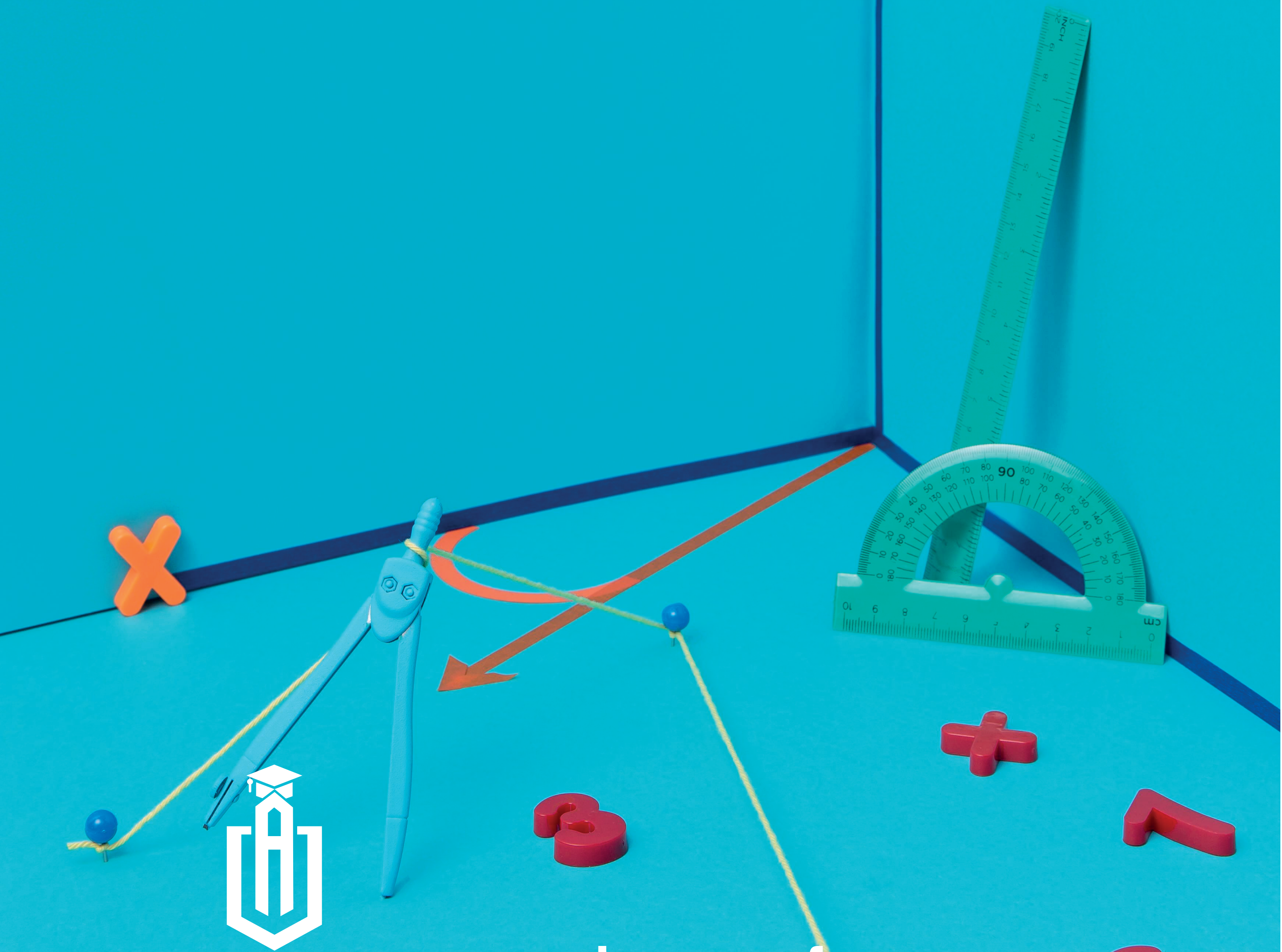
ICSAS

3rd INTERNATIONAL CONFERENCE ON MATHEMATIC

September 19 - 21, 2025

IZMIR

FACE TO FACE and ONLINE



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ICSAS 3rd INTERNATIONAL CONFERENCE ON MATHEMATIC
September 19 - 21, 2025
Izmir

Edited By
Prof. Dr. Elif Akpınar Külekçi

ORGANIIZATION
ACADEMY GLOBAL CONFERENCES

EVALUATION PROCESS
All applications have undergone a double-blind peer review process.

PARTICIPATING COUNTRIES

TURKEY –Georgia – Ghana – Austria- South Korea- Iran - Portugal- Uzbekistan- Pakistan- Netherlands – Japan- Saudi Arabia- India- South Africa- Tunisia- Taiwan-

PRESENTATION
Oral presentation

PERCENTAGE OF PARTICIPATION
More than 50 % of paper are presented by participants from maintained countries.
8 papers from Turkey and 17 paper from other countries.

Members of the organizing committees of the conference perform their duties with an "official assignment letter"

LANGUAGES
Turkish, English, Russian, Persian, Arabic

CONGRESS ORGANIZING BOARD

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TÜRKİYE CUMHURİYETİ'NİN YÜZÜNCÜ YILI

Sayı : E-16710634-03-903.07.02-2300384284 01.12.2023

Konu : Doç.Dr.Elif AKPINAR KÜLEKÇİ'nin Görevlendirilmesi

MİMARLIK VE TASARIM FAKÜLTESİ DEKANLIĞINA

İlgi : 29.11.2023 tarihli ve E-53120705-000-2300381989 sayılı belge.

Fakülteniz Peyzaj Mimarlığı Bölümü öğretim üyelerinden Doç.Dr.Elif AKPINAR KÜLEKÇİ'nin, Yükseköğretim Genel Kurulunun 15.06.2023 tarihli, 10 sayılı oturumunda alınan 2023.10.183 sayılı kararı gereğince Doçentlik Başvuru Şartlarında bulunan ve doçent olacak adaylardan istenen "Diğer uluslararası/ ulusal bilimsel toplantının düzenleme komitesinde resmi olarak görevlendirilmiş üniversite akademisyen temsilcisi bulunması zorunludur." maddesi gereğince, Academy Global Conference & Journals tarafından yapılan kongrelerin düzenleme kurullarında yolluksuz ve gündeliksiz olarak görevlendirilmesi Rektörlüğümüzce uygun görülmüştür.

Bilgilerini ve gereğini rica ederim.

Prof.Dr. Ömer ÇOMAKLI
Rektör

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Meeting ID: 885 7151 8350

Passcode: 202224

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- **Oturum içerisinde en KIDEMLİ olan moderator olarak seçilir. Moderatörün oturum düzenini gözetmesi, akademisyen adaylarını yönlendirmesi beklenmektedir.**
- Oturuma bağlanmadan önce Salon numaranızı adınızın önüne aşağıdaki gibi ekleyiniz. Bu sayede kongre açılışında beklemeden oturumlarınıza gönderilebileceksiniz. Ör. 5 Ahmet Ahmetoglu
- **Sunum süresi 10 dakikadır. Bu sürenin aşılmamasını moderatörler temin edecektir.**
- Sunum sonrası 5 dakikayı geçmeyen soru-cevap, tartışma süresi verilmektedir.
- **Sunumlar TÜRKÇE veya İNGİLİZCE yapılabilmektedir.**
- Kameralar, oturum süresince toplam % 70 oranında açık olmak zorundadır.
- **Sunum yapan katılımcının kamerası açık olmak zorundadır.**
- Sunum yapmak zorunludur. **Herhangi bir nedenle sunum yapmamış olan katılımcıya sertifika verilmesi ve çalışmasının yayınlanması sözkonusu olamaz.**
- Katılımcı, kendi oturumda, oturum bitene kadar bulunmak zorundadır.
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Salon	Moderator		Bildiri No ve Başlığı / Paper ID and Title	Authors
HALL / SALON 1	**Asst. Prof. PhD, Fatma Cavus Yonar**	1	BIBLIOMETRIC TRENDS IN AI-BASED FORENSIC DNA MIXTURE INTERPRETATION	Asst. Prof. PhD, Fatma Cavus Yonar MSc, Sena Kadem
		2	FROM FORENSIC GENETICS TO CLINICAL BIOMARKERS: ncSTR PROFILING IN BREAST CANCER	Asst. Prof. PhD, Fatma Cavus Yonar
		3	ORGAN-SPECİFİC DİSTRİBUTİON OF AMİNO ACİDS AND TRACE ELEMENTS İN VERBASCUM TUNA-EKİMİİ: A PHYTOCHEMİCAL AND PHARMACOLOGİCAL PERSPECTİVE	Assist. Prof. Dr. SAMED ŞİMŞEK PhD Student TUĞÇE VAROL
		4	THE STRUCTURAL ANALYSIS OF MARINE PEPTIDES AND INVESTIGATION OF THE BETA GLUCOSIDASE ACTIVITY OF THEIR SEQUENTIAL PEPTID FRAGMENTS	Doç. Dr. Sibel AVUNDUK Dr. Burcu OMUZBÜKEN Prof. Dr. Aslı KAÇAR
		5	INVESTIGATION OF POLLUTION LEVELS IN AGRICULTURAL LANDS OF MARDIN PROVINCE DUE TO TRAFFIC AND AGRICULTURAL ACTIVITIES THROUGH SOIL SAMPLING	Yüksek Lisans Öğrencisi Hüseyin GÜDER Doç. Dr. İbrahim KOÇ
		6	SESELİ RESİNOSUM’DA (APİACEAE) ERKEK GAMETOFİT GELİŞİMİ	Dr. Öğr. Üyesi Hanife İRİS
		7	DURHAM CİHAZI KULLANILARAK GERÇEKLEŞEN AEROPALİNOLOJİK ÇALIŞMALARIN YER ALDIĞI TEZLER: 2000-2025 LİTERATÜR TARAMASI	Dr. Öğr. Üyesi Hanife İRİS
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HALL / SALON 2	**Asst. Prof. Dr. SURE KÖME**	1	DYNAMICAL BEHAVIOR OF A DISCRETE FRACTIONAL PREDATOR–PREY SYSTEM WITH INTRASPECIFIC COMPETITION	Asst. Prof. Dr. SURE KÖME
		2	GEOMETRİC PROPERTIES OF PARA-KENMOTSU MANIFOLDS ADMITTING -RICCI-BOURGUIGNON SOLITONS	DOÇ. DR. TUĞBA MERT PROF. DR. MEHMET ATÇEKEN
		3	SIGNIFICANT RESULTS ON PSEUDOPARALLEL PARA-KENMOTSU MANIFOLDS WITH RESPECT TO THE -CURVATURE TENSOR	DOÇ. DR. TUĞBA MERT PROF. DR. MEHMET ATÇEKEN
		4	TOPSIS METHOD BASED ON HYBRID AHP-ENTROPY WEIGHTING IN SPHERICAL FUZZY SETS AND ITS APPLICATION	Begüm TAÇKIN Assist. Prof. Dr. Elif GÜNER Prof. Dr. Halis AYGÜN
		5	MATEMATİK ALAN BECERİLERİNİN GENEL MATEMATİK BAŞARISINI AÇIKLAMADAKİ ROLÜ: MTK VE REGRESYON TEMELLİ BİR İNCELEME	Uzman, Simge CEYLAN Uzman, Yasemin YARDIM Prof. Dr. Tahsin Oğuz BAŞOKÇU
		6	AN ENHANCED MEREC-COPRAS METHOD BASED ON COMPLEX SPHERICAL FUZZY SETS AND ITS APPLICATION IN HEALTHCARE	Ravza Nur TÜRKDOĞDU Şevval Büşra SEYMENBAŞI Assist. Prof. Dr. Elif GÜNER Prof. Dr. Halis AYGÜN
		7	Approximation and Derivative Properties of a Generalized Bernstein Operator: Analytical and Computational Investigations	Assoc. Prof. Dr.,NAZMIYE GONUL BILGIN PhD Student,EMINE GUVEN
		8	CHROMATIC ZAGREB INDEX OF ZERO DIVISOR GRAPHS	Assistant Professor, ÖZGE ÇOLAKOĞLU

