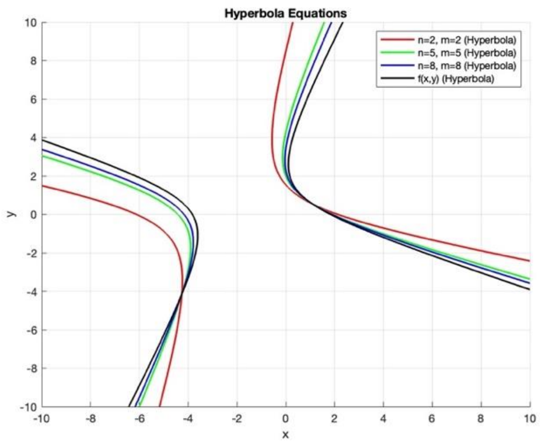

Figure 2.6

b) If we consider the hyperbola indicated by the equation $x^2 + 2xy + 2x + 5y - 7 = 0,$

then $AC = 0$ and

$$C_{n,m}(f; x, y) = \frac{n-1}{n}x^2 + 2xy + 2x + 5y - 7 + \frac{1}{n} - \frac{1}{m} = 0$$

denotes hyperbola for every $n, m \in \mathbb{N}$.

In the $C_{n,m}(f; x, y)$ operator, when $n = 4$, $m = 4$, $n = 10$, $m = 10$, $n = 15$ and $m = 15$, the approach to the $f(x, y)$ function is shown in Figure 2.7. has been given.

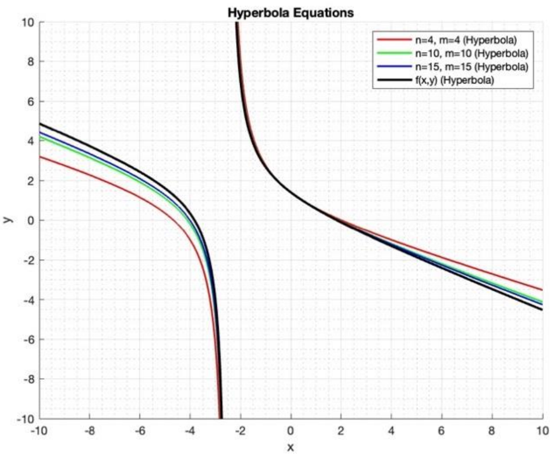

Figure 2.7

c) Considering the hyperbola specified by the equation

$x^2 - y^2 + 2x + 5y - 7 = 0,$

then $AC = -1 < 0$ and

$$C_{n,m}(f;x,y) = \frac{n-1}{n}x^2 - \frac{m-1}{m}y^2 + 2x + 5y - 7 + \frac{1}{n} - \frac{1}{m} = 0$$

the equations specify a hyperbola for every $n, m \in \{2,3,4 \dots\}$.

In the $C_{n,m}(f;x,y)$ operator, the approach to the $f(x,y)$ function when $n = 4, m = 5, ,\ n = 7, m = 10, n = 10$ and $m = 15$ is shown in Figure 2.8. has been given.

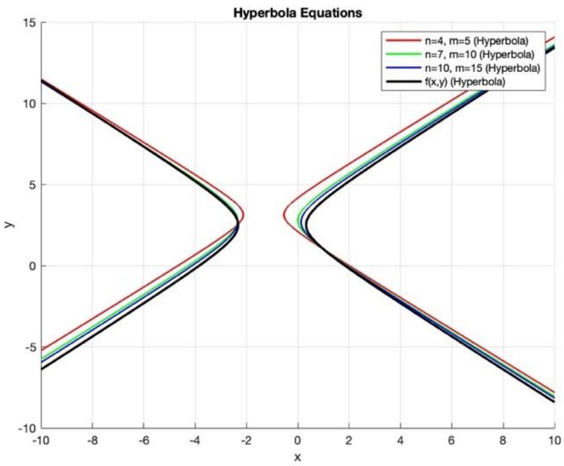

Figure 2.8

$$C_{1,m}(f;x,y) = -\frac{m-1}{m}y^2 + 2x + 5y - 6 - \frac{1}{m} = 0$$

$$C_{n,1}(f;x,y) = \frac{n-1}{n}x^2 + 2x + 5y - 8 + \frac{1}{n} = 0$$

equations denote parabola.

In the $C_{n,m}(f;x,y)$ operator, when $n=1, m=5,10,15$, when $m=1$, $n=5,10,15$, the approach to the $f(x,y)$ function is shown in Figure 2.9. has been given.

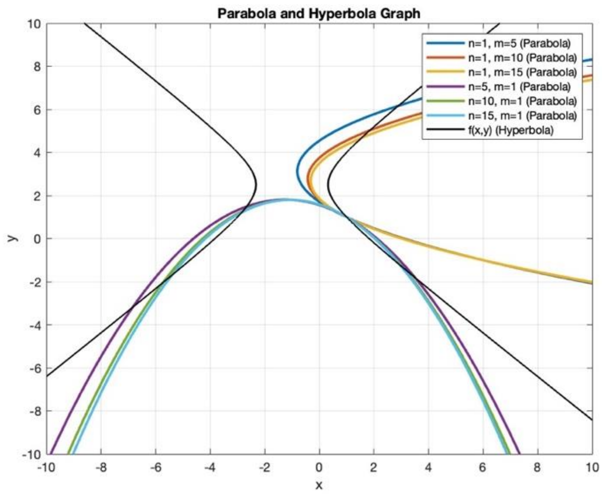

Figure 2.9

Examining the parabola

Theorem 2.8

Let the equation given by (4) specify a parabola. In this situation,

i) If $AC \neq 0$, $C_{n,m}(f;x,y)$ indicates a hyperbola for every $n,m \in \{2,3,4 \ldots\}$.

ii) If $AC = 0$, $C_{n,m}(f;x,y)$ denotes a parabola for each $n,m \in \{2,3,4 \ldots\}$.

Remark 2.5

Let the equation given by (4) specify a parabola and $AC > 0$. In this case, the equation $C_{n,m}(f;x,y)=0$ for every $n,m \in \mathbb{N}$ where $min(n,m)=1$ and $n \neq m$ indicates a hyperbola. Because under these conditions $(B')^2 - 4A'C' = B^2 > 0$.

Example 2.7

a) Considering the parabola specified by the equation

$f(x,y) = 4x^2 + 4xy + y^2 - 3x + 2y = 0$,

then $AC = 4 > 0$.

For every $n,m \in \{2,3,4 \ldots\}$

$$C_{n,m}(f;x,y) = 4\frac{n-1}{n}x^2 + 4xy + \frac{m-1}{m}y^2 - 3x + 2y + \frac{4}{n} + \frac{1}{m} = 0,$$

for every $n,m \in \{2,3,4 \ldots\}$

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$$C_{n,1}(f;x,y) = 4\frac{n-1}{n}x^2 + 4xy - 3x + 2y + \frac{4}{n} + 1 = 0$$

and for each $n, m \in \{2,3,4 \dots\}$

$$C_{1,m}(f;x,y) = 4xy + \frac{m-1}{m}y^2 - 3x + 2y + 4 + \frac{1}{m} = 0$$

equations specify a hyperbola.

In the $C_{n,m}(f;x,y)$ operator, when $n = 1$, $m = 5$, 10 and $m = 1$, $n = 5, 15$, the approach to the $f(x,y)$ function is given in Figure 2.10.

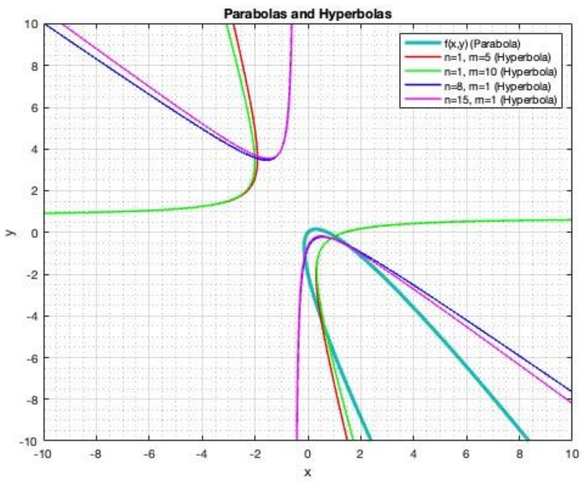

Figure 2.10

b) If we consider the parabola specified by the equation $f(x,y) = y^2 - 3x + 2y = 0$, it is clear that AC=0 and for each $n, m \in \{2,3,4 \dots\}$ the equations

$$C_{n,m}(f;x,y) = \frac{m-1}{m}y^2 - 3x + 2y + \frac{1}{m} = 0$$

specify a parabola.

In $C_{n,m}(f;x,y)$ operator, when $n = 1$, $m = 2$, when $n = 5$, $m = 10$, when $n = 10$, $m = 20$, when $n = 15$, when $m = 15$, the approach to the $f(x,y)$ function is given in Figure 2.11.

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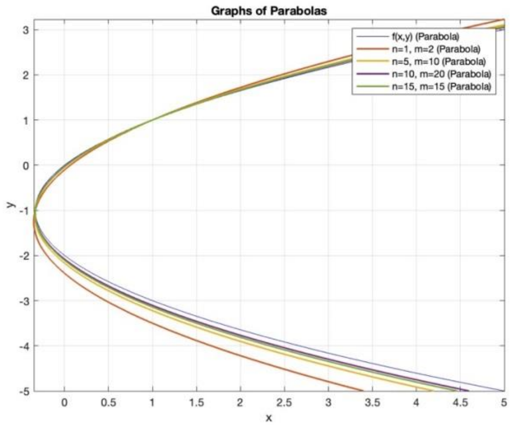

Figure 2.11

CONCLUSION

A two-variable generalization of a univariate Bernstein operator existing in the literature was defined and applications of this operator to conic equations were made. Thanks to the choice of the operator with a double index, it is open to more general studies. Many different operators in the field of approximation theory can be applied to conic equations in this sense, and considering that conic equations are also found in fields other than mathematics, it is thought that an important study has been presented to the literature (Acar and Izgi, 2022), (Bilgin and Eren, 2023), (Acar et al., 2014), (Güven and Bilgin, 2022, a), (Güven and Bilgin, 2022, b). In the next process, the study will be taken to different dimensions through applications of spherical conic equations. New studies can be carried out using the operator we defined above in the study (Tunç and Uzun, 2022) where other geometric properties related to the Bernstein operator are examined. Studies can be conducted on the extent to which the defined operator can be applied to kinetic equations and integro differential equations (Kaytmaz. 2024). Different approach features can be investigated for hybrid operators that will be created by combining special number sequences with the operator we defined (Soykan et al., 2023).

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ρ −STATISTICAL CONVERGENCE OF ORDER α FOR DOUBLE SEQUENCES OF FUZZY NUMBERS

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ABSTRACT

In this study, we introduce notions of ρ −strongly summable of order α, ρ −statistically convergence of order α and ρ −statistically Cauchy of order α for double sequences of fuzzy numbers. We give some results related to these concepts.

Keywords : ρ −statistical, double sequences, statistical convergence

1. INRODUCTION

Fast [1] gave short description of statistical convergence 1951. Schoenberg [2] examined statistical convergence as a method of summability and stated some basic properties of statistical convergence. This concept has been applied by many researchers under different names to measurement theory, locally convex spaces, summability theory, Banach spaces, trigonometric series in Fourier analysis and theory of fuzzy set ([3]-[6]). The concept of statistical convergence depend on the density subsets of the set $\mathbb{N}$. The natural density of a subset K of $\mathbb{N}$ is defined by $\delta(K) = \lim_{n\to\infty} \frac{1}{n} |\{k \le n : k \in K\}|$, if the limit exists, where the vertical bars indicate number of the elements in $\{k \le n : k \in K\}$.

Çolak [7] was introduced the statistical convergence of order α as follows:

The sequence $z = (z_k)$ is said to be statistically convergent of order α to z_0 if there a complex number z_0 such that

$$\lim_{n\to\infty} \frac{1}{n^\alpha} |\{k \le n : |z_k - z_0| \ge \varepsilon\}| = 0.$$

If z is a sequence such that z_k satifies property P for all k except a set of naturaly density zero, then we say that z_k satifies P for "almost all k" and we abbreviate this by "$a.a.k.$"

Fuzzy set theory, which is a very valuable logic with accuracy, was first introduced by Zadeh [8] in 1965. This theory has a wide range of applications such as fuzzy topological spaces, fuzzy measurements, fuzzy mathematical programming and fuzzy logic. The concept of fuzzy number sequence is first encountered in Matloka's paper [9].

Matloka [9] defined the concept of bounded and convergent sequences of fuzzy numbers and studied their some properties. Since then, many studies on sequences of fuzzy numbers have been made and studies on this subject are still ongoing([10]-[16]).

A fuzzy number is fuzzy set $Z: \mathbb{R} \to [0,1]$ with the following properties:

$i)$ Z is normal, that is, there exists an $z_0 \in \mathbb{R}$ such that $Z(z_0) = 1$;

$ii)$ Z is fuzzy convex, that is, for $z, t \in \mathbb{R}$ and $0 \le \lambda \le 1, Z(\lambda z + (1-\lambda)t) \ge \min[Z(a), Z(b)]$;

$iii)$ Z is upper semicontinuous;

$iv)$ $suppZ = cl\{z \in \mathbb{R}: Z(z) > 0\}$, or denoted by $[Z]^0$, is compact, then it is called a fuzzy number. We denote space of all fuzzy numbers by $L(\mathbb{R})$.

α −level set $[Z]^\alpha$ of a fuzzy number is defined by

$$[Z]^\alpha = \begin{cases} \{z \in \mathbb{R}: Z(z) \geq \alpha\}, & \text{if } \alpha \in (0,1] \\ suppZ, & \text{if } \alpha = 0. \end{cases}$$

It is clear that Z is a fuzzy number if and only if $[Z]^\alpha$ is a closed interval for each $\alpha \in [0,1]$ and $[Z]^1 \neq \emptyset$. We denote space of all fuzzy numbers by $L(\mathbb{R})$. The distance between two fuzzy numbers Z and T, we use the metric

$$d(Z,T) = \sup_{0\leq\alpha\leq1} d_H([Z]^\alpha,[T]^\alpha)$$

Let $Z = \left[\underline{Z}^\alpha, \overline{Z}^\alpha\right]$ and $T = \left[\underline{T}^\alpha, \overline{T}^\alpha\right]$ be two fuzzy numbers. Then, the Hausdorff metric is defined by

$$d_H([Z]^\alpha,[T]^\alpha) = \max\left\{\left|\underline{Z}^\alpha - \underline{T}^\alpha\right|, \left|\overline{Z}^\alpha - \overline{T}^\alpha\right|\right\}.$$

It is known that d is a metric on $L(\mathbb{R})$, and $(L(\mathbb{R}), d)$ is a complete metric space.

P-convergence for double sequences of fuzzy numbers was defined by Savaş [17]. A double sequence $Z = (Z_{mn})$ of fuzzy numbers is a function $Z: \mathbb{N} \times \mathbb{N} \to L(\mathbb{R})$. A double sequence $Z = (Z_{mn})$ fuzzy numbers is said to be convergent to the fuzzy number Z_0 in Pringsheim's sense or P-convergent if for each $\varepsilon > 0$ there exists $N \in \mathbb{N}$ such that $d(Z_{mn}, Z_0) < \varepsilon$ whenever $m,n > N$. The number Z_0 is called Pringsheim limit of Z. We denote by $P - \lim Z = Z_0$.

Let $K \subseteq \mathbb{N} \times \mathbb{N}$ and $K(i,j) = \{(m,n): m \leq i \text{ and } n \leq j\}$. The double natural density of K is defined by

$$\delta^2(K) = P - \lim_{i,j} \frac{1}{ij} |K(i,j)|, \text{ if the limit exists.}$$

Savaş and Mursaleen [18] defined the concept of statistical convergence for double sequences of fuzzy numbers. Let $Z = (Z_{mn})$ be a double sequence of fuzzy numbers. Then the double sequence $Z = (Z_{mn})$ of fuzzy numbers, is said to be statistically convergent to the fuzzy number Z_0 if for each $\varepsilon > 0$

$$\lim_{i,j} \frac{1}{ij} |\{(m,n), m \leq i, n \leq j: d(Z_{mn}, Z_0) \geq \varepsilon\}| = 0.$$

The set of all statistically convergent double sequences of fuzzy numbers will be denoted by $S(F)$. In this case, either $S(F) - \lim Z_{mn} = Z_0$ or $Z_{mn} \to Z_0(S(F))$ is used as a notation.

Aral et al. [19] defined the concept of $\overline{\rho}$ −density as follows:

Let $\rho = (\rho_i)$ and $\varrho = (\varrho_j)$ be two non-decreasing sequence for each $i \in \mathbb{Z}^+$ tending to ∞ such that $\limsup_i \frac{\rho_i}{i} < \infty$, $\Delta\rho_i = O(1)$ and $\Delta\rho_i = \rho_{i+1} - \rho_i$ for each $i \in \mathbb{Z}^+$ and $\limsup_i \frac{\varrho_j}{j} < \infty$, $\Delta\varrho_j = O(1)$ and $\Delta\varrho_j = \varrho_{j+1} - \varrho_j$ for each $j \in \mathbb{Z}^+$. Let $K \subseteq \mathbb{N} \times \mathbb{N}$. The $\overline{\rho}$ −density of K defined by

$$\delta_{\overline{\rho}}(K) = P - \lim_{i,j} \frac{1}{\overline{\rho}_{ij}} |\{m \leq i, n \leq j: (i,j) \in K\}|$$

if the limit exists where $\overline{\rho}_{ij} = \rho_i \varrho_j$.

Çakallı [20] defined the concept of ρ −statistically convergence. Subsequently, many authors have done a great deal of work on ρ- statistical convergence ([21]-[28]). The purpose of this paper is to generalize the study of Çakallı [20] and Savaş and Mursaleen [18].

2. MAIN RESULTS

In this section, we introduce notions of ρ −strongly summable of order α, ρ −statistically convergence of order α and ρ −statistically Cauchy of order α for double sequences of fuzzy numbers. We give some results related to these concepts.