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HALL / SALON 3	**Prof. Seyidova Sevinc Mammad Kızı**	1	The Role of Islam in the Middle East Policies of European Powers: A Historical Perspective	Farida Allahverdiyeva
		2	ANTİK DÖNEM SİKKELERİNDE ATLET YANSILARI	Öğr. Gör. Dr. ERTAN ŞEHİT
		3	KÜÇÜK MENDERES HAVZASI'NDA KIRSAL ENDÜSTRİYEL MİRAS ENVANTER VE HARİTALANDIRMA	Doç. Dr Olcay Pullukçuoğlu Yapucu Prof Dr. Aylin Ü. Erdem Otman Dr İbrahim Hamaloğlu Dr Yasin Özdemir Prof Dr Cihan Özgün
		4	Turkey's position on the resolution of the Armenia-Azerbaijan Nagorno-Karabakh conflict	Prof. Seyidova Sevinc Mammad Kızı

ICSAS 1ST INTERNATIONAL CONFERENCE ON MARRIAGE AND FAMILY THERAPY SEPTEMBER 19 - 21, 2025 IZMIR Meeting ID: 885 7151 8350 Passcode: 202224				
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Salon	Moderator		Bildiri No ve Başlığı / Paper ID and Title	Authors
HALL / SALON 4	**Doç. Dr. Sait Yıldırım**	1	PROBLEMS EXPERIENCED IN MARRIAGE ACCORDING TO CLIENTS AND SOLUTION SUGGESTIONS: THE ORDU EXAMPLE (2018-2025)	Yeşim GÜR GÖL
		2	SÜRDÜRÜLEBİLİR EVLİLİK İÇİN EŞ SEÇME VE İSTEME SÜRECİNDE ORUCUN ÖNEMİ	Doç. Dr. SEZAİ BEKDEMİR
		3	SOSYAL MEDYA VE EVLİLİK ALGILARI	Yüksek Lisans Öğrencisi, SÜMEYYE ÜSTÜN ALKAN
		4	MEDIATION AND COUNSELLING IN MUSLIM FAMILIES IN THE ENGLAND (THE EXAMPLE OF THE ISLAMIC SHARIA COUNCIL)	Associate Professor Yusuf EŞİT
		5	YAPISAL AİLE TERAPİSİ KURAMI PERSPEKTİFİNDEN “DANGAL” FİLMİNİN ANALİZİ	Yüksek Lisans Öğrencisi, Kübra UÇARSU
		6	DİJİTAL BAĞIMLILIK ÖZELİNDE TİKTOK UYGULAMASININ ZAMAN YÖNETİMİ VE İLİŞKİ DOYUMU AÇISINDAN EVLİLİKTEKİ ÇATIŞMA DİNAMİKLERİNE ETKİSİNİN İNCELENMESİ	Doç. Dr. Sait Yıldırım
		7	MEVCUT AİLE YAPISINDA YAŞANAN SORUNLARIN X VE Y KUŞAĞI KADINLAR TARAFINDAN DEĞERLENDİRİLMESİ: AZERBAYCAN GADABAY KENTİ ÖRNEĞİ	Doç. Dr. Sait Yıldırım Lisans Öğrencisi Ayan Əliyeva
		8	EVLİLİKTE ALDATMA OLGUSU: NEDENLERİ, TÜRLERİ, VE SONUÇLARI ÜZERİNE BİR DEĞERLENDİRME	Yüksek Lisans Öğrencisi, YASEMİN SEVDE KÖKÇÜ Prof.Dr. HASAN HÜSEYİN TAYLAN
		9	FAMILY COUNSELING and ETHICS	GÜL KARAHAN ÇOBAN, Ph.D.

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HALL / SALON 1	**Assoc. Prof. Dr. Noura Al-Harbi**	1	VIRTUAL BIOLOGY LABORATORY FOR ENHANCED STUDENT LEARNING	Dr. Nurul Huda Dr. Amirul Faizal
		2	INTEGRATING ONLINE LABORATORIES INTO SECONDARY BIOLOGY EDUCATION IN KAZAKHSTAN	Ayan Mukhtar Zhanar Bektemir Dinara Sadykova
		3	BIOLOGICALLY INSPIRED REACTIVE AGENT MODELING USING X-MACHINES	Dimitrios Papadopoulos Elena Nikolaidis Yannis Christodoulou
		4	CRYPTOGENIC FRESHWATER FISH BIODIVERSITY IN BANGLADESH	Dr. Md. Hasan Dr. Kamal Uddin Md. Tarek Hossain
		5	UNIFIED MODELS IN GENOME REARRANGEMENT AND SORTING PRIMITIVES IN BIOINFORMATICS	Dr. Anil Kumar Assis. Prof. Dr. Priya Sharma Rohit Singh
		6	MOLECULAR IDENTIFICATION AND DIVERSITY ASSESSMENT OF AGRICULTURAL PESTS USING DNA BARCODING	Dr. Saud Al-Mutairi Faisal Al-Qahtani Assoc. Prof. Dr. Noura Al-Harbi
		7	ADVANCED MICROFLUIDIC TECHNIQUES FOR BIOMEDICAL AND BIOHEALTH APPLICATIONS	Dr. Zahra Roudsari Dr. Nima Etemad
		8	EVALUATING THE EFFECTIVENESS OF INTERPROFESSIONAL SIMULATION TRAINING ON COLLABORATIVE SKILLS AMONG HEALTHCARE UNDERGRADUATES	Assoc. Prof. Dr. L. Thompson Ramirez, Lec. N. Khamal
		9	SUSTAINABLE POULTRY FEED FORMULATIONS BASED ON ALTERNATIVE ENERGY SOURCES	Dr. Chinedu Okoro Ifeanyi Nwosu Ngozi Eze

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HALL / SALON 2	**Assoc. Prof. Dr. Hiroshi Takeda**	1	DIFFERENTIATION OF GENE EXPRESSION PROFILES IN LIVER AND KIDNEY TISSUES OF PIGS	Assoc. Pro. Dr. Sergey Sokolova
		2	DECISION TREES FOR PREDICTING RISK OF MORTALITY USING ROUTINELY COLLECTED DATA	Dr. Emily Carter Dr. James Walker Dr. Michael Thompson
		3	SELENIUM LEVELS IN AGRICULTURAL SOILS AND CROPS OF THE BALKAN REGION	Mila Stefanovic Dragan Kovacevic
		4	MODELING THE COMPLEX INTERACTIONS BETWEEN miRNA DYSREGULATION AND BREAST CANCER PROGRESSION	Dr. Neda Karimi Assoc. Prof. Dr. Pardis Mousavi
		5	MODELING THE COMPLEX RELATIONSHIP BETWEEN miRNA DYSREGULATION AND BREAST CANCER PROGRESSION	Dr. Olufemi Adewale Dr. Chinedu Okoro
		6	PRELIMINARY ANALYSIS OF REAL-TIME HAND MOVEMENT RECOGNITION FOR UPPER-LIMB PROSTHETICS BASED ON ELECTROMYOGRAPHIC SIGNALS	Assoc. Prof. Dr. Hiroshi Takeda Assist. Prof. Dr. Maria Gonzalez Lec. Ahmed El-Sayed Dr. Jonathan Clarke
		7	ADVANCED SIMULATION PLATFORM FOR MEDICAL IMAGE FUSION AND LEARNING	Assoc. Prof. Dr. Beatriz Oliveira, Assis. Prof. Dr. Inês Soares, Miguel A. Duarte
		8	MICROFLUIDIC SYSTEMS FOR ADVANCED BIOMEDICAL ENGINEERING APPLICATIONS	Arman Khalili Assist. Prof. Dr. Laleh Moradi Lec. Sahar Nematpour

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HALL / SALON 3	**Assoc. Prof. Dr. Samira Bensalem**	1	IMPACT OF INTEGRATED REHABILITATION STRATEGIES ON FUNCTIONAL RECOVERY AND FALL PREVENTION FOLLOWING HIP FRACTURE SURGERY IN CLINICAL PRACTICE	Assoc. Prof. Dr. N. Papadopoulos Assis. Prof. Dr. E. Kallergi Lec. M. Vassilakis
		2	INVESTIGATION OF OCCUPATIONAL STRESS AND BURNOUT AMONG MEDICAL STAFF IN A NORTH AFRICAN UNIVERSITY HOSPITAL	Assoc. Prof. Dr. Samira Bensalem Assist. Prof. Dr. Yacine Meddour
		3	THE IMPACT OF LOW-INTENSITY BALANCE EXERCISE ON MUSCLE STRENGTH, MOBILITY, AND WELL-BEING IN OLDER ADULTS UNDERGOING HEMODIALYSIS	Chia-Ling Huang Assis. Prof. Dr. Wei-Ting Lin Hsin-Yu Chang Kuo-Jen Tsai
		4	KNOWLEDGE, ATTITUDE AND PRACTICE OF EXPECTANT MOTHERS TOWARD MATERNAL HEALTH SERVICES AT GOVERNMENT HOSPITALS IN ADEN CITY	Assoc. Prof. Dr. Samir Al-Mahdi Khaled N. Al-Saqqaf Amal M. Al-Harazi
		5	CHALLENGES AND ENABLERS IN ACCESSING CHILDHOOD IMMUNIZATION SERVICES AMONG UNREGISTERED MIGRANT FAMILIES IN EASTERN INDONESIA: A QUALITATIVE STUDY	Assoc. Prof. Dr. Putri Anindya Santoso Mohammad Rizal Fauzi Lec. Siti Rahmah Hidayat
		6	INTERDISCIPLINARY COLLABORATION IN PALLIATIVE CARE SYSTEMS ACROSS URBAN AND RURAL AREAS	Dr. A. Kovács Lec. S. Thompson Assis. Prof. Dr. H. Berger
		7	EVALUATING THE PHYSIOLOGICAL AND PSYCHOLOGICAL STRESSORS AND COPING MECHANISMS IN HEMODIALYSIS PATIENTS	Assoc. Prof. Dr. Leila M. Hassan Reem A. Al-Farhan Assis. Prof. Dr. Omar S. Rahman Lec. Nour T. Aziz

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HALL / SALON 4	**Assis. Prof. Dr. Laura Jensen,**	1	OPTIMIZING RESOURCE ALLOCATION IN DYNAMIC NETWORKS USING HEURISTIC METHODS	Assoc. Prof. Dr. Elena Vasilev, Pavel Korolenko, Dr. Matteo Ricci
		2	DEEP LEARNING-DRIVEN ANALYSIS FOR EARLY DETECTION OF CROP STRESS	Lec. Amina Koulibaly, Dr. Jonathan Reese
		3	INTELLIGENT DECISION-MAKING IN ORGANIZATIONAL STRATEGIES USING FUZZY LOGIC	Assis. Prof. Dr. Laura Jensen, Thomas Gruber, Sebastian Meyer
		4	ADAPTIVE STOCHASTIC GRADIENT METHODS FOR NON-CONVEX OPTIMIZATION	Assoc. Prof. Dr. Emil Dubois, Jae-Hyun Lee
		5	COSMIC INFLATION AND THE ROLE OF QUANTUM VACUUM FLUCTUATIONS	Lec. Amir Rahimi
		6	EXTENDED RÉNYI ENTROPY AND APPLICATIONS IN COMPLEX SYSTEMS	Assis. Prof. Dr. Carla M. Silva, Dr. José F. Romero, Miguel A. Torres, Ana L. Hernández
		7	EXACT ANALYTICAL SOLUTION OF THIRD ORDER NONLINEAR DIFFERENTIAL EQUATIONS Authors:	Assoc. Prof. Dr. Farid Alimov,
			APPLICATION OF MULTIVARIATE REGRESSION MODELS FOR ECONOMIC TREND PREDICTION: ANALYZING THE IMPACT OF ENERGY PRICES, NATIONAL OUTPUT, AND GDP GROWTH	Dr. Ravi K. Menon, Assoc. Prof. Dr. Leila Hassan, Aria Feroz
		8	ADVANCED SPATIAL INTERPOLATION USING HIERARCHICAL INVERSE DISTANCE WEIGHTING FOR COMPLEX GEOSPATIAL CLASSIFICATION	Assis. Prof. Dr. Karim Ben Salem, Olivier Tremblay, Prof. Dr. Marie-Claire Dubois, Dr. Lucien Gagnon

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HALL / SALON 5	**Assoc. Prof. Dr. Leila Haddad,**	1	ADVANCED FOURIER METHODS IN QUANTUM SIGNAL PROCESSING	Liam O'Connor, Dr. Esi K. Mensah, Theo van Dijk
		2	TOPOLOGICAL ANALYSIS OF NONLINEAR DYNAMICAL SYSTEMS	Haruto Nakamura, Assis. Prof. Dr. Priya S. Nair
		3	MATHEMATICAL FOUNDATIONS OF DEEP REINFORCEMENT LEARNING	Dr. Arjun V. Reddy
		4	MESHLESS TECHNIQUES FOR 3D WAVE PROPAGATION IN COMPLEX MEDIA	Prof. Dr. Farid Al-Husseini, Dr. Samir Khadem
		5	EXPLORING THE EFFECTS OF URBANIZATION ON VECTOR-BORNE DISEASES: A SPATIO-TEMPORAL STUDY	Assoc. Prof. Dr. Leila Haddad, Ahmed R. Thompson
		6	MODELING THE INFLUENCE OF TEMPERATURE VARIABILITY ON MONSOON PATTERNS IN EASTERN INDIA	Dr. Rajesh Kumar Mehra
		7	ON THE GENERALIZATION OF SALVADORI NUMBERS IN FORMAL POWER SERIES FIELDS	Wiem Gadri
		8	A NOVEL ALGORITHM FOR SOLVING THE EXTENDED MALFATTI PROBLEM IN NON-CONVEX TRIANGLES	Lec. Ching-Shuo Chiang

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HALL / SALON 6	**Assoc. Prof. Dr. Marat Akhmetov**	1	ANCIENT TRADE ROUTES AND CULTURAL EXCHANGE IN ASIA	Assoc. Prof. Dr. Hiroshi Tanaka Prof. Dr. Priya Raghavan Lec. Nguyen Van Long
		2	IMPACT OF COLONIALISM ON SOUTHEAST ASIAN POLITICAL STRUCTURES	Assoc. Prof. Dr. Aisha Rahman Prof. Dr. Somchai Chaiyaporn Dr. David Chen
		3	RELIGIOUS TRANSFORMATIONS IN THE MIDDLE EAST FROM 7TH TO 15TH CENTURY	Prof. Dr. Leila Al-Mansouri Assoc. Prof. Dr. Farhad Rezai Lec. Omar Khalid
		4	HISTORY OF SCIENCE AND TECHNOLOGY IN EAST ASIA	Prof. Dr. Min-Joon Park Assoc. Prof. Dr. Ananya Gupta Dr. Chen Wei
		5	REVOLUTIONARY MOVEMENTS IN MODERN CENTRAL ASIA	Assoc. Prof. Dr. Marat Akhmetov Prof. Dr. Dilshod Rasulov Lec. Gulnara Abdiyeva
		6	ART AND ARCHITECTURE IN THE OTTOMAN EMPIRE	Prof. Dr. Leyla Hassan Dr. Samir Al-Khalifa
		7	ENVIRONMENTAL HISTORY AND AGRICULTURAL PRACTICES IN ANCIENT CHINA	Prof. Dr. Li Na Assoc. Prof. Dr. Takeshi Sato Lec. Tran Thi Hoa

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HALL / SALON 7	**Assoc. Prof. Dr. Jad Nasser**	1	THE IMPACT OF CULTURAL BELIEFS AND TRADITIONAL NORMS ON FAMILY THERAPY OUTCOMES IN MIDDLE EASTERN SOCIETIES	Dr. Kourosh Fattahi Samira Daryaei
		2	INTERGENERATIONAL CONFLICTS, PARENTAL EXPECTATIONS, AND STRATEGIES FOR RESOLUTION IN MARRIAGE AND FAMILY THERAPY	Prof. Dr. Elena Kuznetsova Igor Volkov Anastasia Morozova
		3	THE ROLE OF EMOTIONAL INTELLIGENCE, SELF-AWARENESS, AND COPING SKILLS IN IMPROVING MARITAL SATISFACTION AND STABILITY	Assis. Prof. Dr. Aisuluu Tulegenova Yerzhan Akhmetov Bibigul Saparova
		4	PARENTAL STRESS, WORK-LIFE BALANCE, AND THEIR IMPACT ON CHILD DEVELOPMENT AND FAMILY RELATIONSHIPS	Rania Al-Masri Omar Khalaf
		5	ANALYZING COUPLE COMMUNICATION PATTERNS, CONFLICT RESOLUTION TECHNIQUES, AND THERAPEUTIC INTERVENTIONS FOR HEALTHIER RELATIONSHIPS	Huda Nasser
		6	THE INFLUENCE OF RELIGIOUS BELIEFS, SPIRITUALITY, AND PRACTICES ON MARITAL RELATIONSHIPS AND FAMILY DYNAMICS	Dr. Ahmed Salah Dr. Laila Mostafa Nourhan Youssef
		7	MANAGING FINANCIAL STRESS, ECONOMIC PRESSURES, AND RESOURCE ALLOCATION IN FAMILY SYSTEMS THROUGH MARRIAGE AND FAMILY THERAPY	Dr. Hanan Al-Harbi Lec. Dana Al-Mutairi
		8	THE EFFECTS OF TECHNOLOGY, SOCIAL MEDIA, AND DIGITAL COMMUNICATION ON FAMILY INTERACTIONS, COUPLE RELATIONSHIPS, AND THERAPEUTIC OUTCOMES	Mariam Al-Nuaimi Latifa Al-Mazrouei
		9	TRAUMA-INFORMED MARRIAGE COUNSELING: STRATEGIES FOR SUPPORTING FAMILIES EXPERIENCING PSYCHOLOGICAL, EMOTIONAL, AND SOCIAL TRAUMA	Assoc. Prof. Dr. Jad Nasser Dr. Maya Khalil Lec. Rami Bou Saab

ICSAS 2nd INTERNATIONAL CONFERENCE ON ECONOMICS
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Salon	Moderator		Bildiri No ve Başlığı / Paper ID and Title	Authors
HALL / SALON 1	**Doç. Dr. Suna Muğan Ertuğral**	1	THE RELATIONSHIP BETWEEN DIGITALIZATION AND TAX REVENUES: A FOURIER-BASED EMPIRICAL STUDY FOR TÜRKİYE	Assistant Professor Oğuzhan BOZATLI Research Assistant Şeref Can SERİN
		2	Assessing Health Insurance Awareness and Its Correlation with Economic Utilization Among Gujjars and Bakarwals For Seeking Health in Jammu and Kashmir	Iliyas Ahmad Rather Sadam Hussain Pandow Monisa Qadiri
		3	ASYMMETRIC SHOCKS IN INDUSTRIAL PRODUCTION AND ECONOMIC GROWTH: AN EMPIRICAL STUDY ON TURKEY WITHIN THE KALDO-STYLE APPROACH FRAMEWORK	Ögr.Gör.Dr. Atilla ÜNLÜ
		4	FRAGILE SYNCHRONIZATION: GLOBAL, REGIONAL AND COUNTRY-SPECIFIC DYNAMICS OF BUSINESS CYCLE FLUCTUATIONS	Dr. Öğr. Üyesi Ahmet TUNÇ
		5	THE EFFECTS OF WAR ECONOMY ON THE BUDGET: THE CASE OF THE RUSSIA–UKRAINE WAR	Res. Assist. Dr. Ali Fuat URUŞ Dr. Fadime Ayca CEYLAN
		6	ENERJİ VERGİLERİ VE YENİLENEBİLİR ENERJİ İLİŞKİSİ: TÜRKİYE ÜZERİNE BİR UYGULAMA	Dr. Öğr. Üyesi MURAT ALBAYRAK
		7	Silver Economy Perspective: A Sectoral Focus on Silver Tourism	Doç. Dr. Suna Muğan Ertuğral
		8	Turuncu Bayrak Programı ile Gıda İsrafının Azaltılması ve Sürdürülebilir Turizm Uygulamaları: "Tabağını Sen Seç" Yaklaşımı	Doç. Dr. Suna Muğan Ertuğral

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HALL / SALON 2	**Assoc. Prof. Dr. Gülhan SUADİYE**	1	AN ASSESSMENT OF THE FAST MOVING CONSUMER GOODS INDUSTRY	Doktora Öğrencisi Hakan ERTUĞRAL
		2	Evaluation of Bioplastic Production Processes in Terms of Sustainability and Efficiency Using GaBi-Based Life Cycle Analysis: An Interdisciplinary Approach	Chemical Engineer Hakan Ertuğral Computer Engineer Alaeddin Gazi Toprak
		3	DERGİPARK'TA YAYIMLANAN SATIŞ YÖNETİMİ ÇALIŞMALARININ NİTEL ANALİZİ	Dr. Öğr. Üyesi, MUALLA AKÇADAĞ
		4	TÜKETİCİ KARAR SÜREÇLERİNDE NÖROPAZARLAMANIN ROLÜ: NÖROFİZYOLOJİK BİR PERSPEKTİF	Öğrt. Görv. Dr. PINAR COŞKUN
		5	VISITOR PERCEPTIONS IN NATIONAL PARKS AND NATURE PARKS IN ANTALYA PROVINCE: A CONTENT ANALYSIS BASED ON TRIPADVISOR REVIEWS	Assistant Professor Hüseyin KELEŞ Lecturer Dr. Güray KARACIL
		6	Artificial Intelligence in Advertising: Personalization, Predictive Analytics, and Consumer Trust	PhD Candidate EYLEM ŞENCAN
		7	AI-Driven Marketing Strategies: From Consumer Insights to Sustainable Engagement	PhD Candidate EYLEM ŞENCAN
		8	FULL-RANGE LEADERSHIP AS A MULTIDIMENSIONAL APPROACH: THE GLOBAL DEVELOPMENT OF THE LITERATURE	Öğretim Görevlisi Doktor, YAŞAR ŞAHİN
		9	ESG REPORTING: GLOBAL COMPLIANCE THROUGH TURKISH SUSTAINABILITY REPORTING STANDARDS (TSRS)	Assoc. Prof. Dr. Gülhan SUADİYE