Definition 2.1 Let $Z = (Z_{mn})$ be a double sequence of fuzzy numbers and $\alpha \in (0,1]$. Then a sequence $Z = (Z_{mn})$ of fuzzy numbers is said to be $S_{\bar{\rho}}^{\alpha}(F)$ −statistically convergent to Z_0 (or $\bar{\rho}$ −double statistically convergent of order α to Z_0) if there is a fuzzy number Z_0 such that

$$\lim_{i,j} \frac{1}{\bar{\rho}_{ij}^{\alpha}} |\{m \leq i, n \leq j: d(Z_{mn}, Z_0) \geq \varepsilon\}| = 0$$

for each $\varepsilon > 0$. In this case, either $S_{\bar{\rho}}^{\alpha}(F) - \lim Z_{mn} = Z_0$ or $Z_{mn} \to Z_0 \left(S_{\bar{\rho}}^{\alpha}(F)\right)$ is used as a notation. The set of all $S_{\bar{\rho}}^{\alpha}(F)$ −statistically convergent sequences of fuzzy numbers will be denoted with $S_{\bar{\rho}}^{\alpha}(F)$. If $\bar{\rho}_{ij} = ij$ and $\alpha = 1$, $S_{\bar{\rho}}^{\alpha}(F)$ −statistically convergent is coincide double statistical convergence.

Definition 2.2 Let $Z = (Z_{mn})$ be a double sequence of fuzzy numbers and $\alpha \in (0,1]$. Then a sequence $Z = (Z_{mn})$ of fuzzy numbers is said to be $S_{\bar{\rho}}^{\alpha}(F)$-Cauchy sequence of order α if for each $\varepsilon > 0$, there is $N, M \in \mathbb{N}$, such that for all $m, r \geq N$, $n, s \geq M$

$$\lim_{i,j} \frac{1}{\bar{\rho}_{ij}^{\alpha}} |\{m \leq i, n \leq j: d(Z_{mn}, Z_{rs}) \geq \varepsilon\}| = 0.$$

Definition 2.3 Let $Z = (Z_{mn})$ be a double sequence of fuzzy numbers and $\alpha \in (0,1]$. Then a sequence $Z = (Z_{mn})$ of fuzzy numbers is said to be $S_{\bar{\rho}}^{\alpha}(F)$ −strongly summable of order α to Z_0 if there is a fuzzy number Z_0 such that

$$\lim_{i,j} \frac{1}{\bar{\rho}_{ij}^{\alpha}} \sum_{m=1}^{i} \sum_{n=1}^{j} d(Z_{mn}, Z_0) = 0.$$

for each $\varepsilon > 0$. In this case, either $S_{[\bar{\rho}]}^{\alpha}(F) - \lim Z_{mn} = Z_0$ or $Z_{mn} \to Z_0 \left(S_{[\bar{\rho}]}^{\alpha}(F)\right)$ is used as a notation. The set of all $S_{\bar{\rho}}^{\alpha}(F)$ −strongly summable of order α of fuzzy numbers will be denoted with $\left(S_{[\bar{\rho}]}^{\alpha}(F)\right)$.

Theorem 2.1 Let $\alpha \in (0,1]$ and (Z_{mn}), (T_{mn}) be two double sequences of fuzzy numbers, Then
(i) $Z_{mn} \to Z_0 \left(S_{\bar{\rho}}^{\alpha}(F)\right)$ and $\delta \in \mathbb{C}$ implies $(\delta Z_k) \to \delta Z_0 \left(S_{\rho}(F)\right)$,
(ii) $Z_{mn} \to Z_0 \left(S_{\bar{\rho}}^{\alpha}(F)\right)$and $T_{mn} \to T_0 \left(S_{\bar{\rho}}^{\alpha}(F)\right)$ implies $(Z_{mn} + T_{mn}) \to (Z_0 + T_0) \left(S_{\bar{\rho}}^{\alpha}(F)\right)$.

Proof. (i) For $\delta = 0$, the proof is clear. Let $\delta \neq 0$, the proof follows from the inequality

$$\frac{1}{\bar{\rho}_{ij}^{\alpha}} |\{m \leq i, n \leq j: d(\delta Z_{mn}, \delta Z_0) \geq \varepsilon\}| \leq \frac{1}{\bar{\rho}_{ij}^{\alpha}} \left|\left\{ m \leq i, n \leq j: d(Z_{mn}, Z_0) \geq \frac{\varepsilon}{\delta}\right\}\right|$$

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(ii) Let $Z_{mn} \to Z_0 \left(S_{\overline{\rho}}^{\alpha}(F)\right)$ and $T_{mn} \to T_0 \left(S_{\overline{\rho}}^{\alpha}(F)\right)$, we can write

$$\frac{1}{\overline{\rho}_{ij}^{\alpha}} |\{m \le i, n \le j: d(Z_{mn} + T_{mn}, Z_0 + T_0) \ge \varepsilon\}|$$

$$\le \frac{1}{\overline{\rho}_{ij}^{\alpha}} \left|\left\{m \le i, n \le j: d(Z_{mn}, Z_0) \ge \frac{\varepsilon}{2}\right\}\right| + \frac{1}{\overline{\rho}_{ij}^{\alpha}} \left|\left\{ m \le i, n \le j: d(T_{mn}, T_0) \ge \frac{\varepsilon}{2}\right\}\right|$$

for each $\varepsilon > 0$ and thus if $Z_{mn} \to Z_0 \left(S_{\overline{\rho}}^{\alpha}(F)\right)$ and $T_{mn} \to T_0 \left(S_{\overline{\rho}}^{\alpha}(F)\right)$ then $(Z_{mn} + T_{mn}) \to (Z_0 + T_0) \left(S_{\overline{\rho}}^{\alpha}(F)\right)$.

Theorem 2.2 Let $\alpha \in (0,1]$. If a double sequence (Z_{mn}) of fuzzy numbers is $S_{\overline{\rho}}^{\alpha}(F)$ $-$strongly summable of order α to Z_0, then the double sequence is $\overline{\rho}$ $-$double statistically convergent of order α to Z_0.

Proof. Let $I_{ij} = \{(m,n), m \le i, n \le j: d(Z_{mn}, Z_0) \ge \varepsilon\}$.

$$\frac{1}{\overline{\rho}_{ij}^{\alpha}} \sum_{m=1}^{i} \sum_{n=1}^{j} d(Z_{mn}, Z_0) = \frac{1}{\overline{\rho}_{ij}^{\alpha}} \left\{ \sum_{(m,n)\in I_{ij}}^{j} d(Z_{mn}, Z_0) + \sum_{(m,n)\notin I_{ij}}^{j} d(Z_{mn}, Z_0) \right\}$$

$$\ge \frac{1}{\overline{\rho}_{ij}^{\alpha}} \left\{ \sum_{(m,n)\in I_{ij}}^{j} d(Z_{mn}, Z_0) \right\}$$

$$\ge \frac{1}{\overline{\rho}_{ij}^{\alpha}} |\{m \le i, n \le j: d(Z_{mn}, Z_0) \ge \varepsilon\}|.$$

Hence Z is $\overline{\rho}$ $-$double statistically convergent of order α to Z_0.

Theorem 2.3 Let $\alpha \in (0,1]$. If a double sequence (Z_{mn}) of fuzzy numbers is bounded and this double sequence is $\overline{\rho}$ $-$double statistically convergent of order α to Z_0, then this double sequence is $S_{\overline{\rho}}^{\alpha}(F)$ $-$strongly summable of order α to Z_0.

Theorem 2.4 A double sequence (Z_{mn}) of fuzzy numbers is $S_{\overline{\rho}}^{\alpha}(F)$ $-$convergent a necessary and sufficient condition is that (Z_{mn}) is an $S_{\overline{\rho}}^{\alpha}(F)$ $-$Cauchy sequence.

Proof. Suppose that $Z = (Z_{mn})$ is is $S_{\overline{\rho}}^{\alpha}(F)$ $-$convergent. For every $\varepsilon > 0$, we can write

$$\frac{1}{\overline{\rho}_{ij}^{\alpha}} |\{m \le i, n \le j: d(Z_{mn}, Z_{rs}) \ge \varepsilon\}| \le \frac{1}{\overline{\rho}_{ij}^{\alpha}} \left|\left\{m \le i, n \le j: d(Z_{mn}, Z_0) \ge \frac{\varepsilon}{2}\right\}\right|$$

$$+ \frac{1}{\overline{\rho}_{ij}^{\alpha}} \left|\left\{m \le i, n \le j: d(Z_0, Z_{rs}) \ge \frac{\varepsilon}{2}\right\}\right|.$$

Since $S_{\overline{\rho}}^{\alpha}(F) - \lim Z_{mn} = Z_0$, we get

$$\lim_{i,j}\frac{1}{\overline{\rho}_{ij}^{\alpha}}\left|\{m\leq i,n\leq j: d(Z_{mn},Z_{rs})\geq\varepsilon\}\right|\leq \lim_{i,j}\frac{1}{\overline{\rho}_{ij}^{\alpha}}\left|\left\{m\leq i,n\leq j: d(Z_{mn},Z_0)\geq\frac{\varepsilon}{2}\right\}\right|$$

$$+\lim_{i,j}\frac{1}{\overline{\rho}_{ij}^{\alpha}}\left|\left\{m\leq i,n\leq j: d(Z_0,Z_{rs})\geq\frac{\varepsilon}{2}\right\}\right|$$

Hence, Z is an $S_{\overline{\rho}}^{\alpha}(F)$ −Cauchy sequence.
The proof to the contrary is obvious.

Theorem 2.5 If $\frac{\rho_i^{\alpha}}{i}\geq 1$ and $\frac{\varrho_j^{\alpha}}{j}\geq 1$ for all i,j ∈ ℕ, then $S(F)\subset S_{\overline{\rho}}^{\alpha}(F)$.

Proof. Suppose that $S(F)-\lim Z_{mn}=Z_0$, the proof is obtained from the following inequality, for every $\varepsilon>0$

$$\frac{1}{ij}\left|\{m\leq i \text{ and } n\leq j: d(Z_{mn},Z_0)\geq\varepsilon\}\right| = \frac{\overline{\rho}_{ij}^{\alpha}}{ij}\frac{1}{\overline{\rho}_{ij}^{\alpha}}\{m\leq i \ and \ n\leq j: d(Z_{mn},Z_0)\geq\varepsilon\}$$

$$\geq\frac{1}{\overline{\rho}_{ij}^{\alpha}}\{m\leq i \ and \ n\leq j: d(Z_{mn},Z_0)\geq\varepsilon\}$$

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Z_4 −IMAGES OF CYCLIC CODES OVER FINITE RING R

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ABSTRACT

Within the scope of our study, we interest with Z_4 −images of cyclic codes over finite rings and explore their algebraic structures. We also deduce the general form of the generator of a cyclic code and find its minimal spanning sets. By considering Gray images of cyclic codes over R, we obtain new linear codes over Z_4.

Keywords: Cyclic Codes, Codes Over Finite Rings, Z_4 −Cyclic Codes

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SOME APPLICATIONS OF MULTIVARIATE PADÉ APPROXIMANT METHOD

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Abstract

In applied sciences, mathematical models generally appear as differential-algebraic equations or partial differential algebraic equations. Therefore, the search for approximate solutions of differential-algebraic and partial differential-algebraic equations has contributed greatly to the development of mathematical modeling theory and led to the introduction of many approximate solution methods. One of the approximate solution methods used to solve differential equations is the Padé Approach. The history of Padé approximation dates back to the 18th century. The method, which was first used by Henri Eugune Padé (1863-1953), has been used by many famous mathematicians to solve various problems until today. In the univariate Padé approach, the approximate solution function tried to be found is assumed to be equal to the ratio of two polynomials whose coefficients are unknown. Then, the coefficients of the polynomial in the numerator and denominator are found with the help of the coefficients in the Taylor series expansion. It is known that the Padé approach generally provides a better approximation than the truncated Taylor series of the function. In the Multivariate Padé Approach, it appears that the features used for univariate Padé approaches continue to be valid. Moreover, although many features of univariate Padé approaches have been clarified, studies on multivariate models have remained quite limited . Therefore, in our study, we will use the Padé approach for twovariable functions.

Keywords : Padé Approximation, Multivariate Padé Approximation, Partial differential equations,

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Properties of Permuting Tri f −derivation on Lattices

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ABSTRACT

The concept of derivation, which is one of the basic concepts of analysis and applied mathematics, has been transferred to various algebraic structures such as BCK-algebras, subtraction algebras, d-algebras, bitonic algebras after the ring theory in algebra. One of these algebraic structures is lattices. Let L be a lattice and $d: L \to L$ be a function. If the conditions $d(x \wedge y) = (d(x) \wedge y) \vee (x \wedge d(y))$ and $d(x \vee y) = d(x) \vee d(y)$ are satisfied for all $x, y \in L$, then d is called derivation on L. It has been shown that the second condition in this definition is already met for isotone derivations in distributive lattices, and only the first condition is taken into account when defining the derivation. The authors focused on the fixed points of the derivations on the lattices and connected between this set and the lattice ideals. With the help of derivations, it was investigated whether the lattices were distributive or modular. After this study, the generalized derivation, symmetric bi-derivation, permuting tri-derivation and many other derivation types defined on rings were transferred to lattices and the properties of lattices were examined using these derivation types.

Let L be a lattice and $D: L \times L \times L \to L$ be a permuting function. If the equality $D(x \wedge w, y, z) = (D(x, y, z) \wedge w) \vee (x \wedge D(w, y, z))$ is satisfied for all $x, y, z, w \in L$, then D is called permuting tri –derivation on L. Using this definition, a permuting tri f-derivation on L was defined.

Let L be a lattice and $D: L \times L \times L \to L$ be a permuting tri-derivation. If there exists a function $f: L \to L$ such that $D(x \wedge w, y, z) = (D(x, y, z) \wedge f(w)) \vee (f(x) \wedge D(w, y, z))$ for all $x, y, z, w \in L$, then D is called a permuting tri f-derivation on L.

Let L be a lattice and $D: L \times L \times L \to L$ be a permuting tri f-derivation. If for all $x, y, z, w \in L$, $x \leq w$ implies $D(x, y, z) \leq D(w, y, z)$, then D is called isotone permuting tri f-derivation.

Our aim here is to investigate the connections between D and f on the lattice L. In addition, when L lattice is distributive lattice or modular lattice, if D is isotone permuting tri f-derivation, then it is to investigate what properties D provides.

Keywords : Lattice, derivation, permuting tri-derivation, permuting tri f-derivation

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Note: This paper is included in Hilal Ünal's master's thesis prepared at Sivas Cumhuriyet University, Institute of Science and Technology.

LATİSLERDE PERMUTING ÜÇLÜ f −TÜREVLERİN ÖZELLİKLERİ

ÖZET

Analiz ve uygulamalı matematiğin temel kavramlarından biri olan türev kavramı cebirde halka teoriden sonra BCK-cebirleri, fark cebirleri, d-cebirler, bitonik cebirler gibi çeşitli cebirsel yapılara da taşınmıştır. Bu cebirsel yapılardan biri de latislerdir. L bir latis, $d: L \to L$ bir fonksiyon olsun. Her $x, y \in L$ için $d(x \wedge y) = (d(x) \wedge y) \vee (x \wedge d(y))$ ve $d(x \vee y) = d(x) \vee d(y)$ koşulları sağlanıyorsa d ye L üzerinde türevdir, denir. Bu tanımdaki ikinci koşulun dağılmalı latislerde izoton türevler için zaten sağlandığı gösterilmiş olup türev tanımı yapılırken sadece birinci koşul dikkate alınmıştır. Latisler üzerinde türevlerin sabit noktaları üzerinde yoğunlaşılmış ve bu kümeyle latis idealleri arasında bağlantı kurulmuştur. Türev yardımıyla latislerin dağılmalı veya modüler olup olmadığı araştırılmıştır. Bu çalışmadan sonra halkalar üzerinde tanımlanan genelleştirilmiş türev, simetrik ikili türev, permuting üçlü türev ve daha birçok türev çeşidi latisler üzerine taşınmış ve latislerin özellikleri bu türev çeşitleri kullanılarak incelenmiştir.

L bir latis, $D: L \times L \times L \to L$ bir permuting fonksiyon olsun. Her $x, y, z, w \in L$ için $D(x \wedge w, y, z) = (D(x, y, z) \wedge w) \vee (x \wedge D(w, y, z))$ eşitliği sağlanıyorsa Dye L üzerinde permuting üçlü türev denir. Bu tanımdan faydalanılarak L latisi üzerinde permuting üçlü f-türev tanımlanmıştır.

L bir latis, $D: L \times L \times L \to L$ permuting üçlü türev olsun. Her $x, y, z, w \in L$ için $D(x \wedge w, y, z) = (D(x, y, z) \wedge f(w)) \vee (f(x) \wedge D(w, y, z))$ olacak biçimde $f: L \to L$ fonksiyonu varsa D ye L üzerinde permuting üçlü f-türev denir.

L latisi üzerinde D permuting üçlü f-türev olsun. Eğer $x, y, z, w \in L$ için $x \leq w$ iken $D(x, y, z) \leq D(w, y, z)$ ise D ye L üzerinde izoton permuting üçlü f-türev denir.

Bizim burada amacımız L latisi üzerinde D ile f arasındaki bağlantıları araştırmaktır. Ayrıca L latisi dağılmalı latis veya modüler latis olduğunda D permuting üçlü f-türevi izoton ise D nin hangi özellikleri sağladığını araştırmaktır.

Anahtar Kelimeler : Latis, türev, permuting üçlü türev, permuting üçlü f-türev

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CHATTERJEA TYPE CONTRACTION MAPPINGS FOR RECTANGULAR SOFT B-METRIC SPACES

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ABSTRACT

Soft set theory contains alternative tools for state mathematical problems with different viewpoint. The theory is studied by many mathematicians, especially topologist in recent years, so it has lots of applications in different branches of mathematics, engineering, computer science, medical and social science, economics, etc. On the other hand, fixed point theory is one of the rapidly developing subjects of mathematics for years. Various metrics such as soft metric, fuzzy metric, rectangular metric, b-metric, partial metric, s-metric, etc. and different contraction mappings such as Kannan Type, Chatterjea Type, Caristi Type, Jungck Type, Meir-Keeler, F-contraction, cyclic contraction etc. have been studied by mathematicians and many new fixed point theorems have been introduced to the literature. In recent years, new fixed point theorems are expressed and proven by considering soft set theory and fixed point theory together. Thus, a sub-branch called soft fixed point theory began to develop.