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HALL / SALON 3	**Doç. Dr., Ferdi SELİM**	1	İMKAN ANLAYIŞININ KARŞILAŞTIRMALI ANALİZİ (AYNÜLKUDÂT HEMEDÂNÎ, İBN SÎNÂ VE GAZÂLÎ'NİN GÖRÜŞLERİ DOĞRULTUSUNDA)	Rüstem Hacıyev
		2	PLATON'DA EĞİTİM: BİLGİ, ERDEM VE DEVLET İLİŞKİSİ	Yüksek Lisans Öğrencisi NURPERİ TİTİZ
		3	AN EXAMINATION OF THE RELATIONSHIP BETWEEN THE CONCEPTS OF SCIENCE AND IDEOLOGY	Dr. Karani Kağan BADEM
		4	THE CONCEPT OF RACE IN THE CONTEXT OF THE RELATIONSHIP BETWEEN SCIENCE AND IDEOLOGY	Dr. Karani Kağan BADEM
		5	MANTIK FELSEFESİNİN PROBLEMLERİNE YÖNELİK BİR ÇÖZÜMLEME	Doç. Dr. Muhammet Nasih Ece
		6	ANTİNATALİZM: KAVRAMSAL TEMELLER, TARİHSEL GELİŞİM VE FELSEFİ ELEŞTİRİLER	Dr. Öğr. Üyesi Fegani BEYLER
		7	AN EVALUATION OF HAYEK'S CONCEPT OF JUSTICE AND HIS CRITIQUE OF SOCIAL JUSTICE	Doç. Dr., Ferdi SELİM
		8		

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HALL / SALON 4	**Prof. Dr. Habibe ACAR**	1	Spatial Configuration Transformation of Repurposed Traditional Houses: The Case of Traditional Dwellings in Alanya	Metehan DAYI Dr. Öğr. Üyesi Murat ŞAHİN Doç. Dr. Tuba Nur OLĞUN
		2	Architecture and modern life of people	Amina Huseynova
		3	DESIGNING VISITOR ROUTES IN ARCHAEOLOGICAL SITES AS A CONSERVATION STRATEGY	Doç. Dr. Gamze Fahriye PEHLİVAN
		4	THE ROLE OF URBAN LANDSCAPE PLANTS IN THE CONTEXT OF ECOSYSTEM	Research Assistant, RIDVAN TİK
		5	QUALITY IN URBAN SPACE AND ITS EFFECT ON QUALITY OF LIFE	Prof. Dr. Habibe ACAR Assist. Prof. Dr. Aysel YAVUZ Prof. Dr. Nilgün GÜNEROĞLU
		6	LANDSCAPE POTENTIAL OF PLANT TAXON IN TRABZON URBAN GREEN AREAS	Prof. Dr. Nilgün GÜNEROĞLU Prof. Dr. Habibe ACAR Assist. Prof. Dr. Aysel YAVUZ

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HALL / SALON 5	**Doçent, AYŞE SEZER**	1	ARTIFICIAL INTELLIGENCE-SUPPORTED CREATIVE DRAWING APPLICATION: ArtinDraw	Doçent, AYŞE SEZER
		2	ELEKTRİK TÜKETİMİ VE FİYATLANDIRMA ÜZERİNE BİR YAPAY SİNİR AĞLARI UYGULAMASI: ABD ÖRNEĞİ	Dr. Öğretim Üyesi, Murat KAPUSUZ Elektrik ve Elektronik Mühendisi, Enes YILMAZER
		3	PREDICTIVE ANALYTICS FOR DEFECT DETECTION IN MANUFACTURING: A DATA-DRIVEN APPROACH USING MACHINE LEARNING MODELS	Dr. Sameer Jain
		4	ENHANCING REGRESSION MODELING IN MATERIALS SCIENCE: EVALUATING SYNTHETIC DATA AUGMENTATION FOR MECHANICAL DISPLACEMENT PREDICTION	Dr. Öğretim Üyesi, Erhan KAVUNCUOĞLU
		5	DEVELOPMENT OF AN ENTERPRISE-LEVEL CHATBOT	Alim Toprak Fırat Gökşen Çalışkan Ceren Ulus Nazlı Yusufoğlu M. Fatih Akay

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HALL / SALON 1	**Dr. Lara M. Oliveira , Assoc. Prof. Dr. Marco Vitali**	1	SUSTAINABLE URBAN HOUSING DESIGN STRATEGIES FOR RESILIENT COMMUNITIES	Elena Markarian, Assis. Prof. Dr. Dimitrios Kalogeropoulos, Maria Fernanda Castillo, Dr. Arvind Rathore
		2	INTEGRATED FLOOD-RESILIENT INFRASTRUCTURE PLANNING FOR COASTAL CITIES	Dr. Lara M. Oliveira, Assoc. Prof. Dr. Marco Vitali
		3	CROSS-CULTURAL PERSPECTIVES IN MODERN URBAN ARCHITECTURE	João Mendes, Assis. Prof. Dr. Farid Aliev, Nadira Khan
		4	INNOVATIVE FINANCIAL MODELS FOR AFFORDABLE HOUSING IN EMERGING ECONOMIES	Dr. Timothy Adebayo, Chidera Okonkwo, Rizal Hidayat
		5	ARCHITECTURAL STRATEGIES FOR CLIMATE-ADAPTIVE PUBLIC SPACES	Manoj Patel, Dr. Hana Syafiqah
		6	RISK ASSESSMENT AND SAFETY MANAGEMENT IN INDUSTRIAL URBAN ZONES	Dr. Khaya Qonono, Assis. Prof. Dr. Zeynep Alimova, Paulo Ribeiro
		7	DVANCES IN LIGHTING SIMULATION AND VISUAL COMFORT ANALYSIS IN CONTEMPORARY ARCHITECTURE	Assoc. Prof. Dr. Luca Moretti Mahmoud Al-Khatib E. S. Cantrell Sofia Petrova

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HALL / SALON 2	**Assoc. Prof. Dr. Matteo Ricci**	1	ELITE SUSTAINABLE URBAN DEVELOPMENT STRATEGIES: INTEGRATING GREEN SPACES AND SMART INFRASTRUCTURE	Assoc. Prof. Dr. Matteo Ricci, Hana Kim, Prof. Dr. Samuel Osei
		2	REVITALIZING HISTORIC CITY CENTERS THROUGH CONTEMPORARY LANDSCAPE DESIGN AND COMMUNITY ENGAGEMENT	Tomasz Kowalski, Amina Diallo
		3	ADVANCED MATERIALS AND TECHNOLOGIES IN MODERN ARCHITECTURE FOR ENERGY EFFICIENCY	Dr. Isabella Fernandes, Assoc. Prof. Dr. Ahmed Al-Mansouri
		4	BIODIVERSITY-FOCUSED URBAN PARK DESIGN: PROMOTING ECOLOGICAL CORRIDORS AND HABITAT RESTORATION	Prof. Dr. Juan Carlos Ramirez, Prof. Dr. Fatima Noor
		5	SMART MOBILITY AND INTEGRATED TRANSPORTATION PLANNING IN FUTURE CITIES	Dr. Lars Svensson, Mei Ling Zhao
		6	ARCHITECTURAL RESPONSES TO CLIMATE CHANGE: RESILIENT STRUCTURES AND FLOOD MANAGEMENT STRATEGIES	Assoc. Prof. Dr. Olga Ivanova, Dr. Rafael Martinez, lec. Noura Al-Faraj
		7	CULTURAL LANDSCAPES AND URBAN IDENTITY: PRESERVATION, ADAPTATION, AND COMMUNITY PARTICIPATION	Anika Schreiber, Assoc. Prof. Dr. Sahar Mohammadi
		8	INNOVATIVE HOUSING SOLUTIONS FOR DENSE URBAN ENVIRONMENTS: VERTICAL GREENERY AND MULTI-USE SPACES	Lucas Pereira, Prof. Dr. Min-Joon Choi, Assoc. Prof. Dr. Elena Dimitrova

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HALL / SALON 3	**Dr. Jonathan Clarke**	1	THE PHILOSOPHICAL IMPLICATIONS OF ARTIFICIAL INTELLIGENCE IN HUMAN AGENCY AND MORAL RESPONSIBILITY	Emma Johansson Dr. Hiroshi Tanaka Isabella Moretti
		2	EXPLORING THE INTERSECTIONS OF ENVIRONMENTAL PHILOSOPHY AND ETHICS IN THE CONTEXT OF GLOBAL CLIMATE CHANGE RESPONSE	Dr. Carlos Martínez Assoc. Prof. Dr. Amina Said Fiona Campbell
		3	PHENOMENOLOGICAL INQUIRY INTO THE NATURE OF CONSCIOUSNESS AND SUBJECTIVE EXPERIENCE	Dr. Markus Weber Léa Dupont
		4	CRITICAL ANALYSIS OF FEMINIST PHILOSOPHY AND ITS CHALLENGES TO TRADITIONAL METAPHYSICAL THEORIES	Assoc. Prof. Dr. Sofia Dimitriou Dr. Priya Rao Zara Al-Khalil
		5	POLITICAL PHILOSOPHY AND THE ETHICS OF GLOBAL JUSTICE IN AN ERA OF INCREASING INEQUALITY	Dr. David Kim Amara Ndlovu Prof. Dr. Lucia Rossi
		6	MIND-BODY PROBLEM REVISITED: CONTEMPORARY THEORIES IN PHILOSOPHY OF MIND AND COGNITIVE SCIENCE	Dr. Jonathan Clarke
		7	RECONSIDERING THE CONCEPT OF HAPPINESS IN CONTEMPORARY ETHICAL THEORIES AND PSYCHOLOGICAL WELL-BEING	Prof. Dr. Olivia Bennett Dr. Samuel Okoro
		8	THE ROLE OF LANGUAGE IN SHAPING REALITY: A STUDY IN CONTEMPORARY PHILOSOPHICAL LINGUISTICS	Dr. Abdullah Al-Mutairi Assoc. Prof. Dr. Elise Fournier Mateo González