In this paper, at first we recall some basic definitions and properties deal with soft set theory and soft rectangular metric spaces. We attach importance to working with new tools in fixed point theory. So we consider the rectangular soft b-metric space and we restated the definition of rectangular soft be metric by using soft elements and soft parametric scalars. We focuse the Chatterjea type contraction mapping for rectangular b-metric space and we prove some soft fixed point theorems. We draw attention to the role of soft rectangular space inequality in theorems. Hence new results are obtained in rectangular soft b-metric spaces. Also an interesting example is presented as an application.

Anahtar Kelimeler : Soft Metric, Soft Rectangular b-Metric, Chatterjea Type Contraction.

1. Preliminaries

Definition 1.1: [8] A pair (F, A) is said to be a soft set over the universe U , where F is a mapping given by $F : A \to P(U)$. In other words, a soft set over U, is a parameterized family of subsets of the universe U. For any parameter $x \in A, F(x)$ may be considered as the set of $x-$ approximate elements of the soft set (F, A).

Definition 1.2: [7] Let (F,A) and (G,B) be two soft sets over U. We say that (F,A) is a soft subset of (G,B) and denote it by $(F,A)\tilde{\subseteq}(G,B)$ if
(1) $A\subseteq B$, and
(2) $F(a)\subseteq G(a)$, for all $a\in A$.
(F,A) is said to be a soft super set of (G,B), if (G,B) is a soft subset of (F,A). We denote it by $(F,A)\tilde{\supseteq}(G,B)$.

Definition 1.3: [7] Let (F,A) and (G,B) be two soft sets over U. The union of (F,A) and (G,B) is soft set (H,C), where $c=A\cup B$ and for all $e\in C$,

$$f(x)=\begin{cases} F(e), & \text{e} \in \text{A-B} \\ G(e), & \text{e} \in \text{B-A} \\ F(e)\cup G(e), & \text{e} \in \text{A} \cap \text{B} \end{cases}$$

We write $(F,A)\tilde{\cup}(G,B)=(H,C)$.

Definition 1.4: [7] Let (F,A) and (G,B) be two soft sets over U. The intersection of (F,A) and (G,B) is a soft set (H,C), where $C=A\cap B$ and for all $e\in C$, $H(e)=F(e)\cap G(e)$. We write $(F,A)\tilde{\cap}(G,B)=(H,C)$.

Definition 1.5: [7] Let (F,A) be soft set over U. The relative complement of (F,A) is denoted by $(F,A)^c$ and is defined $(F,A)^c=(F^c,A)$, where $F^c:A\to P(U)$ is a mapping given by $F^c(a)=U-F(a)$ for all $a\in A$.

Definition 1.6: [7] Let (F,A) and (G,B) be two soft sets over U. Then $(F,A)-(G,B)$ is a soft set $(x,F(x)):F(x)\notin G(B),x\notin B$.

Definiton 1.7: [7] Let (F,A) be soft set over U. Then
(1) (F,A) is said to be null soft set denoted by $\tilde{\varnothing}$ if for every $a\in A, F(a)=$ null set $\varnothing$.
(2) (F,A) is said to be absolute soft set denoted by $\tilde{A}$ if for every $a\in A, F(a)=U$.

Definition 1.8: [6] Let $A\subseteq E$ be a set of parameters. We say the ordered pair (a,r) is a soft parametric scalar if $r\in\mathbb{R}$ and $a\in A$. The parametric scalar (r,a) called nonnegative if $r\geq 0$. Let (a,r) and (b,r') be two soft parametric scalars. We say (a,r) is no less than (b,r') and we write $(a,r)\pm(b,r')$, if $r\geq r'$.

Definition 1.9: [6] Let $A\subseteq E$ be a set of parameters. Let (a,r) and (b,r') be two soft parametric scalars. Then we define addition between soft parametric scalars and scalar multiplication on soft parametric scalars as follows $(a,r)\hat{e}\,(b,r')=(\{a,b\},r+r')$, and $\lambda(a,r)=(a,\lambda r)$, for every $\lambda\in\mathbb{R}$.

Definition 1.10: [6] Let (F,A) be a soft set over X. We call a function f on (F,A) is parametric scalar valued, if there are functions $f_1:A\to A$ and $f_2:F(A)\to\mathbb{R}$ such that $f(F,A)=(f_1,f_2)(A,F(A))$.

Similarly, we can extent above defined parametric scalar valued function as $f:(F,A)\times(F,A)\to(A,R)$ by $f(A\times A,F(A)\times F(A))=(f_1,f_2)(A\times A,F(A)\times F(A))$, where $f_1:A\times A\to A$ and $f_2:F(A)\times F(A)\to\square$.

Definition 1.11: [6] Let (F,A) be a soft set over X and let $\tilde{\pi}:A\times A\to A$ be parametric function. We say the parametric scalar valued function $\mathcal{D}:(F,A)\times(F,A)\to(A,\square^{+}\{0\})$ a soft metric on (F,A) if $\mathcal{D}$ satisfies in the following conditions:

$(\mathbf{1})\mathcal{D}((a,F(a)),(b,F(b))) \quad \pm \quad (\tilde{\pi}(a,b),0), \textit{and equality holds, whenever } a=b.$

$(\mathbf{2})\mathcal{D}((a,F(a)),(b,F(b))) \quad = \quad \mathcal{D}((b,F(b)),(a,F(a))), \textit{ for all } a,b\in A.$

$(\mathbf{3})\mathcal{D}((a,F(a)),(c,F(c))) \quad \preceq \quad \mathcal{D}((a,F(a)),(b,F(b)))\,\hat{e}\,\mathcal{D}((b,F(b)),(c,F(c))), \textit{ for all } a,b,c\in A.$

We say the pair $((F,A),\mathcal{D})$ is a soft metric space over X.

Definition 1.12: [4] Assume X is a not null set and $d:X\times X\to[0,\infty]$ satisfy the following conditions for all $x,y\in X$ and all distinct $u,v\in X$ each of which is dissimilar from x and y.

i. d(x,y)=0 $\Leftrightarrow$ x=y,
ii. d(x,y)=d(y,x)
iii. d(x,y)$\le$ d(x,u)+d(u,v)+d(v,y).

is named a rectangular metric and the pair (X,d) is named a rectangular metric spaces.(RMS)

Definition 1.13: [3] $X\neq\varnothing$ and an element $x\in X$ is a fixed point of $f:X\to X$ if $f(x)=x.$

Definition 1.14: [3] Let T be a mapping of a metric space M into M. We say that T is a contraction mapping if there exists a number k such that $0<k<1$ and $p(Tx,Ty)\le kp(x,y)$ $(\forall x,y\in M)$.

Definition 1.15: [3] Any contraction mapping of a complete non-empty metric space M into M has a unique fixed point in M.

Definition 1.16: [13] Let $\tilde{\pi}:A\times A\to A$ be a scalar valued parametric function. The parametric scalar valued function $\mathcal{D}_R:(F,A)\times(F,A)\to(A,R^{+}\cup\{0\})$ is called to be a rectangular soft metric on (F,A) if $\mathcal{D}_R$ satisfies the following conditions:

(**RSM1**) $\mathcal{D}_R((a,F(a)),(b,F(b)))\ge(\tilde{\pi}(a,b),0)$, and equality holds whenever $a=b$.

(**RSM2**) $\mathcal{D}_R((a,F(a)),(b,F(b)))=(\tilde{\pi}(a,b),0)\Leftrightarrow$ for all $((a,F(a)),(b,F(b)))\in(F,A)$, (a,F(a))=(b,F(b)) [for all $a,b\in A, a=b$]

(**RSM3**) $\mathcal{D}_R((a,F(a)),(b,F(b)))=\mathcal{D}_R((b,F(b)),(a,F(a)))$, for all $a,b\in A$

(**RSM4**) $\mathcal{D}_R((a,F(a)),(b,F(b)))\le\mathcal{D}_R((a,F(a)),(c,F(c)))+\mathcal{D}_R((c,F(c)),(d,F(d)))$

$$+\mathcal{D}_R((d,F(d)),(b,F(b)))$$

for all $a,b,c,d\in A$

Then we say the pair $((F,A),\mathcal{D}_R)$ is a rectangular soft metric space over X.

Definition 1.17: [13] Let (F,A) be a soft set over \$X.\$ A soft sequence in (F,A) is a function $f:N\rightarrow(F,A)$ by setting $f(n)=(F_n,A)$ such that (F_n,A) is a soft subset of (F,A) for $n\in N,$ and we denote it by $\{(F_n,A)\}_{n=1}^{\infty}.$

Definition 1.18: [13] Let (F,A) be a soft set over X. Let $\mathcal{D}_R$ be a rectangular soft metric on $(F,A),$ $\{(F_n,A)\}_{n=1}^{\infty}$ be a soft sequence in (F,A) and $(x,F(x))\in(F,A).$ Then we say $\{(F_n,A)\}_{n=1}^{\infty}$ converges to $(x,F(x)),$ if for every positive number $\epsilon,$ there exists a naturel number N such that for every natural number n which $n\geq N,$ we have
$\mathcal{D}_R((a,F_n(a)),(x,F(x)))\leq(\tilde{\pi}(a,x),\epsilon).$

Definition 1.19: [13] Let (F,A) be a soft set over X. Let $\mathcal{D}_R$ be a rectangular soft metric on (F,A) and $\{(F_n,A)\}_{n=1}^{\infty}$ be a soft sequence in $(F,A).$ Then we say $\{(F_n,A)\}_{n=1}^{\infty}$ is a Cauchy soft sequence, if for every positive number $\epsilon,$ there exists a natural number N such that for every natural number n,m which $n,m\geq N,$ we have $\mathcal{D}_R((a,F_n(a)),(a,F_m(a)))\leq(\tilde{\pi}(a,a),\epsilon).$

Proposition 1.20: [13] Let $((F,A),\mathcal{D}_R)$ be a rectangular soft metric space over X, and let $\{(F_n,A)\}_{n=1}^{\infty}$ be a convergent soft sequence in $(F,A).$ Then $\{(F_n,A)\}_{n=1}^{\infty}$ is a Cauchy soft sequence.

Theorem 1.21: [13] Let (F,A) be a soft set over X, let $\mathcal{D}_R$ be a meter on (F,A) and $(F_n,A)_{n=1}^{\infty}$ be a soft sequence in (F,A). If $(F_n,A)_{n=1}^{\infty}$ is convergent in (F,A), then it converges to unique element of (F,A).

Definition 1.22: [13] Let (F,A) be a soft set over X, let $\mathcal{D}_R$ be a rectangular soft metric on $(F,A).$ We say that (F,A) is a complete rectangular soft metric space if every Cauchy soft sequence converges in $(F,A).$

Theorem 1.23: [13] Let $((F,A),\mathcal{D}_R)$ and $((F',A'),\mathcal{D}'_R)$ be two rectangular soft metric spaces over X and Y respectively. Let $f=(f_1,f_2):((F,A),\mathcal{D}_R)\rightarrow((F',A'),\mathcal{D}'_R)$ be a soft mapping. Then f is soft continuous if and only if for every $(x,F(x))\in(F,A)$ and every positive number $\epsilon,$ there exists a positive number δ such that for every $(y,F(y))\in(F,A)$

$$\mathcal{D}'_R((f(x,F(x))),f((y,F(y))))\leq(\tilde{\pi}'(\tilde{\pi}(x,y)),\epsilon)\ \textit{whenever}$$
$$\mathcal{D}_R((x,F(x)),(y,F(y)))\leq(\tilde{\pi}(x,y)),\delta).$$

Definition 1.24: [13] Let $((F,A),\mathcal{D}_R)$ be a rectangular soft metric space over X and $f:((F,A),\mathcal{D}_R)\rightarrow((F,A),\mathcal{D}_R)$ be a soft mapping. We say that f is soft contractive if there is a positive number c with $0<c<1$ such that
$\mathcal{D}_R((f(x,F(x))),f((y,F(y))))\leq c\mathcal{D}_R((x,F(x)),(y,F(y))),$ for all $x,y\in A.$

Theorem 1.25: [13] Soft contractive mapping is soft continuous in rectangular soft metric space $((F,A),\mathcal{D}_R)$.

Definition 1.26: [13] Let $((F,A),\mathcal{D}_R)$ be a complete rectangular soft metric space over X and let $f:((F,A),\mathcal{D}_R)\to((F,A),\mathcal{D}_R)$ be a soft mapping. A fixed soft set for f is a soft subset of (F,A) such as $(x,F(x))$ such that $f((x,F(x)))=(x,F(x))$.

Theorem 1.27: [13] **(Banach Contraction Theorem for Rectangular Soft Metric Space)**
Let $((F,A),\mathcal{D}_R)$ be a complete rectangular soft metric space over X
$f:((F,A),\mathcal{D}_R)\to((F,A),\mathcal{D}_R)$ be a rectangular soft contractive mapping. Then f has unique fixed soft set.

Definition 1.28: [12] Let (X,d) be a metric space. Chatterjea proved the existence and uniqueness of fixed points for mappings $T:X\to X$ satisfying

$$d(Tx,Ty)\le\alpha[d(x,Ty)+d(y,Tx)],\quad \forall x,y\in X,\alpha\in[0,\frac{1}{2}].$$

This contraction is known as Chatterjea Contraction.

Definition 1.29: [15] Let X ibe a nonempty set and th mapping $d:X\times X\to[0,\infty)$ satisfies: the following conditions for all $x,y\in X$ and all distinct $u,v\in X$ each of which is dissimilar from x and y.

i. d(x,y)=0 $\Leftrightarrow$ x=y;
ii. d(x,y)=d(y,x) for all $x,y\in X$;
iii. There exists a real number $s\ge1$ such that d(x,y)$\le$ s[d(x,u)+d(u,v)+d(v,y)] for all $x,y\in X$ and all distinct points $u,v\in X\setminus\{u,v\}$.

Then d is called a rectangular b-metric on X and (X,d) is called a rectangular b-metric with coefficient s.

2. CHATTERJEA TYPE CONTRACTION MAPPINGS FOR RECTANGULAR SOFT B-METRIC SPACES

Definition 2.1: Let $\tilde{\pi}:A\times A\to A$ be a scalar valued parametric function. The parametric scalar valued function $\mathcal{D}_R:(F,A)\times(F,A)\to(A,R^+\cup\{0\})$ is called to be a rectangular soft b-metric on (F,A) if $\mathcal{D}_R$ satisfies the following conditions:

(**RSM1**) $\mathcal{D}_R((a,F(a)),(b,F(b)))\ge(\tilde{\pi}(a,b),0)$, and equality holds whenever $a=b$.

(**RSM2**) $\mathcal{D}_R((a,F(a)),(b,F(b)))=(\tilde{\pi}(a,b),0)\Leftrightarrow$ for all $((a,F(a)),(b,F(b)))\in(F,A)$, (a,F(a))=(b,F(b)) [for all $a,b\in A, a=b$].

(**RSM3**) $\mathcal{D}_R((a,F(a)),(b,F(b)))=\mathcal{D}_R((b,F(b)),(a,F(a)))$, for all $a,b\in A$.

(**RSM4**) There exists a real number $s\ge1$ such that

$$\mathcal{D}_R((a,F(a)),(b,F(b))) \le s[\mathcal{D}_R((a,F(a)),(c,F(c)))+\mathcal{D}_R((c,F(c)),(d,F(d)))$$
$$+\mathcal{D}_R((d,F(d)),(b,F(b)))]$$

for all $a,b\in A$ and all distinct points $c,d\in A/\{a,b\}$.

Then we say the pair $((F,A),\mathcal{D}_R)$ is a rectangular soft b-metric space over X.

Theorem 2.2: Let $((F,E),\mathcal{D}_R)$ be a complete rectangular soft b-metric space and $T:((F,E),\mathcal{D}_R)\to((F,E),\mathcal{D}_R)$ be a Chatterjea mapping with a constant $\alpha\in[0,\frac{1}{s+1}]$, where $s\ge 1$. Then

$$\mathcal{D}_R(T^n(x,F(x)),T^{n+1}(x,F(x)))\le(\frac{\alpha}{1-\alpha})^n\mathcal{D}_R((x,F(x)),T(x,F(x))) \qquad \textbf{(1)}$$

for all $(x,F(x))\in(F,E)$ and $n\ge 0$. In particular $\mathcal{D}_R(T^n(x,F(x)),T^{n+1}(x,F(x)))\to 0$ as $n\to\infty$.

Proof: Let $(x,F(x))\in(F,E)$. Then $((x,F(x)),T(x,F(x)))\in((F,E),\mathcal{D}_R)$.

$((x,F(x)),T(x,F(x)))\in((F,E),\mathcal{D}_R)\Rightarrow(T(x,F(x)),T^2(x,F(x)))\in((F,E),\mathcal{D}_R)$.

Therefore, in general $(T\text{^\{^}n\}(x,F(x)),T^{n+1}(x,F(x)))\in((F,E),\mathcal{D}_R)\ \forall n\ge 0$.

If $n=0$, **(1)** is trivally true.

Consider

$$\mathcal{D}_R(T^n(x,F(x)),T^{n+1}(x,F(x)))\le\alpha[\mathcal{D}_R(T^{n-1}(x,F(x)),T^{n+1}(x,F(x)))$$
$$+\mathcal{D}_R(T^n(x,F(x)),T^n(x,F(x)))]$$
$$=\alpha[\mathcal{D}_R(T^{n-1}(x,F(x)),T^{n+1}(x,F(x)))+0]$$
$$\le\alpha[\mathcal{D}_R(T^{n-1}(x,F(x)),T^n(x,F(x)))+\mathcal{D}_R(T^n(x,F(x)),T^{n+1}(x,F(x)))].$$

$$(1-\alpha)\mathcal{D}_R(T^n(x,F(x)),T^{n+1}(x,F(x)))\le\alpha\mathcal{D}_R(T^{n-1}(x,F(x)),T^n(x,F(x))).$$

$$\mathcal{D}_R(T^n(x,F(x)),T^{n+1}(x,F(x)))\le\frac{\alpha}{1-\alpha}\mathcal{D}_R(T^{n-1}(x,F(x)),T^n(x,F(x))).$$

Using induction,

$$\mathcal{D}_R(T^n(x,F(x)),T^{n+1}(x,F(x)))\le(\frac{\alpha}{1-\alpha})^n\mathcal{D}_R((x,F(x)),T(x,F(x))).$$

Here $\alpha\in[0,\frac{1}{s+1}]\Rightarrow\frac{\alpha}{1-\alpha}\le\frac{1}{s}$.

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$$\mathcal{D}_R(T^n(x,F(x)),T^{n+1}(x,F(x))) \le (\frac{1}{s})^n \mathcal{D}_R((x,F(x)),T(x,F(x))),$$

$$(\frac{1}{s})^n \to 0 \text{ as } n \to \infty.$$

Hence, we have, $\mathcal{D}_R(T^n(x,F(x)),T^{n+1}(x,F(x))) \to 0$ as $n \to \infty$.

Theorem 2.3: Let $((F,E),\mathcal{D}_R)$ be a complete rectangular soft b-metric space. Let $T:((F,E),\mathcal{D}_R) \to ((F,E),\mathcal{D}_R)$ be a mapping satisfying

$$\mathcal{D}_R(T(x,F(x)),T(y,F(y))) \le \alpha[\mathcal{D}_R((x,F(x)),T(y,F(y))) + \mathcal{D}_R((y,F(y)),T(x,F(x)))]$$

for all $(x,F(x)),(y,F(y)) \in (F,E)$ $\alpha \in [0,\frac{1}{s+1}]$. Then T has a unique fixed point.

Proof: If $((F,E),\mathcal{D}_R) = \varnothing$ then there is nothing to prove.

If not, let $(x,F(x)) \in (F,E)$ then $((x,F(x)),T(x,F(x))) \in ((F,E),\mathcal{D}_R)$.

Therefore, $(T(x,F(x)),T^2(x,F(x))) \in ((F,E),\mathcal{D}_R)$ i.e. $T(x,F(x)) \in (F,E)$.

$$\begin{aligned}\mathcal{D}_R(T^n(x,F(x)),T^m(x,F(x))) &\le \mathcal{D}_R(T^n(x,F(x)),T^{n+1}(x,F(x))) \\ &+ \mathcal{D}_R(T^{n+1}(x,F(x)),T^{n+2}(x,F(x))) \\ &+ \dots + \mathcal{D}_R(T^{m-1}(x,F(x)),T^m(x,F(x))) \\ &\le \lambda^n \mathcal{D}_R((x,F(x)),T(x,F(x))) + \lambda^{n+1}\mathcal{D}_R((x,F(x)),T(x,F(x))) \\ &+ \dots + \lambda^{m-1}\mathcal{D}_R((x,F(x)),T(x,F(x))) \\ &\le \lambda^n(1+\lambda+\lambda^2+\dots)\mathcal{D}_R((x,F(x)),T(x,F(x))) \\ &= \frac{\lambda^n}{1-\lambda}\mathcal{D}_R((x,F(x)),T(x,F(x))),\end{aligned}$$

where $\lambda = \frac{\alpha}{1-\alpha} < \frac{1}{s} < 1$ (since $s > 1$).

$\Rightarrow \mathcal{D}_R(T^n(x,F(x)),T^m(x,F(x))) \to 0$ as $m,n \to \infty$.

$\{T^n(X,F(x))\}$ is a soft Cauchy sequence in (F,E). But (F,E) is soft complete. Therefore this sequence converges in (F,E). i.e. $T^n(x,F(x)) \to (x^*,F(x^*))$ as $n \to \infty$.

We shall now prove that $(x^*,F(x^*))$ is the fixed point of T.

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$$\begin{aligned} D_R((x^*, F(x^*)), T(x^*, F(x^*))) &\le s[D_R((x^*, F(x^*)), T^n(x, F(x))) \\ &\quad + D_R(T^n(x, F(x)), T^{n+1}(x, F(x))) + D_R(T^{n+1}(x, F(x)), T(x^*, F(x^*)))] \\ &\le s[D_R((x^*, F(x^*)), T^n(x, F(x))) + D_R(T^n(x, F(x)), T^{n+1}(x, F(x))) \\ &\quad + \alpha[D_R(T^n(x, F(x)), T(x^*, F(x^*))) + D_R((x^*, F(x^*)), T^{n+1}(x, F(x)))]] \\ &\le s[D_R((x^*, F(x^*)), T^n(x, F(x))) + \lambda^n D_R((x, F(x)), T(x, F(x))) \\ &\quad + \alpha[D_R(T^n(x, F(x)), T(x^*, F(x^*))) + D_R((x^*, F(x^*)), T^{n+1}(x, F(x)))]] \\ &\le s D_R((x^*, F(x^*)), T^n(x, F(x))) + s\lambda^n D_R((x, F(x)), T(x, F(x))) \\ &\quad + s\alpha[D_R(T^n(x, F(x)), T(x^*, F(x^*))) + D_R((x^*, F(x^*)), T^{n+1}(x, F(x)))]]. \end{aligned}$$

As $n \to \infty, D_R(T^n(x, F(x)), T(x^*, F(x^*))) \to D_R((x^*, F(x^*)), T(x^*, F(x^*)))$.

In the right side $D_R((x^*, F(x^*)), T(x, F(x))) \to 0$ and $D_R((x^*, F(x^*)), T^{n+1}(x, F(x))) \to 0$ as $n \to \infty$.

Also $\lambda^n \to 0$ as $n \to \infty$ (since $\lambda = \dfrac{\alpha}{1-\alpha} < \dfrac{1}{s} < 1$.)

$(1 - s\alpha) D_R((x^*, F(x^*)), T(x^*, F(x^*))) \to 0$ as $n \to \infty$.

$D_R((x^*, F(x^*)), T(x^*, F(x^*))) \to 0$ as $n \to \infty$.

$T(x^*, F(x^*)) = (x^*, F(x^*))$.

i.e. $(x^*, F(x^*))$ is a fixed point of T.

Now we prove that the fixed point is unique.

If possible, let (y*F(y*))$ be another fixed point of T. i.e. $T(y^*, F(y^*)) = (y^*, F(y^*))$.

Consider

$$\begin{aligned} D_R((x^*, F(x^*)), (y^*, F(y^*))) &= D_R(T(x^*, F(x^*)), T(y^*, F(y^*))) \\ &\le \alpha[D_R((x^*, F(x^*)), T(y^*, F(y^*))) + D_R((y^*, F(y^*)), (x^*, F(x^*)))]. \end{aligned}$$

$$D_R((x^*, F(x^*)), (y^*, F(y^*))) \le \alpha[D_R((x^*, F(x^*)), (y^*, F(y^*))) + D_R((y^*, F(y^*)), (x^*, F(x^*)))].$$

$$D_R((x^*, F(x^*)), (y^*, F(y^*))) \le 2\alpha D_R((x^*, F(x^*)), (y^*, F(y^*))).$$

$$\Rightarrow \alpha \ge \frac{1}{2}.$$

This is a contradiction since $\alpha \in [0, \dfrac{1}{s+1}]$ where $s > 1$.

$s > 1 \Rightarrow \dfrac{1}{s+1} < \dfrac{1}{2}$ i.e. $\alpha < \dfrac{1}{2}$.

The fixed point of T is unique.

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Existence Result for p- Laplacian Fractional Boundary Value Problems

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ÖZET (Abstract)

In this study, by using fixed point theorems, we will obtain the existence results for the following p-laplacian fractional boundary value problem,

$$ {}^{c}D_{1^-}^{\beta} \left(\varphi_p\left(D_{0^+}^{\alpha} y(t) \right)\right) = f(t, y(t), K(t), H(t)), \quad t \in (0,1) $$

$$ y(0) = 0, \quad y(1) = \varphi_1 y(\rho_1) $$

$$ D_{0^+}^{\alpha} y(0) = 0, \quad D_{0^+}^{\alpha} y(1) = \varphi_2 \, D_{0^+}^{\alpha} y(\rho_2) $$

Where $1<\alpha , \beta\leq 2$, $0 < \varphi_1 < \varphi_2 < 1$ such that $\rho_2 \varphi_2^{p-1} \neq 1$ and $\rho_1\varphi_1 \neq 1$, $\varphi_P(u) = |u|^{p-2} . u$, $\frac{1}{p} + \frac{1}{q} = 1$, p,q > 1 , $D_0^{\alpha}+$ is left Riemann – Liouville fractional derivable , ${}^{c}D_{1^-}^{\beta}$ is right caputo fractional derivable and f : [0,1] x R x R x R → R is a given function satisfying some assumptions that will be specified later. f is a continuous functions. 'Establishing the Green's function associated with the above boundary value problem, we will give the sufficient conditions to ensure the existence of solutions for this problem.

Anahtar Kelimeler : Fractional Equaitons , fixed point teorems , existence of solutions.

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AĞLARDA ETKİN UZAKLIK ÇEKİM MERKEZLİĞİ MODELİ İÇİN ALGORİTMA TASARIMI

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ÖZET

Karmaşık ağlar, bileşenlerin ve bu bileşenler arasındaki etkileşimlerin, sırasıyla, tepeler ve ayrıtlar olarak modellendiği gerçek hayattaki çok sayıda karmaşık sistemi temsil eder. Karmaşık ağlarda ağın yapısal özelliklerini, işlevlerini ve dinamik sürecini ağdaki diğer tepelerden daha fazla etkileyen anahtar tepeler vardır. Karmaşık ağlarda etkili tepelerin belirlenmesi için etkin uzaklık çekim merkezliği modeli önerilmiştir. Etkin uzaklık çekim merkezliği modeli ağdaki bir tepenin lokal bilgisini ve ağın global bilgisini kapsamlı olarak ele alır. Etkin uzaklık çekim merkezliği modelinin uygunluğunu ve etkinliğini göstermek için çeşitli hesaplamalar ve literatürdeki diğer ölçümlerle kıyaslamalar yapılmıştır. Elde edilen deneysel sonuçlar önerilen yöntemin mevcut yöntemlere göre daha sağlam ve makul olduğunu göstermiştir. Bu çalışmada, çizgenin etkili tepelerini belirlemek için etkin uzaklık çekim merkezliği modelini kullanan bir algoritma önerilmiştir.

Anahtar Kelimeler : Karmaşık ağlar, etkin uzaklık, çekim merkezliği modeli, algoritma tasarımı, çizge algoritmaları.

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1. GİRİŞ

Karmaşık ağlar, ulaşım, elektrik, bilgisayar sistemleri, sosyal ve biyolojik sistemler gibi gerçek hayattaki çeşitli karmaşık sistemlerim yapısını ve dinamik davranışını incelemek için yaygın olarak kullanılmaktadır. Karmaşık ağlar karmaşık sistemlerdeki nesneler arasındaki ilişkileri etkili bir biçimde açıklayabilen modellerdir. Karmaşık ağlar, sırasıyla, V ve E kümelerinin tepe ve ayrıt kümelerini temsil ettiği bir G=(V,E) çizgesi ile gösterilebilir. Karmaşık ağlarda tepeler, genler, istasyonlar, havaalanları, bilgisayarlar ve bireyler gibi çeşitli bileşenleri, ayrıtlar da farklı bileşenler arasındaki etkileşimleri temsil eder.

Karmaşık ağlarda ağın yapısal özelliklerini, işlevlerini ve dinamik sürecini ağdaki diğer tepelerden daha fazla etkileyen anahtar tepeler vardır. Örneğin, az sayıda etkili tepenin bozulması ağdaki diğer tepelerin aşamalı olarak bozulmasına ve hatta tüm ağın işlev dışı kalmasına neden olabilir. Karmaşık ağlardaki etkili tepelerin belirlenmesi ağ bilimindeki en önemli ve temel araştırma konularından biridir. Karmaşık ağlardaki etkili tepelerin belirlenmesi ağların araştırılmasını ilerletmek, sistem tasarımını optimize etmek ve ağ güvenliğini arttırmak için çok önemlidir. Önemli tepelerin korunması ve kullanılması karmaşık sistemin güvenliğini ve işlevsel etkinliğini sağlayabilir. Örneğin, elektrik şebekesinde etkili tepelerin belirlenmesi güç sistemindeki arızaların ve kesintilerin tahmin edilmesine ve önlenmesine yardımcı olabilir. Güç sistemindeki kritik tepeler belirlenebilirse bu tepeler daha iyi korunabilir ve böylece güç sisteminin güvenilirliği ve kararlılığı arttırılabilir.

Karmaşık ağlarda etkili tepelerin belirlenmesi için etkin uzaklık çekim merkezliği modeli önerilmiştir (Shang, Deng, ve Cheong, 2021). Etkin uzaklık çekim merkezliği modeli ağdaki bir tepenin lokal bilgini ve ağın global bilgisini kapsamlı olarak ele alır. Gerçek hayatta kullanılan ağlarda tepeler arasında dinamik bir etkileşim vardır. Öklid uzaklığı sadece tepelerin statik topolojik uzaklığına odaklanır dolayısıyla ağdaki gömülü dinamik bilgi akışını etkin bir şekilde araştıramaz. Etkin uzaklık ise ağlardaki tepeler arasındaki dinamik bilgi etkileşimini tam olarak dikkate alır. Etkin uzaklığın Öklid uzaklığından en önemli farkı asimetrik olmasıdır. Etkin uzaklık çekim merkezliği modeli ağın gömülü dinamik yapısını ve ağın etkin işleyişine hakim olan tepeler arasındaki dinamik etkileşim bilgisini ortaya çıkarabilir. Dinamik bilgi ile açık statik bilginin kombinasyonu karmaşık ağlardaki önemli tepeleri daha iyi tespit edebilir. Etkin uzaklık çekim merkezliği modelinde kullanılan kümülatif merkezlik puanı hesaplama yöntemi kararsız yapının neden olduğu tanımlama hatasını azaltır. Etkin uzaklık çekim merkezliği modelinin uygunluğunu ve etkinliğini göstermek için çeşitli hesaplamalar ve literatürdeki diğer ölçümlerle kıyaslamalar yapılmıştır. Elde edilen deneysel sonuçlar önerilen yöntemin mevcut yöntemlere göre daha sağlam ve makul olduğunu gösterilmiştir.

Bu çalışmada etkin uzaklık çekim merkezliği modeli için polinom zamanlı bir algoritma önerilmiştir.

2. ETKİN UZAKLIK ÇEKİM MERKEZLİĞİ MODELİ

2.1. Etkin Uzaklık Çekim Merkezliği Modelinin Tanımı

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Bir G çizgesinin herhangi bir v tepesine bitişik olan ayrıtların sayısına v tepesinin derecesi denir ve deg(v) ile gösterilir. Bir G çizgesinde herhangi iki tepeyi birbirine bağlayan yollardan en az ayrıta sahip olan yola en kısa yol denir. Bir G çizgesinde herhangi iki tepe arasındaki en kısa yolda bulunan ayrıt sayısına iki tepe arasındaki uzaklık denir. u,v€V(G) olmak üzere G çizgesinde u ve v tepeleri arasındaki uzaklık d(u,v) ile gösterilir (Buckley ve Harary, 1990; Chartrand ve Lesniak, 1996).

Birbirine komşu olan u ve v tepeleri için u tepesinden v tepesine etkin uzaklık değeri

$D_{v|u}=1\text{-}\log 2(P_{v|u})$

olarak tanımlanır. Burada, $P_{v|u}$ u tepesinden v tepesine olan olasılıktır ve

$P_{v|u}=1/\deg(u) \quad (u\neq v)$

ile hesaplanır. Eğer u=v ise, $D_{v|u}=0$ olur. Birbirine komşu olmayan tepeler için, bu tepeler arasındaki etkin uzaklık $D_{y|x}=D_{z|x}+D_{y|z}$ benzeri olacak şekilde elde edilebilir. Eğer u ve v tepeleri arasında birden fazla yol var ise bu tepeler arasındaki en kısa yol kullanılır. $D^{i}_{v|u}$, u tepesinden v tepesine farklı etkin uzaklıkları temsil etmek üzere u tepesinden v tepesine etkin uzaklık

$D_{v|u}=\min\{ D^{1}_{v|u}, D^{2}_{v|u}, D^{3}_{v|u},\dots \}$

olarak belirlenir (Brockmann ve Helbing, 2013). u ve v tepeleri arasındaki etkileşim puanı

$W_{etkileşim}(u,v)=\deg(u)\deg(v)/D^{2}_{v|u}$

olarak tanımlanır. u tepesinin EffG merkezlik puanı

$C_{EffG}(u)=\sum_{v\in V,v\neq u} W_{etkileşim}(u,v)= \sum_{v\in V,v\neq u} \deg(u)\deg(v)/D^{2}_{v|u}$

olarak hesaplanır (Shang, Deng, ve Cheong, 2021).

2.2. Etkin Uzaklık Çekim Merkezliği Modeli Algoritması

n tepeli bir çizgenin her bir tepesinin EffG merkezlik puanını hesaplayan bir algoritma tasarlanmıştır. G çizgesinin komşuluk matrisi a[i,j] olmak üzere, eğer i ve j tepeleri G çizgesinde birbirine komşu ise a[i,j]=1 değerini alır; eğer i ve j tepeleri G çizgesinde birbirine komşu değil ise veya i=j ise o zaman a[i,j]=0 değerini alır. Bu algoritma, çizgenin komşuluk matrisinden yararlanarak ve Floyd-Warshall algoritmasını (Floyd, 1962) kullanarak çizgenin etkin uzaklık matrisini oluşturur. Sonrasında, çizgenin etkin uzaklık matrisinden çizgenin her bir tepesi için EffG merkezlik puanını hesaplar. Algoritmanın kabakodu aşağıda verilmiştir.

```
int main()
{

        int i,j,k,a[n+1][n+1]={0},deg[n+1]={0};
        float d[n+1][n+1]={0},CEffG[n+1]={0};
        FILE *f;

        f=fopen("a.txt","r");

        for(i=1;i<=n;i++)
                for(j=1;j<=n;j++)
                {
                        fscanf(f,"%d ",&a[i][j]);
                        deg[i]+=a[i][j];
```

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```
        }


    for(i=1;i<=n;i++)
        for(j=1;j<=n;j++)
        {
            if(i!=j && a[i][j]==0) a[i][j]=∞;
            if(i==j) d[i][j]=0;
            else if (a[i][j]==1) d[i][j]=1.0+float(log2(deg[i]));
        }

    for(k=1;k<=n;k++)
        for(i=1;i<=n;i++)
            for(j=1;j<=n;j++)
            {
                if ((a[i][j]!=0 || a[i][j]!=1) && (a[i][j] > (a[i][k] + a[k][j])))
                {
                    a[i][j]=a[i][k]+a[k][j];
                    d[i][j]=d[i][k]+d[k][j];
                }
                else if ((a[i][j]!=0 || a[i][j]!=1) && (i!=k) && (j!=k) && (a[i][j]
== (a[i][k] + a[k][j])))
                {
                    if(float(1.0+log2(deg[i])+1.0+log2(deg[k])) < d[i][j])
                    d[i][j]=d[i][k]+d[k][j];

                }
            }

    for(i=1;i<=n;i++)
    {
        for(j=1;j<=n;j++)
            printf("%2d ",a[i][j]);
        printf("\n");
    }
    printf("\n\n\n");

    for(i=1;i<=n;i++)
    {
        for(j=1;j<=n;j++)
            printf("%9.6f ",d[i][j]);
        printf("\n");
    }

    printf("\n\n");
    for(i=1;i<=n;i++)
    {
        for(j=1;j<=n;j++)
            if (i!=j) CEffG[i]+=(deg[i]*deg[j])/pow(d[i][j],2);
```

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```
		printf("(%d): %9.6f \n",i,CEffG[i]);
	}

}
```

Önerilen algoritma polinom zamanlıdır. Floyd-Warshall algoritması n tepeli bir çizge için O(n^3) zaman karmaşıklığına sahiptir dolayısıyla tasarlanan algoritmanın zaman karmaşıklığı da O(n^3) olur.