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HALL / SALON 4	**Assoc. Prof. Dr. Milena Popescu**	1	FREEDOM AND COLLECTIVE RESPONSIBILITY: PHILOSOPHICAL EXAMINATIONS OF INDIVIDUAL AUTONOMY WITHIN SOCIAL STRUCTURES	Assoc. Prof. Dr. Tomas Becker prof. Dr. Lucia Fernandes lec. Kwesi Boateng
		2	REALITY AND PERCEPTION: ONTOLOGICAL INQUIRIES THROUGH THE LENS OF MODERN SCIENCE AND ANCIENT PHILOSOPHY	Assoc. Prof. Dr. Matteo Bianchi lec. Jonas Fitzgerald
		3	ETHICS IN TECHNOLOGY: THE IMPACT OF ARTIFICIAL INTELLIGENCE AND BIOTECHNOLOGICAL ADVANCES ON MORAL VALUES	Dr. Sven Eriksson lec. Chike Obi
		4	TIME AND EXISTENCE: REFLECTIONS ON PAST, PRESENT, AND FUTURE IN EXISTENTIAL PHILOSOPHY	Assoc. Prof. Dr. Sofia Petrović prof. Dr. Kenji Saito Dr. Diego Alvarez
		5	LANGUAGE AND REALITY: THE ROLE OF SEMANTIC STRUCTURES IN THOUGHT AND MEANING-MAKING	lec. Fatoumata Diop Dr. Heinrich Wagner Assoc. Prof. Dr. Milena Popescu
		6	ART AND AESTHETICS: PHILOSOPHICAL ANALYSIS OF BEAUTY, CREATIVITY, AND SOCIETAL INFLUENCE	Prof. Dr. Ricardo Campos Ifeoma Nwosu Dr. Tatiana Kovalchuk
		7	POLITICS AND JUSTICE: ANALYZING EQUALITY, RIGHTS, AND POWER DYNAMICS IN POLITICAL PHILOSOPHY	Assoc. Prof. Dr. Emil Novak Dr. Charlotte Moreau lec. Jabari Diallo
		8	CONSCIOUSNESS AND THE PHILOSOPHY OF MIND: DEEP INVESTIGATIONS INTO THE MIND-BODY PROBLEM, PERCEPTION, AND SELF-AWARENESS	Dr. Priya Nair lec. Maximilian Krüger Assoc. Prof. Dr. Ivana Stanković

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HALL / SALON 5	**Assoc. Prof. Dr. Diego Morales**	1	THE MORAL FOUNDATIONS OF GLOBALIZATION: ETHICAL CHALLENGES IN AN INTERCONNECTED WORLD	Giulia Bianchi
		2	EXISTENTIAL ANXIETY AND HUMAN AGENCY: NAVIGATING MEANING IN CONTEMPORARY LIFE	Dr. Henrik Johansen Assoc. Prof. Dr. Freja Lund Lars Pedersen
		3	KNOWLEDGE AND TRUTH: EPISTEMOLOGICAL PERSPECTIVES ON BELIEF, JUSTIFICATION, AND EVIDENCE	Prof. Dr. Amina Hassan Dr. Omar El-Sayed Salma Farouk
		4	THE PHILOSOPHY OF EMOTION: UNDERSTANDING THE ROLE OF FEELINGS IN REASONING AND ETHICAL DECISION-MAKING	Sophie Dubois Julien Lefevre Camille Moreau
		5	CIVIC VIRTUE AND DEMOCRATIC RESPONSIBILITY: PHILOSOPHICAL EXAMINATION OF CITIZENSHIP AND ETHICAL GOVERNANCE	Alejandro Ramirez Valeria Torres Assoc. Prof. Dr. Diego Morales
		6	THE CONCEPT OF IDENTITY: SELF, SOCIETY, AND THE FLUIDITY OF PERSONAL AND CULTURAL CONSTRUCTION	Ekaterina Smirnova
		7	SCIENCE AND MORALITY: THE INTERSECTION OF TECHNOLOGICAL ADVANCEMENT AND ETHICAL OBLIGATION	Hiroshi Nakamura Yuki Sato Kenji Fujimoto
		8	SPACE, TIME, AND COSMIC PHILOSOPHY: HUMAN UNDERSTANDING OF THE UNIVERSE AND EXISTENTIAL SIGNIFICANCE	Chinedu Eze Assoc. Prof. Dr. Funke Adewale

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HALL / SALON 6	**Assoc. Prof. Dr. Bambang Saputra**	1	INNOVATIVE STRATEGIES FOR SUSTAINABLE BUSINESS GROWTH IN EMERGING AFRICAN MARKETS	Assoc. Prof. Dr. Abdulrahman Umar Dr. Siti Nuraini Lecturer Fatima Zeghichi
		2	THE IMPACT OF DIGITAL TRANSFORMATION ON SUPPLY CHAIN RESILIENCE IN INDONESIAN MANUFACTURING SECTORS	Rizal Maulana Assoc. Prof. Dr. Aisha Bensalem Emmanuel Okoro
		3	EFFECTIVE LEADERSHIP MODELS FOR MANAGING REMOTE WORKFORCES IN TAJIKISTAN'S SERVICE INDUSTRY	Dr. Dilshod Saidov Assoc. Prof. Dr. Hening Wulandari Grace Mbatha
		4	CORPORATE SOCIAL RESPONSIBILITY AND ITS INFLUENCE ON BRAND LOYALTY IN NIGERIAN CONSUMER MARKETS	Kadija Ali Assoc. Prof. Dr. Bambang Saputra Modupe Adebayo
		5	STRATEGIC HUMAN RESOURCE MANAGEMENT PRACTICES TO IMPROVE EMPLOYEE RETENTION IN ALGERIAN SMEs	Amine Belkacem. Indah Kusumawati Assis. Prof. Dr. Tinashe Dzvairo
		6	THE ROLE OF ENTREPRENEURIAL FINANCE IN FOSTERING INNOVATION IN AFRICAN STARTUPS	Dr. Halima Oumar Assis . Prof. Dr. Dewi Handayani Rashid Alizoda
		7	ANALYSIS OF CONSUMER BEHAVIOR TRENDS IN DIGITAL MARKETING CAMPAIGNS WITHIN INDONESIAN E-COMMERCE PLATFORMS	Salah Eddine Belmokhtar Adeleke Johnson
		8	EXPLORING THE CHALLENGES AND OPPORTUNITIES OF WOMEN ENTREPRENEURS IN TAJIKISTAN'S ECONOMY	Assoc. Prof. Dr. Gulchekhra Rahimi Dr. Hendri Setiawan Dr. Mariam Sow

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HALL / SALON 7	**Dr. Yacine Mansouri Assoc. Prof. Dr. Hariyanti Putri**	1	INNOVATIVE RISK MANAGEMENT FRAMEWORKS FOR FINANCIAL INSTITUTIONS IN ALGERIA	Dr. Yacine Mansouri Assoc. Prof. Dr. Hariyanti Putri
		2	THE INFLUENCE OF ORGANIZATIONAL CULTURE ON EMPLOYEE PERFORMANCE IN NIGERIAN TELECOMMUNICATION COMPANIES	Assoc. Prof. Dr. Dwi Anggraini
		3	THE EFFECTIVENESS OF SUPPLY CHAIN SUSTAINABILITY INITIATIVES IN TAJIKISTAN'S INDUSTRIAL SECTORS	Dr. Murodjon Rasulov Dr. Sari Dewi Assoc. Prof. Dr. Femi Adewale
		4	STRATEGIC ALLIANCES AND BUSINESS PERFORMANCE IN INDONESIAN TECHNOLOGY FIRMS	Indra Wijaya Dr. Abderrahmane Cherifi Dr. Nana K. Mensah
		5	THE IMPACT OF LEADERSHIP STYLE ON ORGANIZATIONAL CHANGE IN AFRICAN AGRIBUSINESS COMPANIES	Dr. Jamilah Baba Assoc. Prof. Dr. Lusiana Putri Prof. Dr. Idriss N'Dour
		6	CUSTOMER SATISFACTION AND LOYALTY IN THE CONTEXT OF NIGERIAN RETAIL INDUSTRY	Prof. Dr. Nkechi Okeke Dr. Rini Anjani Assis. Prof. Dr. Mahmoud Bekhti
		7	E-GOVERNANCE AND BUSINESS TRANSPARENCY: A STUDY ON TAJIKISTAN'S PUBLIC SECTOR	Dr. Shokhrukh Yuldashev Dr. Dewi Ayu Assoc. Prof. Dr. Oumar Sylla

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HALL / SALON 8	**Assoc. Prof. Dr. Ahmed Mazouz Prof. Dr. Rebecca Mwangi**	1	THE ROLE OF CORPORATE GOVERNANCE IN BUSINESS SUSTAINABILITY AMONG ALGERIAN COMPANIES	Dr. Safia Boudiaf Dr. Agung Priyanto Assoc. Prof. Dr. Kwame Asante
		2	STRATEGIES FOR ENHANCING INNOVATION CULTURE IN INDONESIAN FAMILY BUSINESSES	Dian Kusumaningrum Dr. Amina Bourahla
		3	SOCIAL ENTREPRENEURSHIP AND COMMUNITY DEVELOPMENT IN AFRICAN URBAN AREAS	Dr. Maimuna Ahmed Anggi Retno Salah Jamal
		4	THE IMPACT OF DIGITAL MARKETING ON CUSTOMER BUYING BEHAVIOR IN TAJIKISTAN'S FMCG SECTOR	Dr. Farzona Safarova Assoc. Prof. Dr. Widya Lestari Dr. Tunde Balogun
		5	INTERNATIONAL TRADE STRATEGIES FOR SMALL AND MEDIUM ENTERPRISES IN NIGERIA	Dr. Emeka Chukwu Yuliana Dewi Abderrahmane Bensaid
		6	THE DEVELOPMENT OF BUSINESS INTELLIGENCE SYSTEMS IN ALGERIAN MANUFACTURING INDUSTRY	Nourredine Kabache Dr. Ani Natalia Fisayo Kehinde
		7	WORKFORCE DIVERSITY AND ITS EFFECT ON CORPORATE INNOVATION IN INDONESIAN COMPANIES	Dr. Putra Hidayat Prof. Dr. Fatima Ziani Assoc. Prof. Dr. Samuel Karanja
			STRATEGIC PLANNING AND COMPETITIVE ADVANTAGE IN THE AFRICAN FINANCIAL SERVICES SECTOR	Dr. Amina Touré Dr. Arif Rahman
			SOCIAL MEDIA STRATEGIES FOR BRAND DEVELOPMENT IN TAJIKISTAN'S SMALL BUSINESSES	Dr. Sari Widya Dr. Peter Okoye
		8	THE ROLE OF CUSTOMER RELATIONSHIP MANAGEMENT IN ENHANCING BUSINESS PERFORMANCE IN INDONESIA	Dr. Dian Anggraini Assoc. Prof. Dr. Ahmed Mazouz Prof. Dr. Rebecca Mwangi