3. SONUÇLAR VE DEĞERLENDİRME

Bu çalışmada, bir çizgenin etkili tepelerini belirlemek için etki uzaklık çekim merkezliği modelini kullanan ve çizgenin tepelerinin EffG merkezlik puanlarını hesaplayan polinom zamanlı bir algoritma tasarlanmıştır.

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ON THE SOLUTION OF TIME FRACTIONAL WAVE PROBLEM WITH NEUMANN BOUNDARY CONDITIONS

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ABSTRACT

The primary aim of this research is to acquire the exact solution of time fractional wave problem (TFWP) by means of separation of variables method. The Caputo fractional derivative is taken in TFWP with respect to time. The boundary conditions are given as homogenous boundary conditions. The proposed method enables us to establish the solution in terms of fractional trigonometric functions.

Keywords: Time Fractional Wave Equation, Separation of Variables Method, Caputo Fractional Derivative, Fractional Trigonometric Functions

1. INTRODUCTION

Establishing the exact solution of fractional differential equations are open problems. A number of methods have been developed and modified to determine exact or approximate solutions to fractional differential equations. This kind of differential equations have been used widely in the modeling of various problems in physics, engineering, biology, chemistry and mathematics (Choi & Chan , 1992) (Demir, Bayrak, Bulut, Ozbilge, & Cetinkaya, 2023) (Diethelm, 1997) (Kilbas, Srivastava, & Trujillo, 2006) (Li & Zeng, Finite difference methods for fractional differential equations, 2012) (Li & Zeng, Numerical methods for fractional calculus, 2015) (Li & Wang, 2022) (Odibat & Momani, 2009) (Oldham & Spanier, 1974) (Podlubny, 1999) (Salman, Mohd, & Muhammad, 2023) (Tian, Yang, Zhang, & Xu, 2023) (Tomasek, 2023) (Akyol, Cetinkaya, & Demir , 2021) (Cetinkaya & Demir , 2020a) (Cetinkaya & Demir, 2023a) (Cetinkaya & Demir, 2021a) (Cetinkaya & Demir, 2019a) (Cetinkaya & Demir, 2020b)

(Cetinkaya & Demir, 2023b) (Cetinkaya & Demir, 2022a) (Cetinkaya & Demir, 2022b) (Cetinkaya & Demir, 2020c) (Cetinkaya & Demir, 2021b) (Cetinkaya & Demir, 2022c) (Cetinkaya & Demir, 2021c) (Cetinkaya & Demir, 2022d) (Cetinkaya & Demir, 2024a) (Cetinkaya & Demir, 2021d) (Cetinkaya & Demir, 2022e) (Cetinkaya & Demir, 2021e) (Cetinkaya & Demir, 2023c) (Cetinkaya & Demir, 2021f) (Cetinkaya & Demir, 2021g) (Cetinkaya & Demir, 2022e) (Cetinkaya & Demir, 2023d) (Cetinkaya & Demir, 2020d) (Cetinkaya & Demir, 2022f) (Cetinkaya, Bayrak, Demir, & Baleanu, 2022) (Cetinkaya, Demir , & Kodal Sevindir, 2020e) (Cetinkaya, Demir, & Baleanu, 2021h) (Cetinkaya, Demir, & Kodal Sevindir, 2021g) (Cetinkaya, Demir, & Kodal Sevindir, 2020f) (Cetinkaya, Demir, & Kodal Sevindir, 2020ı) (Demir, Bayrak, Bulut, Ozbilge, & Cetinkaya, 2023) (Kodal Sevindir, Çetinkaya, & Demir, 2021). Therefore, fractional differential problems attract the attention of many researchers in different fields of science. This interest leads to development of various fractional derivatives such as Caputo, Riemann-Liouville, conformable fractional derivatives and etc.

In this research, time fractional wave problem with Neumann boundary conditions is taken into consideration. The separation of variables method is utilized to construct the solution in the series form in terms of fractional trigonometric functions.

The following problem is taken in hand to obtain the solution:

$$ {}^{C}D_t^{\alpha}\left({}^{C}D_t^{\alpha}\big(u(x,t)\big)\right) = c^2 u_{xx}(x,t),\ 0 < t < T,\ 0 < x < l,\ 0 < \alpha < 1 $$

$$ u(x,0) = \Phi(x),\quad {}^{C}D_t^{\alpha}\big(u(x,0)\big) = \Psi(x), $$

$$ u_x(0,t) = u_x(1,t) = 0. $$

2. PRELIMINARY RESULTS

This section is devoted to fundamental definitions used in this research (Kilbas, Srivastava, & Trujillo, 2006).

Riemann-Liouville fractional integral of order $\alpha > 0$ for a real valued function $f(t)$ is defined as

$$ I_t^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} f(s)\,ds. $$

Liouville-Caputo fractional derivative of order $\alpha > 0$ in terms of Riemann-Liouville fractional integral is given as

$$\frac{\partial^\alpha f(t)}{\partial t^\alpha} = I_t^{m-\alpha}\left[\frac{\partial^m f(t)}{\partial t^m}\right] = \begin{cases} \frac{1}{\Gamma(m-\alpha)}\int_0^t (t-y)^{m-\alpha-1}\frac{\partial^m f(y)}{\partial y^m}dy, m-1<\alpha<m, \\ \frac{\partial^m f(t)}{\partial t^m}, \alpha = m. \end{cases}$$

Moreover, two parametrized Mittag-Leffler function is defined in series form as follows:

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+\beta)}, Re(\alpha) > 0, z, \beta \in \mathbb{C}$$

where α and β denote parameters. This function has the following useful features in terms of Caputo fractional derivative

i) ${}^C D_t^\alpha\left(E_{\alpha,1}(t^\alpha)\right) = E_{\alpha,1}(t^\alpha)$,

ii) ${}^C D_t^{n\alpha}\left(E_{\alpha,1}(kt^\alpha)\right) = k^n E_{\alpha,1}(kt^\alpha)$ where $0 < \alpha < 1$, k is a constant and $n \in \mathbb{N}$.

Moreover, Mittag-Leffler function leads to the following fractional trigonometric functions

$$\sin_\alpha(ct^\alpha) = \frac{E_{\alpha,1}(ict^\alpha) - E_{\alpha,1}(-ict^\alpha)}{2i} = \sum_{n=0}^{\infty}\frac{(-1)^n (ct^\alpha)^{2n+1}}{\Gamma\left((2n+1)\alpha+1\right)},$$

$$\cos_\alpha(ct^\alpha) = \frac{E_{\alpha,1}(ict^\alpha) + E_{\alpha,1}(-ict^\alpha)}{2} = \sum_{n=0}^{\infty}\frac{(-1)^n (ct^\alpha)^{2n}}{\Gamma(2n\alpha+1)}.$$

where $i = \sqrt{-1}$.

3. MAIN RESULTS

The separation of variables method assumes that the solution can be written as a multiplication of two functions with different variables:

$$u(x,t;\alpha) = X(x)T(t;\alpha). \quad (1)$$

Plugging (1) into the time fractional wave equation yields the following:

$$X(x)\,{}^C D_t^{2\alpha}\left(T(t,\alpha)\right) = c^2 X''(x)T(t;\alpha).$$

Arranging the obtained equation leads to the following:

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$$-\frac{X''(x)}{X(x)} = -\frac{{}^{C}D_t^{2\alpha}(T(t,\alpha))}{c^2 T(t;\alpha)} = \lambda.$$

Taking $\lambda = -\frac{X''(x)}{X(x)}$ produces the following reduced problems

1) $X''(x) + \lambda X(x) = 0,$

   $X'(0) = X'(l) = 0$

where the boundary conditions are obtained from the boundary conditions of TFWP as follows:

$X'(0)T(t) = X'(l)T(t) = 0, \forall t > 0$ imply that $X'(0) = X'(l) = 0$.

2) ${}^{C}D_t^{2\alpha}\big(T(t,\alpha)\big) + \lambda c^2 T(t;\alpha) = 0.$

   $T(0;\alpha) = \Phi(x), \;\; {}^{C}D_t^{\alpha}\big(T(0;\alpha)\big) = \Psi(x)$

The characteristic equation of the first reduced problem is obtained as

$$r^2 + \lambda = 0.$$

For the solution of the reduced problem, the following cases are considered:

Case 1. Taking $\lambda = 0$ yields two coincident roots $r_1 = r_2$ which leads to the following solution:

$$X(x) = k_1 x + k_2,$$

$$X'(x) = k_1.$$

The boundary condition $X'(0) = 0$ yields

$$k_1 = 0$$

which leads to the following solution

$$X(x) = k_2.$$

Second reduced problem has the following solution for $\lambda = 0$:

$$T(t,\alpha) = d_1 \frac{t^\alpha}{\alpha\Gamma(\alpha)} + d_2.$$

Hence, the following solution is obtained for TFWP

$$u(x,t;\alpha) = X(x)T(t;\alpha) = k_2 d_1 \frac{t^\alpha}{\alpha\Gamma(\alpha)} + k_2 d_2 = e_1 \frac{t^\alpha}{\alpha\Gamma(\alpha)} + e_2.$$

Case 2. Taking $\lambda < 0$ yields two distinct real roots $r_{1,2} = \pm\sqrt{-\lambda}$ which leads to the following solution to first reduced problem

$$X(x) = c_1 e^{-\sqrt{-\lambda}x} + c_2 e^{\sqrt{-\lambda}x}.$$

The boundary conditions $X'(0) = 0$ yields the result $c_1 = c_2$. The other boundary condition $X'(l) = 0$ yields the result $c_1 = c_2 = 0$. Hence, the solution of the first reduced problem becomes $X(x) = 0$ which leads to no solution to TFWP.

Case 3. Taking $\lambda > 0$ yields two distinct complex roots r_1, r_2 which leads to the following solution to first reduced problem

$$X(x) = c_1 \cos\left(\sqrt{\lambda}x\right) + c_2 \sin\left(\sqrt{\lambda}x\right).$$

$$X'(x) = -c_1\sqrt{\lambda}\sin\left(\sqrt{\lambda}x\right) + c_2\sqrt{\lambda}\cos\left(\sqrt{\lambda}x\right).$$

Boundary condition $X'(0) = 0$ leads to the following

$X'(0) = c_2\sqrt{\lambda} = 0$ which yields $c_2 = 0$.

Boundary condition $X'(l) = 0$ leads to the following

$X'(l) = -c_1\sqrt{\lambda}\sin\left(\sqrt{\lambda}l\right)$ which yields $\sqrt{\lambda} = \frac{n\pi}{l}$. Therefore, the following solutions are obtained

$$X_n(x) = c_1 \cos\left(\frac{n\pi}{l}x\right), n \in \mathbb{N}.$$

Second reduced problem has the following solution for $\lambda > 0$:

${}^{C}D_t^{2\alpha}\left(T(t,\alpha)\right) + c^2\lambda T(t,\alpha) = 0$ which have the following solutions

$$T_n(t;\alpha) = A_n \cos_\alpha\left(c\sqrt{\lambda}t^\alpha\right) + B_n \sin_\alpha\left(c\sqrt{\lambda}t^\alpha\right), n \in \mathbb{N}.$$

Therefore, the solutions to TFWP are obtained as follows:

$$u_n(x,t;\alpha) = X_n(x)T_n(t;\alpha) = \left(A_n \cos_\alpha\left(c\frac{n\pi}{l}t^\alpha\right) + B_n \sin_\alpha\left(c\frac{n\pi}{l}t^\alpha\right)\right)\cos\left(\frac{n\pi}{l}x\right), n = 1,2,3,\ldots$$

The superposition principle leads to the following general solution to TFWP

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$$u(x,t;\alpha) = e_1 \frac{t^\alpha}{\alpha\Gamma(\alpha)} + e_2 + \sum_{n=1}^{\infty} \left(A_n \cos_\alpha \left(c\frac{n\pi}{l} t^\alpha \right) + B_n \sin_\alpha \left(c\frac{n\pi}{l} t^\alpha \right) \right) \cos\left(\frac{n\pi}{l} x\right)$$

which satisfies the boundary conditions. In order to satisfy the initial conditions the unknown coefficients A_n and $B_n, n = 1,2,3, \ldots$ are determined suitably from the following equations:

$$u(x,0;\alpha) = e_2 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{l} x\right) = \Phi(x),$$

$${}^C D_t^\alpha \big(u(x,0;\alpha)\big) = e_1 + \sum_{n=1}^{\infty} B_n c \frac{n\pi}{l} \cos\left(\frac{n\pi}{l} x\right) = \Psi(x).$$

By using the inner product the unknown coefficient and ortagonality of the functions $\cos\left(\frac{m\pi}{l} x\right), \sin\left(\frac{m\pi}{l} x\right)$, the unknown coefficients are obtained as follows:

$$e_1 = \frac{1}{l} \int_0^l \Psi(x)\, dx,$$

$$e_2 = \frac{1}{l} \int_0^l \Phi(x)\, dx,$$

$$A_m = \frac{2}{l} \int_0^l \Phi(x) \cos\left(\frac{m\pi}{l} x\right) dx, \; m = 1,2,3, \ldots$$

$$B_m = \frac{2}{cm\pi} \int_0^l \Psi(x) \cos\left(\frac{m\pi}{l} x\right) dx, \; m = 1,2,3, \ldots$$

4. ILLUSTRATIVE EXAMPLE

This section is devoted to application of the proposed method for TFWP.

Example: Consider the following TFWP

$${}^C D_t^\alpha \left({}^C D_t^\alpha \big(u(x,t)\big) \right) = u_{xx}(x,t),\; 0 < t < T,\; 0 < x < l,\; 0 < \alpha < 1$$

with following initial and boundary conditions, respectively

$$u(x,0) = \cos(5\pi x), \quad {}^C D_t^\alpha \big(u(x,0)\big) = 0,$$

$$u_x(0,t) = u_x(1,t) = 0.$$

The exact solution of this problem is established as

$$u(x,t) = \cos(5\pi x) \cos_\alpha(5\pi t^\alpha).$$

Taking $\alpha = 1$ leads to the following wave problem

$u_{tt}(x,t) = u_{xx}(x,t)$, $0 < t < T$, $0 < x < l$, $0 < \alpha < 1$

$u(x,0) = \cos(5\pi x)$, $u_t(x,0) = 0$,

$u_x(0,t) = u_x(1,t) = 0.$

Utilizing separation of variables method allows us to determine the solution $u(x,t;\alpha)$ in the following series form

$$u(x,t;\alpha) = e_1 \frac{t^\alpha}{\alpha\Gamma(\alpha)} + e_2 + \sum_{n=1}^{\infty}\left(A_n \cos_\alpha\left(c\frac{n\pi}{l}t^\alpha\right) + B_n \sin_\alpha\left(c\frac{n\pi}{l}t^\alpha\right)\right)\cos\left(\frac{n\pi}{l}x\right)$$

where the unknown coefficients are determined as

$$e_1 = \frac{1}{l}\int_0^l \Psi(x)\,dx = 0,$$

$$e_2 = \frac{1}{l}\int_0^l \Phi(x)\,dx = \frac{1}{1}\int_0^1 \cos(5\pi x)\,dx = 0,$$

$$A_m = \frac{2}{l}\int_0^l \Phi(x)\cos\left(\frac{m\pi}{l}x\right)dx = 2\int_0^1 \cos(5\pi x)\cos(m\pi x)\,dx = \begin{cases}0, m \neq 5\\ 1, m = 5\end{cases},$$

$$B_m = \frac{2}{cm\pi}\int_0^l \Psi(x)\cos\left(\frac{m\pi}{l}x\right)dx = 0,\ m = 1,2,3,\ldots$$

which leads to the following exact solution

$$u(x,t;\alpha) = \cos_\alpha(5\pi t^\alpha)\cos(5\pi x).$$

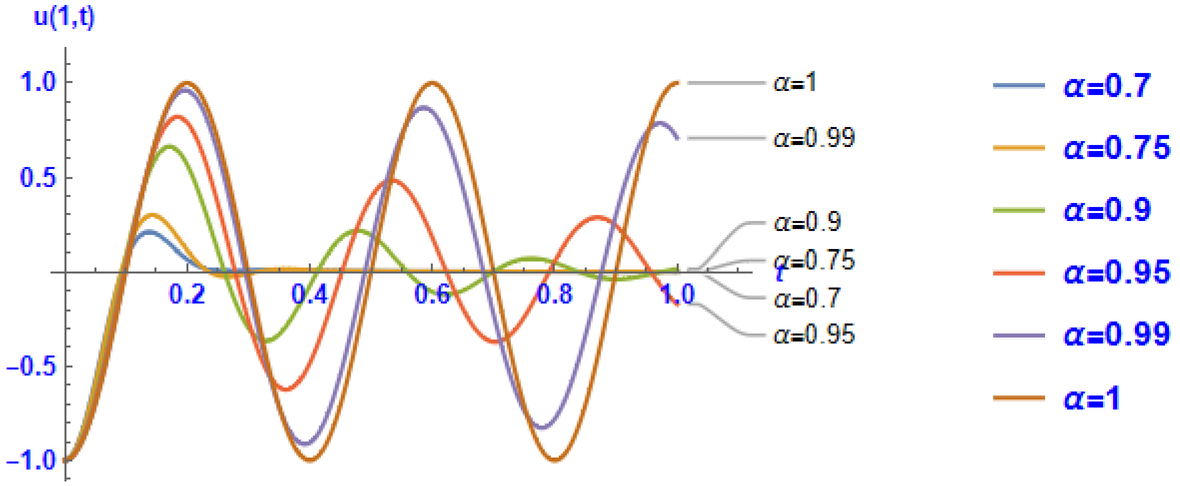

Figure 1. The graphs of exact solution for various values of α at $x = 1$.