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HALL / SALON 1	**Doç. Dr. Sait YILDIRIM**	1	PROBLEMS AND RISK FACTORS FACED BY ELDERLY PEOPLE AFFECTED BY DISASTERS FROM A SOCIAL WORK PERSPECTIVE	Merve KURT Rümeysa ÇAYLI Rümeysa HAZIRLAR Dr. Öğr. Üyesi Işıl AVŞAR ARIK
		2	LOCAL SOLUTIONS TO THE GLOBAL CLIMATE CRISIS: THE CASE OF PAKISTAN	Arş. Gör. SÜMEYYE AYDOĞDU KUZUCU
		3	BAĞIMLILIKLA MÜCADELEDE PSİKOSOSYAL DESTEK YÖNTEMLERİ	Öğr.Gör. Abdulkadir Ilgaz Prof.Dr. Oğuzhan Zengin
		4	DİJİTAL BAĞIMLILIK: PSİKOLOJİK ETKİLERİ VE BAŞ ETME STRATEJİLERİ	Prof.Dr. Oğuzhan Zengin Öğr.Gör. Abdulkadir Ilgaz
		5	BIBLIOMETRIC ANALYSIS OF STUDIES ON CHILD LABOR IN THE LAST FIVE YEARS	Doktora Öğrencisi Nermin Turan Doç. Dr. Sait Yıldırım Prof. Dr. Senayi Dönmez
		6	EXAMİNİNG THE EFFECT OF TEACHERS' EARTHQUAKE PREPAREDNESS ON THEİR ATTİTUDES TOWARD EARTHQUAKES: THE IĞDIR TUZLUCA EXAMPLE	Sinem ŞENGÜL Doç. Dr. Sait YILDIRIM
		7	CAREER PATH MAP FOR SOCIAL WORK STUDENTS: AN EDUCATIONAL SEMINAR MODEL SUPPORTING CAREER ORIENTATION AND SPECIALIZATION	Master's Student Aslıhan UYAR Master's Student Habibe TANDOĞAN AKAR
		8	BREAKİNG THE CYCLE OF CHİLD LABOR: A MARKOV CHAİN ANALYSİS OF MİCROFİNANCE, FAMİLY SUPPORT, AND SOCİAL WORK INTERVENTİONS İN POVERTY DYNAMİCS	Lecturer HAMDİ AYYILDIZ Lecturer, PhD MERVE KAYA BUZKIRAN
		9	SUÇA SÜRÜKLENEN ÇOCUKLARA YÖNELİK ÖNLEYİCİ SOSYAL HİZMETTE KUŞAKLARARASI YAKLAŞIM VE TOPLUMSAL DAYANIKLILIK	Öğr. Gör. Dr. MERVE KAYA BUZKIRAN Öğr. Gör. HAMDİ AYYILDIZ

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HALL / SALON 2	**Doç. Dr. Fatma AŞILIOĞLU**	1	KAMUSAL YEŞİL ALAN KALİTE KRİTERLERİ KAPSAMINDA MAHALLE PARKLARININ DEĞERLENDİRİLMESİ İÇİN NİCEL BİR YÖNTEM	Doç. Dr. Fatma AŞILIOĞLU Öğr. Gör. Hande ASLAN Dr. Reva ŞERMET ACAR
		2	KENTSEL YEŞİL ALANLARIN TERAPİ BAHÇESİ POTANSİYELİNİN DEĞERLENDİRİLMESİ VE HORTİKÜLTÜREL TERAPİ YAKLAŞIMI	Öğr. Gör. Hande ASLAN Doç. Dr. Fatma AŞILIOĞLU Dr. Reva ŞERMET ACAR
		3	MEKANIN KOLEKTİF YENİDEN ÜRETİMİ: LEFEBVRE'NİN ŞEHİR HAKKI PERSPEKTİFİ İLE GÜNÜMÜZ KENT SORUNLARI	Arş. Gör. Pınar Özge PARLAK Prof. Dr. Banu Çiçek KURDOĞLU
		4	SU, BELLEK ve SÜRDÜRÜLEBİLİRLİK: WROCŁAW HYDROPOLİS ÖRNEĞİ	Arş. Gör. Pınar Özge PARLAK Prof. Dr. Banu Çiçek KURDOĞLU
		5	LANDSCAPE NARRATIVES AT THE INTERSECTION OF PLACE AND REPRESENTATION: PEARL S. BUCK'S THE MOTHER AND TOLSTOY'S ANNA KARENINA	Prof. Dr. SERAP YILMAZ Res.Asst. Dr. ABDULLAH ÇİĞDEM
		6	THE ROLE OF SECTIONS AND ELEVATIONS IN THE ERA OF DIGITAL AND AI-ASSISTED DESIGN	Res.Asst. Dr. ABDULLAH ÇİĞDEM Prof. Dr. SERAP YILMAZ
		7	EFFECTS OF GREEN INITIATIVES ON AIR POLLUTION REDUCTION IN URBAN AREAS	Res. Assist. Dr. Seyhan SEYHAN Prof. Dr. Elif BAYRAMOĞLU
		8	EVALUATION OF NATURE-BASED SOLUTION PROPOSALS IN TERMS OF URBAN SURFACE RUNOFF	Res. Assist. Dr. Seyhan SEYHAN Prof. Dr. Elif BAYRAMOĞLU

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HALL / SALON 3		1	THE RELATIONSHIP BETWEEN JOB SATISFACTION AND INTENTION TO LEAVE AMONG NURSES: A META-ANALYSIS STUDY	Master's-Prepared Nurse Esma ASLAN SEKİ
		2	YAPAY ZEKA ÇAĞINDA HEMŞİRELİK METAPARADİGMALARINA YENİDEN BAKIŞ	Öğr. Görevlisi, ÖZNUR TUĞBA ÇELEBİ DURSUN Dr. Öğr. Üyesi, NURDAN YALÇIN ATAR
		3	YAPAY ZEKA TEKNOLOJİLERİNİN HEMŞİRELİK BAKIMINA ENTEGRASYONU	Dr. Öğr. Üyesi, NURDAN YALÇIN ATAR Öğr. Görevlisi, ÖZNUR TUĞBA ÇELEBİ DURSUN
		4	Review of Nursing Studies Related to Health Literacy	Züleyha AYKUT Meryem YAVUZ Van GIERSBERGEN
		5	Scientific Mapping of Artificial Intelligence in Nursing: Bibliometric Analysis	Züleyha AYKUT Meryem YAVUZ Van GIERSBERGEN
		6	THE EFFECT OF HEMSBALL ACTIVITY ON BALANCE, KINESOPHOBIC ATTITUDE, AND FRAILTY IN OLDER PEOPLE: A RANDOMIZED CONTROLLED TRIAL PROTOCOL	Assoc. Prof. Dr. Fatma Zehra GENÇ Assoc. Prof. Dr. Şükran İRİBALCI
		7	YENİDOĞAN BESLENMESİ VE BEYİN GELİŞİMİ İLİŞKİSİ	Arş. Gör. Büşra KÜTÜK KURT Prof. Dr. Aynur AYTEKİN ÖZDEMİR
		8	ÇOCUKLARDA NONFARMAKOLOJİK AĞRI YÖNETİMİ: AİLE MERKEZLİ BAKIM	Arş. Gör. Büşra KÜTÜK KURT Prof. Dr. Aynur AYTEKİN ÖZDEMİR

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HALL / SALON 4	**Dr. Öğretim Üyesi,Buket AKCAN ALTINKAYNAK**	1	EVALUATION OF THE EXPRESSION LEVELS OF GENES ASSOCIATED WITH SALT STRESS IN COTTON	PhD student, SHADER ALIZADE
		2	Development of technology for obtaining drug nanoparticles from medicinal plants using green synthesis method	Kamala Gahramanova Ismat Ahmadov
		3	STUDYING THE INTERACTION OF GLYCYRRHIZIN AND P53 PROTEIN FAMILY USING MOLECULAR DOCKING	Leyla Elman GALANDARLI Dr. Gulnara Ahmad AKVERDIEVA
		4	MANAGEMENT OF OBESITY FROM A NUTRIGENOMIC PERSPECTIVE: GENETIC REGULATION WITH FUNCTIONAL COMPOUNDS	Dr. Öğretim Üyesi,Buket AKCAN ALTINKAYNAK Dr. Öğretim Üyesi, Yahya ALTINKAYNAK
		5	EFFECTS OF ZİNC SULFATE NANOPARTİCLES ON EPS STRUCTURE AND GENE EXPRESSİON İN STAPHYLOCOCCUS AUREUS BİOFİLMS	Research Assistant Ayşe ÜSTÜN BAŞKUT

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HALL / SALON 1	**Assis. Prof. Rustambek Sharipov Prof. Dr. Andini Kusuma**	1	REGULATION OF EPIGENETIC MODIFICATIONS IN METABOLIC SYNDROME AND DIABETES: MECHANISTIC INSIGHTS AND THERAPEUTIC POTENTIAL	Dewi Kartika Dr. Sofia Benali Dr. Olalekan Adeyemi
		2	ADVANCES IN ENZYME KINETICS AND ALLOSTERIC REGULATION FOR DRUG DEVELOPMENT IN INFECTIOUS DISEASES	Dr. Sari Widjaja Abdelhak Imene Aisha Nnamdi
		3	MOLECULAR MECHANISMS OF PROTEIN AGGREGATION AND ITS ROLE IN NEURODEGENERATIVE DISORDERS	Prof. Dr. Ratna Sari Assoc. Prof. Karim Bensalem Dr. Chinedu Umeh
		4	INTERACTION OF NON-CODING RNAS IN GENE EXPRESSION REGULATION AND CANCER BIOLOGY	Prof. Dr. Zukhra Yuldasheva Dr. Irfan Pratama
		5	STRUCTURAL BIOLOGY OF MEMBRANE TRANSPORTERS: IMPLICATIONS FOR DRUG RESISTANCE IN BACTERIA	Assis. Prof. Rustambek Sharipov Prof. Dr. Andini Kusuma
		6	METABOLOMIC PROFILING AND BIOMARKERS DISCOVERY IN CARDIOVASCULAR DISEASES USING MASS SPECTROMETRY	Dr. Javokhir Khodjaev Dian Anggraini Lila Haddad Dr. Folashade Adeyinka
		7	CRISPR-CAS9 APPLICATIONS IN GENOME EDITING: CURRENT CHALLENGES AND FUTURE DIRECTIONS IN THERAPEUTICS	Azizbek Tursunov Assoc. Prof. Kamal Souilah
		8	ROLE OF MITOCHONDRIAL DYSFUNCTION IN AGING AND AGE-RELATED DISEASES: A MOLECULAR BIOLOGY PERSPECTIVE	Assoc. Prof. Daler Hasanov Prof. Dr. Sinta Rahmawati