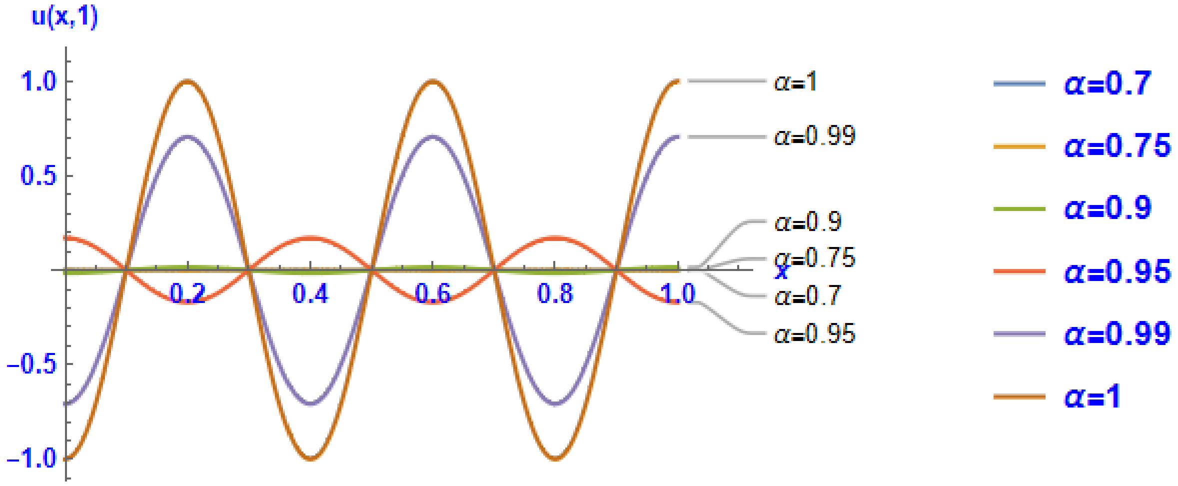

Figure 2. The graphs of exact solution for various values of α at $t = 1$.

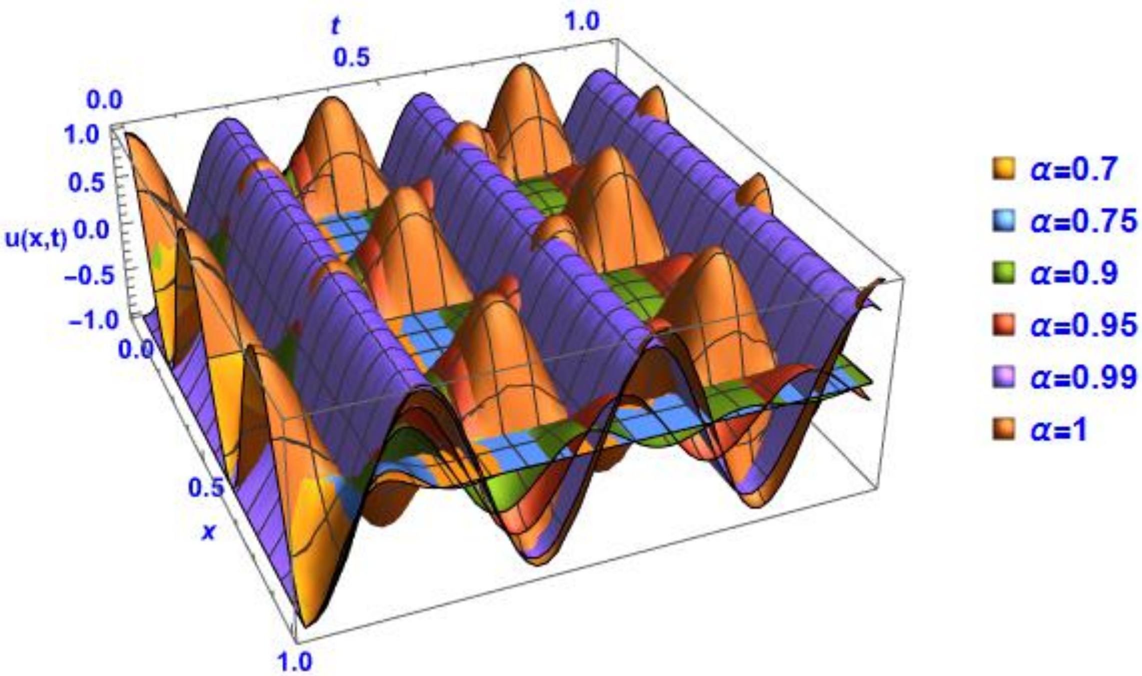

Figure 3. 3D graphs of exact solution for various values of α.

It is clear from Fig. (1) and (3) that as α tends to 1 the amplitude of the solutions gets greater. Moreover, the amplitude of the solutions gets lower as t increases.

Furthermore, it can be seen from Fig. (2) that as α tends to 1 the amplitude of the solution gets greater. However, the amplitude of the solutions does not change as x increases.

5. CONCLUSION

The exact solutions of TFWP with homogenous Neumann boundary conditions were established for various values of fractional order α by utilizing separation of variables method. The proposed method leads TFWP to the two reduced problem. The general solution were obtained by multiplication of solutions of reduced problems and initial and boundary conditions in the series form in terms of fractional trigonometric functions.

In the future work, various fractional wave problems will be studied by separation of variable or other suitable methods.

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EULER METHOD FOR INITIAL VALUE PROBLEMS OF LINEAR FRACTIONAL DIFFERENTIAL SYSTEM

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ABSTRACT

The primary goal of this research is to construct numerical solutions to initial value problems of system including linear fractional differential equations with constant coefficients by means of Fractional Forward Euler Method (FFEM). Some examples are presented to illustrate the implementation of FFEM. Moreover, the accuracy and effectiveness of the method are verified by the presented examples. The numerical solutions are obtained by MATLAB codes.

Keywords: Initial Value Problems of Fractional Differential Equations System, Caputo Fractional Derivative, MATLAB, Euler Method

1. INTRODUCTION

The fractional differential problems have a significant importance due to their wide applications in a number of fields in science (Alpar, Berger, Rysbaiuly, & Belarbi, 2024) (Benetti, Silveira, & Caldas, 2023) (Akyol, Cetinkaya, & Demir , 2021) (Cetinkaya & Demir , 2020a) (Cetinkaya & Demir, 2023a) (Cetinkaya & Demir, 2021a) (Cetinkaya & Demir, 2019a) (Cetinkaya & Demir, 2020b) (Cetinkaya & Demir, 2023b) (Cetinkaya & Demir, 2022a) (Cetinkaya & Demir, 2022b) (Cetinkaya & Demir, 2020c) (Cetinkaya & Demir, 2021b) (Cetinkaya & Demir, 2022c) (Cetinkaya & Demir, 2021c) (Cetinkaya & Demir, 2022d) (Cetinkaya & Demir, 2024a) (Cetinkaya & Demir, 2021d) (Cetinkaya & Demir, 2022e) (Cetinkaya & Demir, 2021e) (Cetinkaya & Demir, 2023c) (Cetinkaya & Demir, 2021f) (Cetinkaya & Demir, 2021g)

(Cetinkaya & Demir, 2022e) (Cetinkaya & Demir, 2023d) (Cetinkaya & Demir, 2020d) (Cetinkaya & Demir, 2022f) (Cetinkaya, Bayrak, Demir, & Baleanu, 2022) (Cetinkaya, Demir , & Kodal Sevindir, 2020e) (Cetinkaya, Demir, & Baleanu, 2021h) (Cetinkaya, Demir, & Kodal Sevindir, 2021g) (Cetinkaya, Demir, & Kodal Sevindir, 2020f) (Cetinkaya, Demir, & Kodal Sevindir, 2020ı) (Demir, Bayrak, Bulut, Ozbilge, & Cetinkaya, 2023) (Kodal Sevindir, Çetinkaya, & Demir, 2021). The mathematical models including fractional differential equations system reflects the behavior of various processes properly and accurately. Especially, for systems with memory and hereditary properties, the system of fractional differential equations is the best choice to model them. Moreover, fractional differential models provide more degrees of freedom which is a significant advantage. The choice of correct fractional derivative also contribute to the analysis of the considered processes.

There are numerous studies for the existence and uniqueness of initial value problems of fractional differential systems in the literature (Samko, Kilbas , & Marichev, 1993) (Podlubny, 1999) (Delbosco, & Rodino, 1996) (Diethelm & Ford, Analysis of fractional differential equations, 2002) (Lakshmikantham & Vatsala, 2008). Moreover, stability analysis of them have been done (Deng, Li, & Lü, 2007) (Tavazoei & Haeri, 2009). There are some studies in the literature about the exact solutions of system of fractional differential equations (Lakshmikantham & Vatsala, 2008) (Bonilla, Rivero, & Trujillo, 2007). Moreover, various mathematical methods have been developed and utilized to construct approximate and numerical solutions of them.

The focus of this research is that to establish numerical solutions to fractional initial value problems of systems by means of FFEM. MATLAB codes for FFEM have been developed and applied in this study.

The following fractional initial value problem of system in Caputo sense is taken into account:

$$\begin{cases} {}^{C}D_t^{\alpha} y(t) = Ay(t), y \in \mathbb{R}^n, t \in [0,T] \\ \qquad y(0) = y_0, y_0 \in \mathbb{R}^n \end{cases} \tag{1}$$

where $\alpha = [\alpha_1, \alpha_2, \dots, \alpha_n]$ with $0 < \alpha_i \leq 1$ for $i = 1,2, \dots, n$.

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The Caputo fractional derivative is defined as

$$ {}^{C}D_t^{\alpha} y(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} y'(s)ds, \ \alpha \in (0,1). $$

2. FRACTIONAL FORWARD EULER METHOD (FFEM)

FFEM is widely used to establish numerical solutions to differential equations. In the case of fractional initial value problem of systems in Caputo sense, the following formula is obtained by using discretization of Caputo fractional derivative:

$$ y_{n+1}^{i} = y_0^{i} + h^{\alpha} \sum_{j=0}^{k} b_{j,k+1} A_{i\times n} y^{n}(t_j), \ k = 0,1,\ldots,N-1, \ i = 1,2,\ldots,n $$

where $b_{j,k+1} = \frac{(k-j+1)^{\alpha}-(k-j)^{\alpha}}{\Gamma(1+\alpha)}$, $k = 0,1,\ldots,N-1$, $j = 0,1,\ldots,k$.

For the asymptotic stability for the fractional initial value problems of systems the following theorem is presented:

Theorem: The autonomous system of fractional differential equations ${}^{C}D_t^{\alpha} y(t) = Ay(t)$ for $\alpha = \alpha_1 = \alpha_2 = \cdots = \alpha_n$ is asymptotically stable if and only if $|\arg(spec(A))| > \alpha\frac{\pi}{2}$. Moreover, the components of the state tends to zero (Odibat, 2010).

3. ILLUSTRATIVE EXAMPLES

Example 1: Let us consider the following fractional initial value problem:

$$ \begin{pmatrix} {}^{C}D_t^{\alpha} y_1(t) \\ {}^{C}D_t^{\alpha} y_2(t) \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 4 & -3 \end{pmatrix} \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix}, \ \alpha \in (0,1], \ t \in [0,T] \qquad , $$

with the initial condition

$$ \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 1.2 \\ 4.2 \end{pmatrix}. $$

The unique of this fractional initial value problem of system is established as

$$ \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} = \frac{1}{5} \begin{pmatrix} 1 \\ 1 \end{pmatrix} E_{\alpha,1}(t^{\alpha}) + \begin{pmatrix} 1 \\ 4 \end{pmatrix} E_{\alpha,1}(-2t^{\alpha}). $$

where $E_{\alpha,\beta}(t) = \sum_{k=0}^{\infty} \frac{t^k}{\Gamma(\alpha k+\beta)}$ is two parametrized Mittag-Leffler function (Diethelm, 1997).

The numerical solutions established by FFEM are given in Figure (1-4) for various values of α and h.

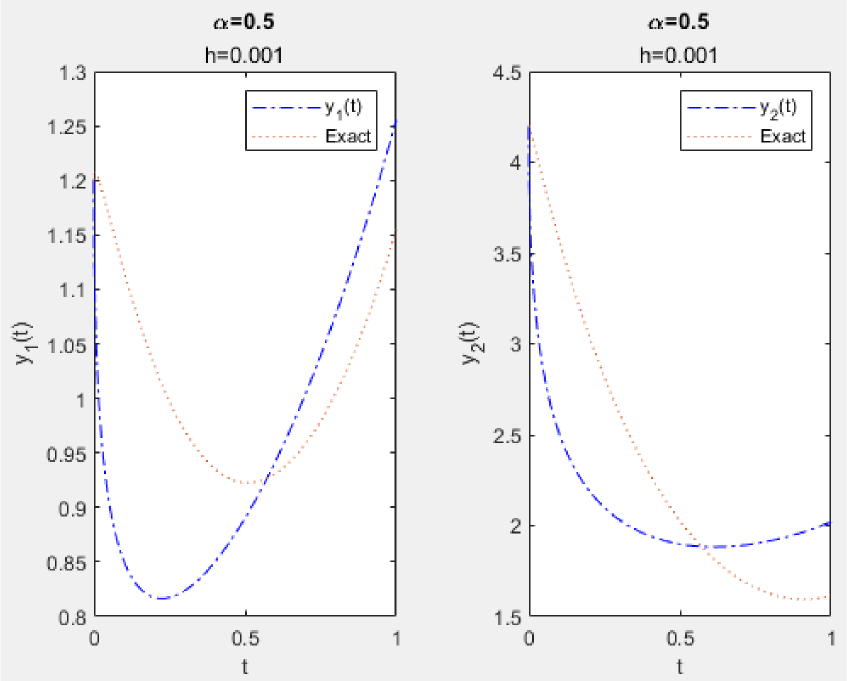

Figure 1. Numerical solutions of Example 1 for $h = 0.001$ at $\alpha = 0.5$.

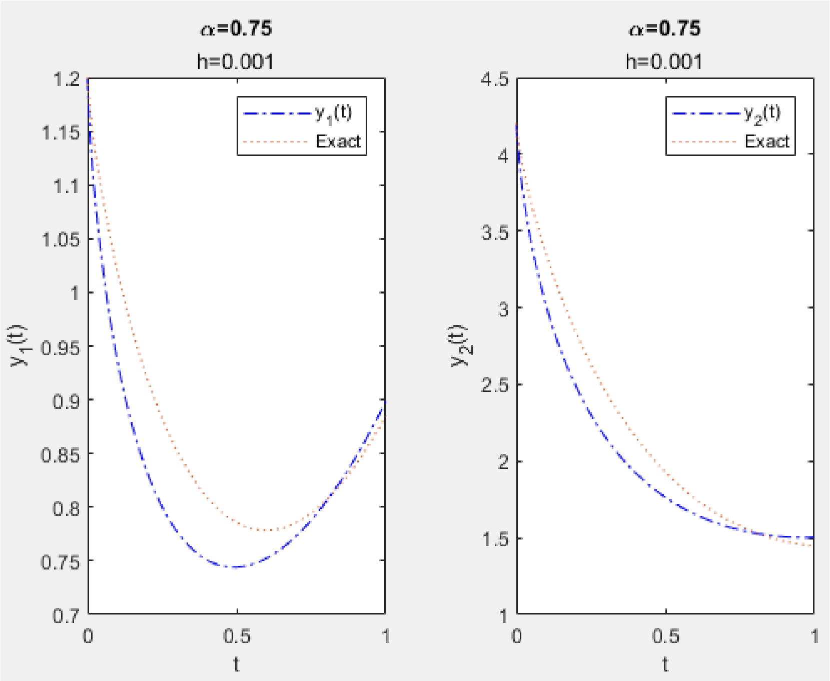

Figure 2. Numerical solutions of Example 1 for $h = 0.001$ at $\alpha = 0.75$.

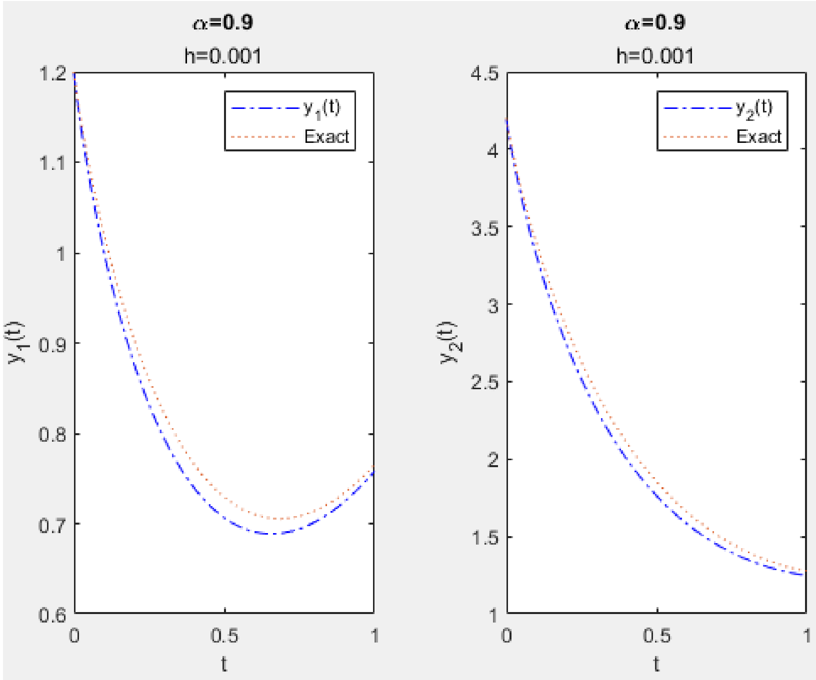

Figure 3. Numerical solutions of Example 1 for $h = 0.001$ at $\alpha = 0.9$.

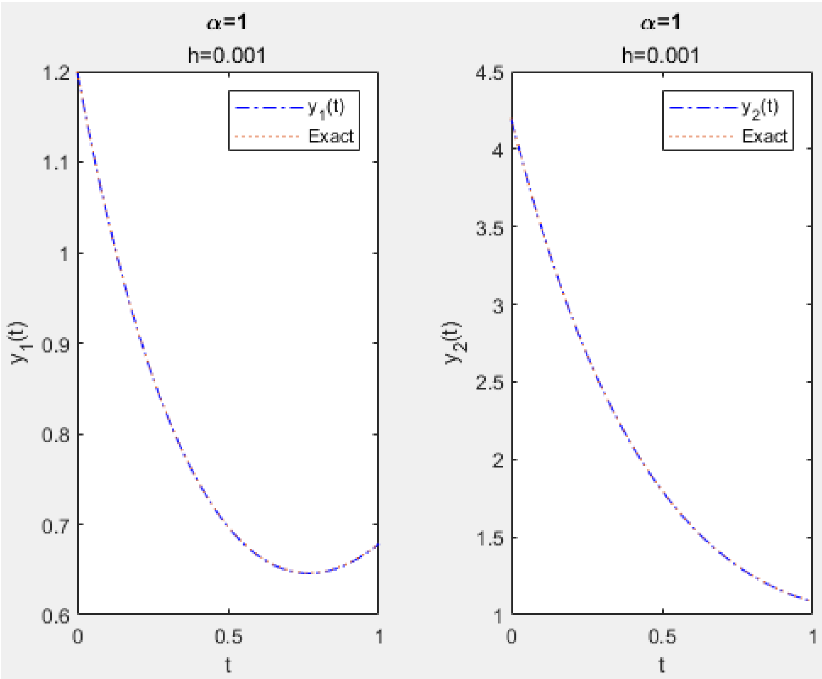

Figure 4. Numerical solutions of Example 1 for $h = 0.001$ at $\alpha = 1$.

Example 2: Let us consider the following fractional initial value problem:

$$\begin{pmatrix} {}^{C}D_t^{\alpha} y_1(t) \\ {}^{C}D_t^{\alpha} y_2(t) \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix}, \alpha \in (0,1], t \in [0, T] \ ,$$

with the initial condition

$$\begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$

The unique of this fractional initial value problem of system is established as

$$\begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} E_{\alpha,1}(t^{\alpha}) + \begin{pmatrix} 1 \\ 1 \end{pmatrix} E_{\alpha,1}(3t^{\alpha}).$$

The numerical solutions established by FFEM are given in Figure (5-8) for various values of α and h.

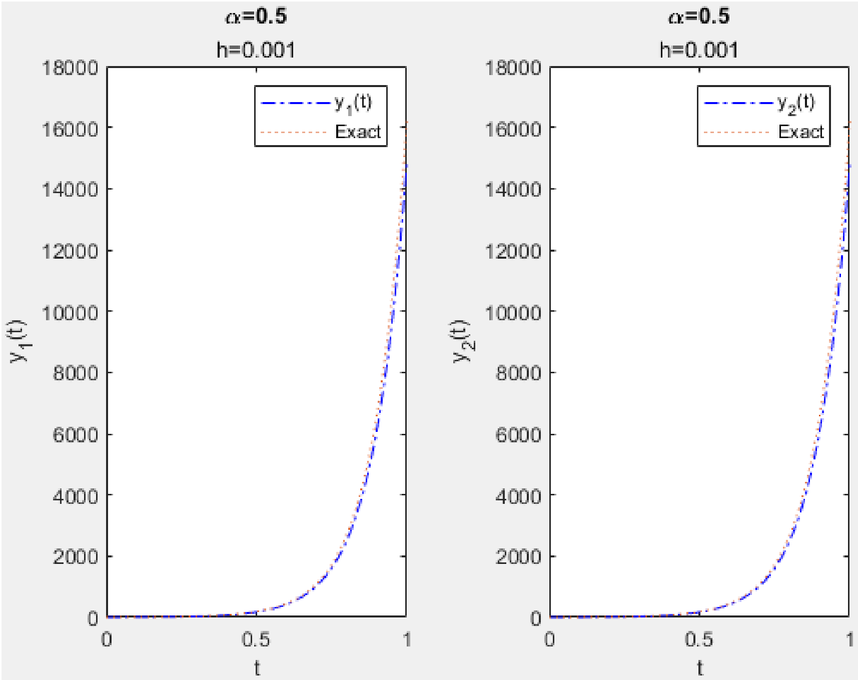

Figure 5. Numerical solutions of Example 2 for $h = 0.001$ at $\alpha = 0.5$.

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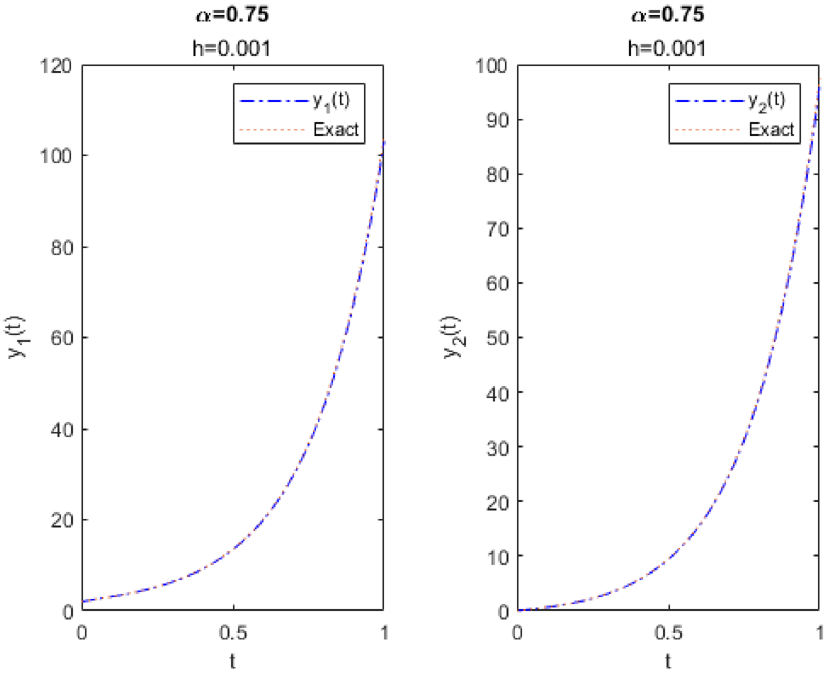

Figure 6. Numerical solutions of Example 2 for $h = 0.001$ at $\alpha = 0.75$.

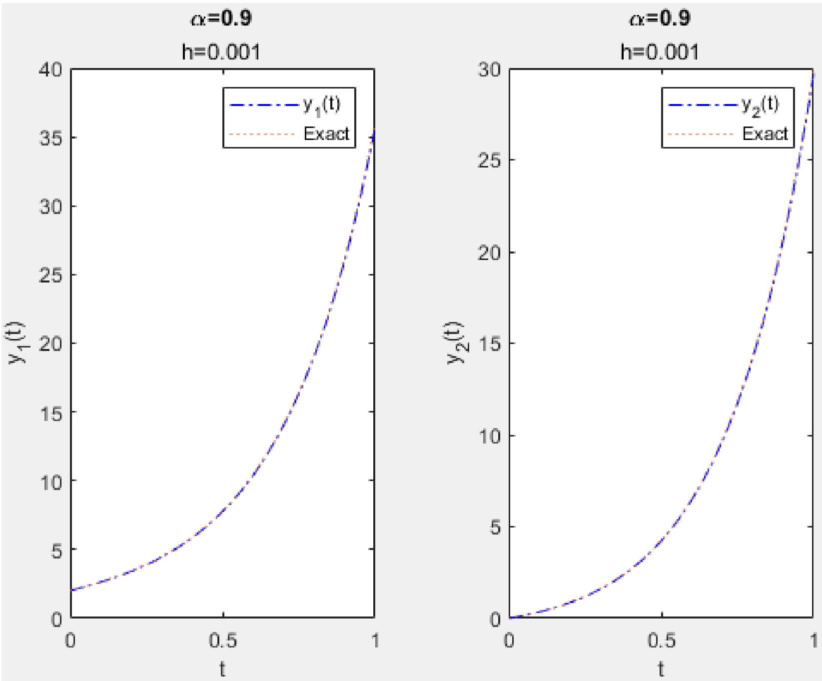

Figure 7. Numerical solutions of Example 2 for $h = 0.001$ at $\alpha = 0.9$.

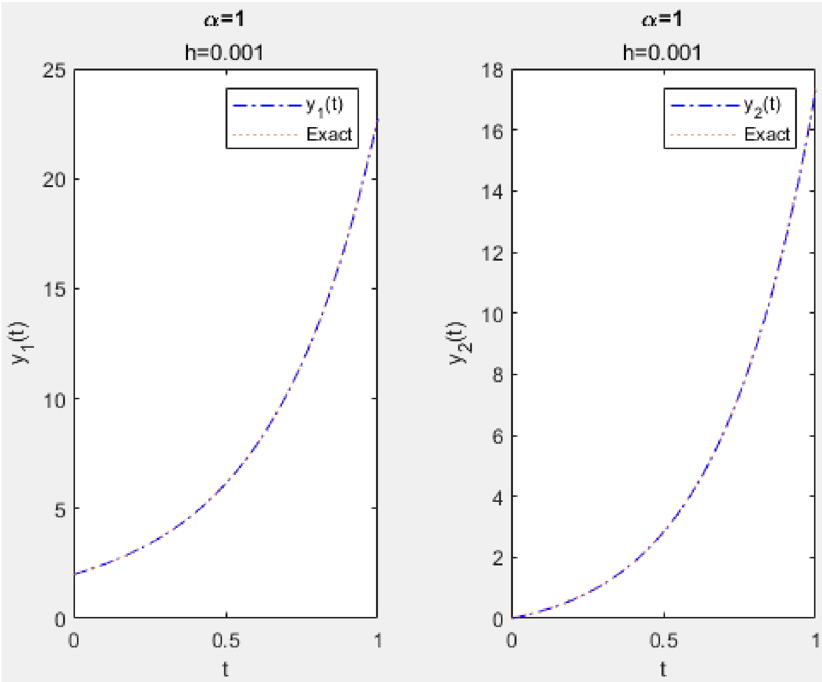

Figure 8. Numerical solutions of Example 2 for $h = 0.001$ at $\alpha = 1$.

Table 1. Percentage of absolute errors for different values of t and α at $h = \frac{1}{1000}$ for Example 2.

	$\alpha = 0.5$		$\alpha = 0.75$		$\alpha = 0.9$		$\alpha = 1$	
t	$y_1(t)$	$y_2(t)$	$y_1(t)$	$y_2(t)$	$y_1(t)$	$y_2(t)$	$y_1(t)$	$y_2(t)$
0.1	%1.11	%2.16	%0.13	%0.55	%0.049	%0.31	%0.026	%0.22
0.2	%1.96	%2.65	%0.23	%0.60	%0.098	%0.346	%0.057	%0.25
0.3	%2.87	%3.31	%0.35	%0.68	%0.13	%0.340	%0.092	%0.28
0.4	%3.79	%4.05	%0.48	%0.76	%0.20	%0.41	%0.13	%0.31
0.5	%4.71	%4.86	%0.60	%0.84	%0.26	%0.46	%0.17	%0.34
0.6	%5.61	%5.69	%0.74	%0.94	%0.33	%0.51	%0.21	%0.37
0.7	%6.51	%6.56	%0.88	%1.04	%0.40	%0.56	%0.25	%0.40
0.8	%7.41	%7.44	%1.01	%1.15	%0.47	%0.61	%0.30	%0.44
0.9	%8.32	%8.33	%1.15	%1.26	%0.53	%0.66	%0.026	%0.22

1	%9.23	%9.24	%1.28	%1.37	%0.60	%0.71	%0.057	%0.25

Table 2. Percentage of absolute errors for different values of t and h at $\alpha = 1$ for Example 2

	$h=\frac{1}{10}$		$h=\frac{1}{100}$		$h=\frac{1}{1000}$		$h=\frac{1}{10000}$	
t	$y_1(t)$	$y_2(t)$	$y_1(t)$	$y_2(t)$	$y_1(t)$	$y_2(t)$	$y_1(t)$	$y_2(t)$
0.1	%2.4	%22.34	%0.26	%2.25	%0.026	%0.22	%0.0026	%0.022
0.2	%2.9	%25.14	%0.56	%2.52	%0.057	%0.25	%0.0057	%0.025
0.3	%3.52	%28.14	%0.90	%2.80	%0.092	%0.28	%0.0092	%0.028
0.4	%4.32	%31.34	%1.28	%3.10	%0.13	%0.31	%0.013	%0.031
0.5	%5.32	%34.74	%1.68	%3.42	%0.17	%0.34	%0.017	%0.034
0.6	%6.59	%38.36	%2.12	%3.74	%0.21	%0.37	%0.021	%0.037
0.7	%8.22	%42.21	%2.57	%4.08	%0.25	%0.40	%0.025	%0.040
0.8	%10.30	%46.29	%3.04	%4.43	%0.30	%0.44	%0.030	%0.044
0.9	12.96	%50.60	%3.52	%4.79	%0.35	%0.47	%0.035	%0.047
1	16.37	%55.17	%4.01	%5.16	%0.40	%0.51	%0.04	%0.051

4. CONCLUSION

The initial value problems of fractional differential equations system in Caputo sense is considered and their numerical solutions are constructed by FFEM. The order of fractional derivatives are equal to each other in this study. A theorem is given for their asymptotic stability. Illustrative examples are demonstrated for the implementation of the method and its MATLAB codes. The obtained results prove the accuracy and effectiveness of FFEM and its MATLAB codes.

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KESİRLİ TÜREV İÇEREN HADAMARD TİPİ ÇOK NOKTALI SINIR DEĞER PROBLEMİNİN ÇÖZÜMLERİNİN VARLIĞI

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ÖZET

Bu çalışmada, p-Laplasyen operatörlü Hadamard kesirli türevli çok noktalı sınır değer problemi, $n-1<\alpha\leq n$ ve $n=[\![\alpha]\!]+1$ olmak üzere ,

$$(\varphi_p({}^{H}D_{1^+}^{\alpha}u(t))' + p(t)f\left({}^{H}D_{1^+}^{\alpha-1}u(t), {}^{H}D_{1^+}^{\alpha-2}u(t), \ldots, {}^{H}D_{1^+}^{\alpha-n-2}u(t), u(t)\right) = 0, \qquad t \in (1,\infty)$$

$$u^{(k)}(1) = 0, \;\; k = 0,1,2,\ldots,n-2,$$

$${}^{H}D_{1^+}^{\alpha}u(1) = 0, \;\; {}^{H}D_{1^+}^{\alpha-1}u(\infty) = \int_1^{\infty} g(t)u(t)\frac{ds}{s} + \sum_{i=1}^{m}\lambda_i\, {}^{H}I_{1^+}^{\beta_i}u(\eta)$$

incelenmiştir.

Burada ${}^{H}D_{1^+}^{\alpha}$, Hadamard kesirli türevi, ${}^{H}I_{1^+}^{\beta_i}$ ise Hadamard kesirli integralini ifade etmektedir. Ayrıca f fonksiyonu $f:\mathbb{R}^{n-1} \to \mathbb{R}$ belirli varsayımları sağlayan sürekli bir fonksiyondur. Bu çalışmada, sınır değer problemi integral denklem olarak ifade edilmiş ve Green fonksiyonu elde edilmiştir.Daha sonra Hadamard kesirli türevli sınır değer problemininin çözümlerinin varlığını garantilemek için yeterli koşullar elde edilmiştir.Elde edilen sonuçlarla ilgili örnek verilmiştir.

Anahtar Kelimeler : Hadamard kesirli türev, p-Laplasyen,Green fonksiyonu

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ÖKLİD UZAYINDA MANNHEİM EĞRİLERİ VE MANNHEİM EĞRİ ÇİFTLERİ

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ÖZET

Bu çalışma bazı özel eğrilerin Mannheim eğrisi ve Mannheim eğri çifti olması durumlarını inceliyoruz. İlk olarak 3-boyutlu Öklid uzayında Frenet çatısına göre Mannheim eğrilerinin ve Mannheim eğri çiftlerinin tanımları verilecektir. İkinci olarak bu tip eğrilerin eğrilik ve torsiyon yardımıyla çeşitli karakterizasyonları incelenecektir. Daha sonra bazı özel eğrilerin Mannheim eğri ve Mannheim eğri çifti olması için teoremler ispatlar ve bu eğrilerin örnekleri verilecektir.

Anahtar Kelim: Öklid uzayı, Mannheim eğrileri, Özel eğriler, Mannheim eğri çiftleri.

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STABILITY AND DYNAMICS OF A HUMAN-MOSQUITO MALARIA MODEL WITH INFECTED IMMIGRANTS

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Abstract:

This study investigates the stability dynamics of a modified SEIR model for malaria incorporating infected immigrants. The model differentiates between human and mosquito populations, employing an SEIR framework for humans and an SI model for mosquitoes. Humans transition through susceptible, exposed, infected, and recovered states following mosquito bites, while mosquitoes remain infected until death after biting an infected human. The basic reproduction number, R0R_0R0, is computed using the next-generation matrix approach. Stability analysis of the equilibrium points is conducted using the Lyapunov function, demonstrating the global stability of these points. The results provide insights into the long-term behavior of the malaria transmission model and its implications for public health strategies.