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HALL / SALON 2	**Prof. Dr. Laila Abdallah**	1	IMPACT OF TELEHEALTH IMPLEMENTATION ON MATERNAL AND NEWBORN HEALTH OUTCOMES IN RURAL MIDDLE EASTERN COMMUNITIES	Assoc. Prof. Dr. Layla Hassan Omar Al-Najjar Sara Al-Mutairi
		2	STRATEGIES TO ENHANCE MENTAL HEALTH SUPPORT FOR PREGNANT AND POSTPARTUM WOMEN IN MIDDLE EASTERN HEALTHCARE SETTINGS	Dr. Fatima Suleiman Khaled Al-Farsi
		3	IMPLEMENTING TELEHEALTH IN RURAL AND REMOTE COMMUNITIES FOR PRIMARY CARE	Prof. Dr. Alejandro Ruiz, Dr. Fatima Khan, Assis. Prof. Dr. Morten Larsen
		4	ASSESSMENT OF PAIN MANAGEMENT PRACTICES IN PEDIATRIC CARE UNITS ACROSS MIDDLE EASTERN COUNTRIES	Dr. Rania Khalil Youssef Mahmoud Dr. Leila Mansour
		5	THE ROLE OF NURSES IN CHRONIC DISEASE MANAGEMENT: A CROSS-COUNTRY STUDY IN THE MIDDLE EAST	Prof. Dr. Ahmed Farouk Dr. Salma Nabil Prof. Dr. Dalia Karam
		6	EVALUATION OF NURSING EDUCATION CURRICULA IN THE MIDDLE EAST: ALIGNMENT WITH GLOBAL HEALTHCARE NEEDS AND CULTURAL CONTEXTS	Prof. Dr. Samir Abbas
		7	PEDIATRIC NURSING AND FAMILY-CENTERED CARE: IMPROVING CHILDHOOD HEALTH OUTCOMES	Assoc. Prof. Dr. Helena Schmidt, Dr. Rafael Costa, lec. Priya Sharma
		8	EFFECTIVENESS OF COMMUNITY-BASED HEALTH PROMOTION PROGRAMS LED BY NURSES IN MIDDLE EASTERN URBAN AREAS	Dr. Sara Nasser Assoc. Prof. Dr. Fadi Hassan Prof. Dr. Laila Abdallah
		9	PUBLIC HEALTH NURSING AND EPIDEMIOLOGICAL STRATEGIES FOR INFECTIOUS DISEASE PREVENTION	Prof. Dr. Nnedi Okeke, Dr. Hye-Jin Park, Assis. Prof. Dr. Marco Rossi

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HALL / SALON 3	**Assoc. Prof. Dr. Maria Fernandez**	1	ELDERLY CARE STRATEGIES AND CHRONIC DISEASE MANAGEMENT IN NURSING PRACTICE	Assoc. Prof. Dr. Maria Fernandez, John O'Connor, lec. Aisha Al-Mansouri
		2	ADVANCES IN MIDWIFERY INTERVENTIONS TO REDUCE MATERNAL AND NEONATAL MORTALITY	Prof. Dr. Lillian Mbatha, Assis. Prof. Dr. Kenji Yamamoto, Sofia Petrescu
		3	MENTAL HEALTH NURSING: INTEGRATED APPROACHES FOR ANXIETY AND DEPRESSION TREATMENT	Dr. Samuel Thompson, Assoc. Prof. Dr. Amina Hassan, lec. Clara Nielsen
		4	CULTURAL COMPETENCY AND ITS EFFECT ON PATIENT SATISFACTION IN MIDWIFERY SERVICES IN THE MIDDLE EAST	Mona Al-Shammari Assoc. Prof. Dr. Jamal Ibrahim
		5	IMPACT OF WORKPLACE STRESS AND BURNOUT ON NURSING STAFF PERFORMANCE IN MIDDLE EASTERN HOSPITALS	Assoc. Prof. Dr. Nour El-Din Basma Saleh Dr. Saeed Al-Harith
		6	NUTRITIONAL INTERVENTIONS AND PATIENT EDUCATION IN HOSPITAL SETTINGS	Dr. Abdul Rahman, Prof. Dr. Camille Dubois, Assis. Prof. Dr. Tomasz Kowalski
		7	ADVANCED PRACTICES IN PAIN MANAGEMENT AND PALLIATIVE CARE	Assoc. Prof. Dr. Ingrid Johansen, Dr. Michael Brown, lec. Leila Haddad
		8	UTILIZATION OF E-LEARNING PLATFORMS TO IMPROVE CONTINUING PROFESSIONAL DEVELOPMENT AMONG MIDDLE EASTERN NURSES AND MIDWIVES	Dr. Hanan Al-Muhanna Dr. Wafa Saeed

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HALL / SALON 4	**Doç. Dr. Fatma AŞILIOĞLU**	1	SUSTAINABLE URBAN DESIGN STRATEGIES FOR RESILIENT CITIES IN RESPONSE TO CLIMATE CHANGE	Emily Carter Prof. Dr. Hiroshi Tanaka
		2	INTEGRATING GREEN INFRASTRUCTURE INTO HIGH-DENSITY URBAN ENVIRONMENTS FOR ENHANCED ECOLOGICAL AND SOCIAL BENEFITS	Dr. Marco Rossi Assoc. Prof. Dr. Aisha Al-Khalid
		3	THE ROLE OF HISTORICAL PRESERVATION IN MODERN URBAN PLANNING: BALANCING CULTURAL HERITAGE WITH CONTEMPORARY DEVELOPMENT	Assis. Prof. Dr. Isabelle Dubois
		4	DESIGNING SMART CITIES: THE IMPACT OF TECHNOLOGY AND DATA-DRIVEN DECISION MAKING ON URBAN LIVING SPACES	Olivia Bennett Dr. Wei Zhang Assoc. Prof. Dr. Jamal Hassan Ricardo Silva
		5	LANDSCAPE ARCHITECTURE FOR URBAN WELL-BEING: PROMOTING PSYCHOLOGICAL AND PHYSICAL HEALTH THROUGH PUBLIC SPACES	Prof. Dr. Hanna Karlsson lec. Fatima Noor Assoc. Prof. Dr. Samuel Adeyemi Dr. Miguel Herrera
		6	ADAPTIVE REUSE OF INDUSTRIAL SPACES: TRANSFORMING FORMER FACTORIES INTO VIBRANT URBAN HUBS	Dr. Katarzyna Nowak Assoc. Prof. Dr. Mohammed Al-Farouq
		7	URBAN MOBILITY AND PUBLIC SPACE DESIGN: CREATING PEDESTRIAN-FRIENDLY CITIES AND ENHANCING TRANSPORT CONNECTIVITY	lec. Laura Gonzalez Assoc. Prof. Dr. Tarek Mahmoud Dr. Michael Thompson
		8	CLIMATE-RESPONSIVE ARCHITECTURE: INTEGRATING PASSIVE DESIGN STRATEGIES IN URBAN BUILDINGS TO REDUCE ENERGY CONSUMPTION	Assoc. Prof. Dr. Nadia Rahman Prof. Dr. Johannes Bauer

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HALL / SALON 5	**Dr. Layla Hassan Prof. Dr. Carlos Rivera**	1	THE IMPACT OF COMMUNITY-BASED INTERVENTIONS ON MENTAL HEALTH OUTCOMES IN URBAN POPULATIONS	. Helena Bergström Dr. Samuel Adeyemi Maria Gonzales
		2	EXPLORING THE ROLE OF SOCIAL WORKERS IN ADDRESSING DOMESTIC VIOLENCE IN MULTICULTURAL SETTINGS	Ahmed Al-Mutairi Dr. Fiona MacLeod Ricardo Fernandez
		3	SOCIAL WORK APPROACHES TO SUPPORTING REFUGEE CHILDREN IN EDUCATIONAL ENVIRONMENTS	Noor El-Sayed Tomislav Novak Dr. Lian Zhang
		4	THE EFFECTIVENESS OF ELDERLY CARE PROGRAMS IN PROMOTING SOCIAL INCLUSION AND WELL-BEING	Dr. Aisha Khan Prof. Dr. Jean-Pierre Leblanc
		5	ADDRESSING SUBSTANCE ABUSE THROUGH HOLISTIC SOCIAL WORK INTERVENTIONS	Dr. Layla Hassan Prof. Dr. Carlos Rivera
		6	THE ROLE OF SOCIAL WORK IN COMBATING YOUTH HOMELESSNESS IN MAJOR CITIES	Dr. Keiko Tanaka Daniel Mwangi
		7	INTEGRATING TECHNOLOGY IN SOCIAL WORK PRACTICES FOR REMOTE COMMUNITIES	Anika Johansson Dr. Mariana Costa
		8	SOCIAL WORK STRATEGIES FOR PROMOTING MENTAL HEALTH IN POST-CONFLICT SOCIETIES	Assoc. Prof. Dr. Abdul Rahman Dr. Sofia Marques Tom Havers
		9	THE INFLUENCE OF CULTURAL COMPETENCY TRAINING ON SOCIAL WORK EFFECTIVENESS	Dr. Chandra Iyer Prof. Dr. Michael O'Connor

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HALL / SALON 6	**Dr. Nabilah Rahman**	1	SUPPORTING SURVIVORS OF HUMAN TRAFFICKING THROUGH INTERDISCIPLINARY SOCIAL WORK	Fatima El-Tayeb Dr. Tobias Klein Ana Pereira
		2	ADDRESSING FOOD INSECURITY THROUGH SOCIAL WORK INTERVENTIONS IN URBAN AND RURAL AREAS	Johan Svensson Lucia Ricci
		3	THE IMPACT OF POLICY ADVOCACY ON IMPROVING CHILD WELFARE SERVICES	Emily Thompson Assoc. Prof. Dr. Jovan Milosevic
		4	PROMOTING GENDER EQUALITY THROUGH SOCIAL WORK INITIATIVES IN DEVELOPING COUNTRIES	Dr. Amira Khalid Assoc. Prof. Dr. Viktor Horvath
		5	EXPLORING TRAUMA-INFORMED CARE PRACTICES IN SOCIAL WORK SETTINGS	Rachel Adams Helga Lund
		6	THE ROLE OF SOCIAL WORK IN DISASTER RESPONSE AND EMERGENCY MANAGEMENT	Anna Kowalski Dr. Mateo Jimenez
		7	SUPPORTING MENTAL HEALTH OF IMMIGRANT FAMILIES THROUGH COMMUNITY SOCIAL WORK PROGRAMS	Dr. Nabilah Rahman
		8	EVALUATING THE EFFECTIVENESS OF SOCIAL WORK INTERVENTIONS IN ADDRESSING CHRONIC POVERTY	Assoc. Prof. Dr. Peter Johansson Sofia Ibrahim Miguel Santos
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HALL / SALON 7	**Assoc. Prof. Dr. Mehrbon Tursunov**	1	ECONOMIC IMPACTS OF DIGITAL TRANSFORMATION ON SMALL AND MEDIUM ENTERPRISES IN DEVELOPING ECONOMIES	Nazarbek Sharipov Dr. Yasmine Boukhalfa
		2	ANALYSIS OF AGRICULTURAL SUBSIDY EFFECTS ON RURAL POVERTY AND FOOD SECURITY IN AFRICAN COUNTRIES	Assoc. Prof. Dr. Idrissa Koulibaly Dr. Rustam Ashurov
		3	MODELING THE EFFECTS OF REMITTANCES ON ECONOMIC GROWTH AND INCOME DISTRIBUTION IN EMERGING MARKETS	Prof. Dr. Hasan Rakhmonov Assoc. Prof. Dr. Jamila Cheriet
		4	THE ROLE OF MICROFINANCE IN EMPOWERING WOMEN ENTREPRENEURS IN RURAL AFRICA: A CASE STUDY APPROACH	Fatima Hassan Prof. Dr. Dodik Santoso
		5	IMPACT OF INFRASTRUCTURE DEVELOPMENT ON FOREIGN DIRECT INVESTMENT INFLOWS IN CENTRAL ASIAN ECONOMIES	Dilshod Rahmonov Dr. Rahayu Handayani
		6	EXPLORING THE RELATIONSHIP BETWEEN EDUCATION INVESTMENT AND ECONOMIC PRODUCTIVITY IN AFRICAN URBAN CENTERS	Assoc. Prof. Dr. Mehrbon Tursunov
		7	EVALUATION OF MONETARY POLICY EFFECTIVENESS IN CONTROLLING INFLATION IN DEVELOPING ASIAN AND AFRICAN COUNTRIES	Rustamov Akbar Aminata Diop
		8	AN EMPIRICAL STUDY ON THE CAUSALITY BETWEEN ENERGY CONSUMPTION AND ECONOMIC GROWTH IN SUB-SAHARAN AFRICA	Dr. Tsegaye Kebede Hendra Surya Aicha Chafai
		9	ASSESSING THE IMPACT OF GLOBAL TRADE POLICIES ON LOCAL MANUFACTURING INDUSTRIES IN EMERGING ECONOMIES	Prof. Dr. Jamaliddin Sobirov Assoc. Prof. Dr. Dini Nurhasanah
		10	SOCIAL PROTECTION PROGRAMS AND THEIR EFFECTS ON INCOME INEQUALITY IN AFRICAN AND ASIAN DEVELOPING COUNTRIES	Dr. Halima Adeyemi Prof. Dr. Bambang Pranoto Dr. Fouad Benali