Keywords: Susceptible, exposed, infected, recovered, infected immigrants, basic reproduction number, Lyapunov function

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RELIABILITY ASSESSMENT OF DATA CENTERS AT KIGALI INSTITUTE OF SCIENCE AND TECHNOLOGY USING LRU ALGORITHM

A. M. Hassan, Nadia Faye, Kofi Mensah

North Africa University

Abstract:

This study investigates the reliability and performance of data centers at Kigali Institute of Science and Technology (KIST), Kigali, Rwanda. The data center is comprised of three servers: one dedicated to database management, one as a redundant backup, and one for local client interactions. Various reliability metrics, including availability, reliability, mean time to failure (MTTF), and cost-benefit analysis, are evaluated to understand the system's robustness. The analysis identifies potential failure modes, including router malfunctions, redundant server issues, and switch failures, which could lead to system downtime. Local server failures result in partial system failure, while complete failure scenarios include cooling system breakdowns, power outages, or natural disasters such as earthquakes and fires. Failures are addressed using the Least Recently Used (LRU) technique, and failure rates are modeled with exponential time distribution, with repairs following general and Gumbel-Hougaard family copula distributions.

Keywords: Reliability, Availability, Gumbel-Hougaard Family Copula, MTTF, Data Center.

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INTEGRATING PYTHON PROGRAMMING WITH ANALYTIC GEOMETRY CONCEPTS

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Abstract:

This paper introduces an innovative application developed in Python that illustrates the benefits of integrating foundational mathematical concepts with programming skills. The application focuses on the basics of analytic geometry, specifically conic sections and quadrics, including their transformation to standard forms. Python is selected due to its straightforward syntax, which enhances readability, and its capability for code reusability—key features for mathematicians who leverage established problems to address new challenges. The goal is twofold: first, to advocate for using mathematical concepts as a gateway to learning programming, and conversely, to demonstrate how programming can facilitate the understanding of mathematical principles through practical projects. Additionally, the application serves as a practical tool, presenting itself as an independent Python package dedicated to analytic geometry.

Keywords: Analytic geometry, conic sections, Python programming, quadrics.

ADVANCED IMPLICIT EULERIAN APPROACH FOR MODELING HIGHLY DEFORMABLE ELASTIC MEMBRANES IN NEWTONIAN FLUIDS"

Lina Oliveira, Haruto Tanaka, Anwar Ahsan

University of Ljubljana, Slovenia

Abstract

This study introduces an advanced implicit Eulerian method designed to model the interaction between highly deformable elastic membranes and Newtonian fluids. The method focuses on accurately representing large deformations in elastic membranes by incorporating a simplified model where surface strain energy is a function of membrane stretching. Utilizing a fully Eulerian framework, the method tracks membrane deformations through a level set approach and updates the modified surface tension tensor. The nonlinear equations derived are efficiently solved using a Newton-Raphson technique, demonstrating quadratic convergence. We implement a monolithic solver and present extensive numerical experiments to validate the model and highlight the accuracy of our approach. Our results indicate that stability is preserved even for substantially larger time steps.

Keywords

Fluid-membrane interaction, deformation modeling, Eulerian method, finite element analysis, Newton-Raphson method, implicit scheme.

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ADVANCED ANALYTICAL TECHNIQUES FOR COROTATIONAL MAXWELL FLUIDS IN WIRE COATING PROCESSES

Li Wei Zhang, Olufemi Adewale, Elsa Müller

Institute of Theoretical Physics, University of Vienna, Austria

Abstract:

This study introduces innovative analytical approaches for solving the non-linear equations governing the flow of a corotational Maxwell fluid used in wire coating processes. The paper utilizes the Enhanced Homotopy Perturbation Method (EHPM) and the Advanced Homotopy Asymptotic Method (AHAM) to derive precise analytic approximations for the velocity field in a non-dimensional framework. A comparative analysis of solutions obtained via these methods, varying the non-dimensional parameter l10, demonstrates that the EHPM provides superior accuracy and usability. The EHPM solution's accuracy can be further refined by optimizing the choice of auxiliary functions within the same order of approximation.

Keywords: Wire coating processes, Corotational Maxwell fluid, Enhanced Homotopy Perturbation Method, Advanced Homotopy Asymptotic Method

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AN ANALYSIS OF STOCHASTIC INTEGRALS IN CATASTROPHIC EVENT MODELS

Dr. Leila Martins, Dr. Huan Zhao, Dr. Amina Osei

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Abstract:

This study explores stochastic integrals within the context of catastrophic event models. Specifically, we examine the dynamics of population models influenced by catastrophic mutations and derive the differential equations governing the joint generating functions for mutant populations and their integrals. We analyze the first-order moments of the dual mutation processes and the second-order moments for a unidirectional mutation model. Additionally, this paper investigates the asymptotic behavior of these integrals as they relate to the limiting distributions of mutant populations over time.

Keywords: Stochastic integrals, population models, catastrophic mutations, asymptotic analysis.

OPTIMAL BLOCK DESIGN STRATEGIES FOR MAIN EFFECTS IN EXPERIMENTAL STUDIES

Lian Chen, Fatima Ahmed

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Abstract:

This paper explores advanced methodologies for constructing optimal blocked main effects plans when investigating mmm two-level factors using nnn experimental runs distributed across bbb blocks, which may vary in size. We assume that block sizes are uniformly even for all blocks and focus on the scenario where n≡2 (mod 4)n \equiv 2 \ (\text{mod} \ 4)n≡2 (mod 4). The study presents optimal designs based on type 1 and type 2 criteria within a design class that ensures orthogonal estimation of main effects relative to block effects. This orthogonal estimation is frequently sought in practical applications. Additionally, we examine E-optimal designs within a broader class of blocked main effects plans where factors are not equally represented at high and low levels across all blocks. Construction techniques utilizing Hadamard matrices and Kronecker products are introduced to facilitate the development of these optimal designs.

Keywords: Block design, E-optimality, Hadamard matrix, Kronecker product, Type 1 and Type 2 optimality.

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PERFORMANCE ANALYSIS AND MODELING OF LOADING FACTORS IN CENTRIFUGAL COMPRESSOR IMPELLERS

Dr. Maria de Souza, Prof. Wei Liu

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Abstract:

Understanding the performance of loading factors is crucial for accurately modeling the gas dynamic performance curves of centrifugal compressors. This study explores the relationship between loading factors and flow coefficients at the impeller exit, emphasizing that the performance characteristics are independent of compressibility criteria. We introduce two key parameters for simulating loading factor performances: the loading factor at zero flow rate and the angle between the ordinate and the performance line. Our results show that, for non-viscous flow conditions, the calculated loading factor performances are linear and align closely with experimental data. We analyzed the performance of several impellers with varying blade exit angles, blade thickness, blade count, and blade height ratios, along with two distinct blade mean line configurations. Notable trends include the relatively minor impact of blade thickness and a more significant influence of geometric parameters on impellers with larger blade exit angles. We propose approximating equations for both parameters and outline the next phase of our research, which will involve simulating experimental performances based on these equations.

Keywords: Centrifugal compressor, loading factor, gas dynamic performance curve, performance modeling.

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ADVANCED DISCRETE EVOLUTIONARY SPLINES FOR MODELING OCCLUSION IN TEMPOROMANDIBULAR DISORDERS

Sofia Mendes, Hiroshi Tanaka,

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Abstract:

This study explores the modeling of occlusion in patients with temporomandibular disorders (TMD) using an advanced discrete evolutionary spline approach. The problem is formulated as an evolutionary partial differential equation, incorporating specific boundary conditions relevant to TMD cases. We establish the existence and uniqueness of the solution for this model and present a convergence analysis of the proposed discrete variational splines. To validate our method, we perform a stress analysis of the jaw occlusion in TMD using finite element methods in the FreeFem++ software. The results demonstrate the effectiveness and accuracy of the proposed approach in capturing the complexities of TMD occlusion.

Keywords: Evolutionary PDE, discrete splines, finite element analysis, temporomandibular disorders, numerical modeling.

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ENHANCING FORECAST ACCURACY THROUGH NORMALIZATION OF REALIZED VOLATILITY IN LONG-MEMORY MODELS

M. A. Silva, L. Zhang, and T. O. Nkosi
School of Economics, Shandong University, China

Abstract:

Accurately modeling realized volatility using high-frequency returns has gained popularity due to its unbiased and efficient nature as a volatility estimator. This study explores the efficacy of normalizing the logarithms of realized volatilities within a fractionally integrated long-memory Gaussian framework. The Gaussian assumption facilitates parameter estimation through the Whittle approximation; however, this assumption may not hold in finite samples, necessitating the normalization of financial series. By analyzing empirical data from the S&P 500 and DAX indices, this paper evaluates the performance of a linear volatility model enhanced by normalization as a pre-treatment compared to traditional models. The findings reveal that incorporating normalization significantly improves forecast accuracy, outperforming existing models in both statistical and economic assessments.

Keywords: Long-memory, Gaussian process, Whittle estimator, normalization, volatility, value-at-risk.

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ENHANCING LOUDSPEAKER DESIGN PARAMETERS THROUGH AIR VISCOSITY DAMPING OPTIMIZATION

Julia Martin, Chen Wei, Ahmed El-Sayed, Sofia Ivankova, Paulo Silva

Department of Acoustics, Sofia University, Bulgaria

Abstract:

This research focuses on optimizing the design parameters of cone loudspeakers by incorporating the effects of air viscosity damping. To address the challenges associated with fine-tuning loudspeaker designs, we developed an advanced acoustic analysis software that factors in air viscosity effects. The software aims to improve the design process by optimizing parameters such as cone material and edge construction, using vibration displacement of the cone paper as the primary objective function. The analysis demonstrated that our design approach achieves high precision compared to initial predictions. These findings indicate that, despite the complexities inherent in parameter design, our approach—guided by empirical insights and optimized algorithms—simplifies the design process significantly.

Keywords: Air viscosity, design optimization, loudspeaker acoustics, advanced analysis.

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PROPERTIES OF QUASI-CONFORMALLY FLAT LP-SASAKIAN MANIFOLDS WITH CONSTANT COEFFICIENT

Elena Rodrigues, Aiden Schmidt,

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Abstract:

This paper investigates the characteristics of quasi-conformally flat LP-Sasakian manifolds with a constant coefficient α. We establish that a quasi-conformally flat LP-Sasakian manifold MMM (where n>3n > 3n>3) with a fixed coefficient α is an η-Einstein manifold. Additionally, we demonstrate that if the scalar curvature tensor of such a manifold is constant, then MMM possesses constant curvature. These results enhance our understanding of the geometric properties and curvature conditions of LP-Sasakian manifolds in this specific context.

Keywords: LP-Sasakian manifolds, constant coefficient α, quasi-conformal curvature tensor, η-Einstein manifold, scalar curvature

A NOVEL APPROACH TO NUMERICAL SOLUTIONS FOR REACTION-DIFFUSION SYSTEMS ON CLOSED SURFACES

Dr. Niazi Barakat, Dr. Anna Zhen, Dr. Hassan Ghanem

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Abstract:

Reaction-diffusion equations play a crucial role in various fields such as mathematical biology, material science, and physics. Despite their significance, developing efficient numerical techniques for these systems on curved surfaces remains a challenging problem. This paper introduces a novel geometric approach for solving reaction-diffusion equations on closed surfaces using an O(r)-LTL configuration method. This method integrates local tangential lifting techniques with configuration equations to provide an effective estimation of differential quantities on curved surfaces. Since accurately approximating the Laplace-Beltrami operator is essential for solving these equations, we employ both the local tangential lifting technique and a generalized finite difference approach to approximate these operators. Our method is not only conceptually straightforward but also practical to implement.

Keywords: Closed surfaces, high-order methods, numerical solutions, reaction-diffusion systems.

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AN ADVANCED NUMERICAL TECHNIQUE FOR DIFFUSION AND CAHN-HILLIARD EQUATIONS ON DYNAMIC SPHERICAL GEOMETRIES

Li Xianjun, Maria Orellana

Department of Applied Mathematics, University of Lublin, Poland

Abstract:

This study introduces a novel numerical geometric approach for accurately assessing the divergence of vector fields on dynamically evolving spherical surfaces. Conservation principles, central to both physical and mathematical frameworks, are often compromised by conventional numerical methods for diffusion equations. Our proposed technique integrates the divergence theorem with an advanced generalized finite difference scheme, ensuring adherence to conservation laws on discrete closed surfaces. This methodology is further applied to solve Cahn-Hilliard equations on spherical surfaces undergoing evolution, demonstrating notable stability and accuracy in numerical simulations.

Keywords: Conservation principles, diffusion modeling, Cahn-Hilliard equations, dynamic spherical surfaces.

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ANALYSIS OF DYNAMIC STABILITY IN AN EXTENDED MODEL OF THE ENDOCRINE FEEDBACK SYSTEM

Dr. Alejandro Silva , Dr. Mei Ling Tan

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Abstract:

This study presents a mathematical framework to examine the dynamics of an extended model of the hypothalamus-pituitary-thyroid (HPT) axis, incorporating additional couplings and delay elements. The model integrates two distinct types of couplings and incorporates delays to account for the time required for hormonal transport. The feedback mechanisms within the system regulate thyroid hormone secretion, while the introduced delays represent time lags. The impact of these delayed feedback mechanisms on the system's stability is analyzed. Analytical findings are supported by numerical simulations demonstrating various dynamical behaviors, including normal thyroid function and several pathological conditions such as hyperthyroidism.

Keywords: Mathematical modeling, ordinary differential equations, endocrine feedback, stability analysis.

Characterization of (λ, μ)-Fuzzy Subgroup Structures in Operator-Groups

Mei Lin, Akira Tanaka, and João Oliveira

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Abstract:

This study explores the framework of (λ, μ)-fuzzy subgroups and (λ, μ)-fuzzy normal subgroups within the context of groups with operators. We focus on their properties and classifications, utilizing M-group homomorphism as a foundational tool. The paper aims to expand the understanding of these fuzzy structures and their applications in various algebraic settings.

Keywords: Fuzzy subgroup structures, (λ, μ)-fuzzy subgroups, (λ, μ)-fuzzy normal subgroups, M-group homomorphism.

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INNOVATIVE DESIGN OF FRACTIONAL ORDER CONTROLLERS FOR VIBRATION REDUCTION IN AIRCRAFT WING STRUCTURES

Leila Martins, Yassir Bouaziz, Elena Kovač, Nikoleta Petrovic
University: Faculty of Engineering, University of Novi Sad, Serbia

Abstract:

Aircraft wing structures are critical for maintaining the stability, lift, and maneuverability of airplanes. The aerodynamic profile of the wing can be effectively modeled as a smart beam. To mitigate vibrations, this study explores the design and implementation of fractional order controllers using piezoelectric actuators positioned on the beam's surface. A novel graphical method for tuning these controllers in the frequency domain is proposed. The effectiveness of this approach is demonstrated through practical experiments conducted on a scaled laboratory model.

Keywords: Fractional order controllers, piezoelectric actuators, smart beam, vibration control.

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ROBUST VARIOGRAM FITTING USING THE MODIFIED HUBER NORM

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Abstract:

In the field of geostatistics, precise estimation of variogram points is crucial, particularly for developing robust methods. While parametric approaches for variogram fitting, which define kriging weights, are well-studied, their robustness has been less explored. This paper introduces a novel approach by applying the modified Huber norm as a robust alternative to the conventional non-linear weighted least squares for variogram fitting. We begin by outlining the principles of variogram estimation and fitting, then propose the modified Huber estimator as a robust method. The performance of the modified Huber estimator is evaluated using a contaminated spatial data set to demonstrate its robustness. We also perform simulations to assess the estimator's performance under various levels of contamination. Results indicate that the modified Huber norm consistently outperforms the weighted least squares method across all contamination levels, including the absence of contamination. This highlights the effectiveness of the modified Huber norm in providing robust variogram fitting.

Keywords: Modified Huber Norm, robust estimation, Variogram fitting

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EVALUATING THE EFFECTIVENESS OF STRATIFIED DOUBLE MEDIAN RANKED SET SAMPLING FOR POPULATION MEAN ESTIMATION

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Abstract:

This study explores the effectiveness of the Stratified Double Median Ranked Set Sampling (SDMRSS) method for estimating population means. We compare SDMRSS with traditional sampling techniques, including Simple Random Sampling (SRS), Stratified Simple Random Sampling (SSRS), and Stratified Ranked Set Sampling (SRSS). The results demonstrate that SDMRSS provides an unbiased estimate of the population mean and exhibits superior efficiency compared to SRS, SSRS, and SRSS. Additionally, SDMRSS significantly enhances the precision of mean estimation for specific sample sizes. Practical applications of SDMRSS are discussed, with real-life examples corroborating the theoretical findings.

Keywords: Efficiency, double median ranked set sampling, median ranked set sampling, ranked set sampling, stratified sampling.

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ADVANCING THE EXTENDED TRAPEZOIDAL TECHNIQUE FOR NUMERICAL RESOLUTION OF VOLTERRA INTEGRO-DIFFERENTIAL EQUATIONS

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Abstract:

Volterra integro-differential equations frequently model complex real-world phenomena. Given the difficulty or even infeasibility of obtaining analytical solutions for such equations, researchers often turn to numerical methods to achieve high accuracy. Traditional numerical approaches for ordinary differential equations are adapted for this purpose. This study introduces an advanced variant of the trapezoidal method tailored for solving Volterra integro-differential equations. Detailed formulae for both the integral and differential components of the equations are provided. The numerical experiments conducted demonstrate that this enhanced trapezoidal method effectively addresses first-order Volterra integro-differential equations with considerable precision.

Keywords: Accuracy, extended trapezoidal method, numerical solution, Volterra integro-differential equations.

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A NOVEL COMPUTATIONAL APPROACH FOR HYPER-ELASTIC STRUCTURAL ANALYSIS USING LAGRANGIAN-HAMILTONIAN FRAMEWORK

Dr. Ana Oliveira, Prof. Yassir Malek, Dr. Lin Zhang

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Abstract:

This study explores a novel computational approach based on the Lagrangian-Hamiltonian framework for simulating hyper-elastic structural dynamics. The governing equations of motion are derived using the principle of least action, and the deformation gradient is calculated via the Weighted Least Squares method. The investigation employs hyper-elasticity models, specifically Saint Venant-Kirchhoff and a compressible variant akin to Mooney-Rivlin, for computing the second Piola-Kirchhoff stress tensor. The stability and accuracy of the numerical model are evaluated by reproducing critical stress fields under static and dynamic conditions. The findings indicate that while the Hamiltonian-based model demonstrates satisfactory performance in handling intense extensional stress fields, it encounters certain instabilities during violent collisions, likely due to zero-energy singular modes.

Keywords: Principle of least action, computational methods, hyper-elasticity, stability analysis