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HALL / SALON 8	**Prof. Dr. Nizom Umedov Assoc. Prof. Dr. Yayan Zuhri**	1	EXPLORING THE ROLE OF DIGITAL FINANCE IN PROMOTING INCLUSIVE GROWTH IN LOW-INCOME AFRICAN COUNTRIES	Assoc. Prof. Dr. Mukhlis Juraev Dr. Dian Puspitasari Boubaker Djerah Ahmed Boussouf
		2	INFLUENCE OF FOREIGN AID ON ECONOMIC DEVELOPMENT AND GOVERNANCE: COMPARATIVE STUDY OF SELECT AFRICAN AND CENTRAL ASIAN COUNTRIES	Assoc. Prof. Dr. Riana Kartika
		3	TRADE LIBERALIZATION AND ITS IMPACT ON POVERTY REDUCTION IN AFRICAN AGRICULTURAL SECTORS	Prof. Dr. Nizom Umedov Mirna Sari
		4	ANALYSIS OF THE INFORMAL ECONOMY'S CONTRIBUTION TO GDP AND EMPLOYMENT IN AFRICAN URBAN AREAS	Assis. Prof. Dr. Salem Benhamadi
		5	FISCAL DECENTRALIZATION AND ITS EFFECTS ON REGIONAL ECONOMIC DEVELOPMENT IN TAJIKISTAN AND INDONESIA	Dr. Tonia Abubakar Rustam Nizomov Dr. Ratna Puspita Lotfi Bouazizi Sami Saidi
		6	THE ECONOMIC CONSEQUENCES OF CLIMATE CHANGE ADAPTATION POLICIES IN NORTHERN AFRICA AND CENTRAL ASIA	Assoc. Prof. Dr. Yayan Zuhri
		7	THE ROLE OF ENTREPRENEURSHIP AND INNOVATION IN DRIVING ECONOMIC GROWTH IN AFRICAN AND INDONESIAN EMERGING MARKETS	Prof. Dr. Azizbek Rahmatov Dr. Dewi Anggraini Abdelkader Zerari
		8	MODELING THE IMPACT OF CORRUPTION ON ECONOMIC PERFORMANCE IN AFRICAN COUNTRIES AND TAJIKISTAN	Komiljon Usmonov Siti Aisyah Dr. Lamine Oumar

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HALL / SALON 1	**Prof.Dr.Özlem Tokuşoğlu**	1	MAD HONEY: BIOACTIVE COMPONENTS AND EFFECTS ON HUMAN HEALTH	Dr. Ekin VAROL Prof. Dr. Banu YÜCEL
		2	IMPORTANCE OF HONEY AND FRESH BEE POLLEN IN HEALTHY DIETS	Prof. Dr. Banu YÜCEL Dr. Ekin VAROL
		3	TÜRKİYE İLE İSPANYA'YA AİT GASTRONOMİK UNSURLARIN KARŞILAŞTIRILMASI	Doç. Dr. EMRE HASTAOĞLU
		4	TÜRKİYE'DEKİ SİMİTLER VE ÖZELLİKLERİ	Doç. Dr. EMRE HASTAOĞLU
		5	PORTAKAL VE LİMON YAPRAKLARININ BİYOAKTİF ÖZELLİKLERİ VE FENOLİK BİLEŞİKLERİNDEKİ DEĞİŞİMLER ÜZERİNE KURUTMANIN ETKİSİ	Yüksek Lisans Öğrenci, MELİHA ÇETİN Doç. Dr., NURHAN USLU Doktora Öğrenci, HAVVANUR YILMAZ Prof. Dr., MEHMET MUSA ÖZCAN
		6	THE ASSESMENTS ON FOOD CHEMISTRY ON VALUE-ADDED PLANT-BASED FOOD EFFERVESCENT SUPPLEMENTS	Prof.Dr.Özlem Tokuşoğlu
		7	MICROENCAPSULATED RASPBERRY LAMBERTIANIN C (LC) POWDER AND RASPBERRY LC BASED CHEWING GEL CANDY	Prof.Dr.Özlem Tokuşoğlu
		8	GASTRONOMİ TURİZMİ VE SÜRDÜRÜLEBİLİR DESTİNASYON PAZARLAMASI	DAMLA KARADAYI PROF. DR. NESRİN M. BAHÇELERLİ

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HALL / SALON 2	**Assist. Prof. Hilal OK ERGÜN**	1	METAL EŞYA, MAKİNE, ELEKTRİKLİ CİHAZLAR ve ULAŞIM ARAÇLARI SEKTÖRÜNDE FİNANSAL PERFORMANSININ ENTROPİ AĞIRLIKLANDIRMALI TOPSIS YÖNTEMİ İLE DEĞERLENDİRİLMESİ	Dr. Osman Nuri AKARSU
		2	THE RELATIONSHIP BETWEEN AUDIT COMMITTEE STRUCTURE AND FINANCIAL PERFORMANCE: EVIDENCE FROM THE BIST-100 INDEX	Dr. Öğr. Üyesi Yavuz KILINÇ
		3	ACCOUNTİNG OF MURABAHA FİNANCİNG: A COMPARATİVE STUDY OF TURKİSH ACCOUNTİNG STANDARDS AND INTEREST-FREE FİNANCE ACCOUNTİNG STANDARDS	Dr. Öğr. Üyesi Mehmet Murat GUTNU Yüksek Lisans Öğrencisi Abdullah ADANIR
		4	COUNTERFEITING'S RIPPLE EFFECT ON SUSTAINABILITY: INSIGHTS AND STRATEGIES FOR CHANGE	Dr. Ali Galip Ayvat Talat Yörük Levent Köseoğlu Assoc. Prof. Dr. Pınar Ayvat
		5	FINANCIAL DEVELOPMENT AND NATURAL RESOURCES MANAGEMENT: EVIDENCE FROM TÜRKİYE	Assist. Prof. Hilal OK ERGÜN
		6	THE IMPACT OF STRATEGIC MANAGEMENT PRACTICES ON EMPLOYEE PERFORMANCE IN CUSTOMS CONSULTANCY FIRMS	Master Student,FERİZA DEMİR GİTMİŞOĞLU Assist. Prof. Dr., FATİH MEHMET BULUT

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HALL / SALON 3	**Prof. Dr. Faruk ÖZDEMİR**	1	Ismāil Rusūhī Ankarāwī’s Maqāṣid al-ʿāliyya fī Sharh al-Tāiyya and Its Impact on Ottoman Thought	Doç. Dr. Zeliha Öner
		2	HADİSLERİN TARİHSEL OLAYLAR İLE KARŞILAŞTIRILMASINDA KULLANILAN YÖNTEMLER	Doç. Dr. FUAT İSTEMİ
		3	KONJOKTÜRÜN HADİS TASNİFİNE YANSIMASI	Doç. Dr. FUAT İSTEMİ
		4	THE ROLE OF CIVIL INITIATIVES IN PREVENTING GLOBAL INJUSTICES: THE EXAMPLE OF THE HILF AL-FUDUL	Prof. Dr. Faruk ÖZDEMİR
		5	KIND WORD AS A UNIVERSAL HUMAN AND MORAL VALUE: A STUDY FROM THE PERSPECTIVE OF ITS POWER TO TRANSFORM HOSTILITY INTO FRIENDSHIP AND ESTABLISH PEACE	Prof. Dr. Faruk ÖZDEMİR
		6	MANTIK VE İLAHİYAT	Doç. Dr. Muhammet Nasih Ece

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HALL / SALON 4	**Dr. Öğr. Üyesi HAKAN AVAN**	1	TECHNOLOGY-SUPPORTED FAMİLY-CENTERED NURSİNG INTERVENTİONS İN CHİLDREN WİTH CHRONİC ILLNESSES: A COMPREHENSİVE NARRATİVE REVİEW FOCUSED ON TYPE 1 DİABETES	Dr. Öğr. Üyesi HAKAN AVAN
		2	SCREEN ADDİCTİON AND SLEEP QUALİTY İN CHİLDHOOD: A REVİEW OF NURSİNG INTERVENTİONS	Dr. Öğr. Üyesi HAKAN AVAN
		3	STİGMATİSİNG ATTİTUDES OF HEALTHCARE PROFESSİONALS TOWARDS INDİVİDUALS WİTH AIDS AND INFLUENCİNG FACTORS: A DESCRİPTİVE-CROSS-SECTİONAL STUDY	Doç. Dr. Meryem Türkan IŞIK Doç. Dr. Rana CAN ÖZDEMİR Öğretim Görevlisi İbrahim Duman
		4	TAZELENME ÜNİVERSİTESİNE KATILAN YAŞLI BİREYLERİN BAŞARALI YAŞLANMA DÜZEYİNİN BELİRLENMESİ	Öğr. Gör. İbrahim DUMAN
		5	DIABETES CARE IN A CHANGING CLIMATE: FROM RISK TO ADAPTATION, FROM ADAPTATION TO RESILIENCE	Öğr. Gör., Merve MURAT MEHMED ALİ Prof. Dr., Selda ÇELİK
		6	THE DUAL FACE OF INSULIN: DIABULIMIA AND ITS MANAGEMENT IN ADOLESCENT DIABETES CARE	Öğr. Gör., Merve MURAT MEHMED ALİ Prof. Dr., Selda ÇELİK
		7	NURSES’ INFORMATİCS COMPETENCİES: A COMPREHENSİVE LİTERATURE REVİEW	Prof. Dr. Meryem YAVUZ Van GİERSBERGEN Nurse Nebihat TEKİN
		8	INFORMATICS NURSING IN GRADUATE NURSING PROGRAMS IN TÜRKIYE	Prof. Dr. Meryem YAVUZ Van GİERSBERGEN Nurse Nebihat TEKİN

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HALL / SALON 1	**Prof. Dr. Aigerim Tursynova**	1	IMPROVING PATIENT OUTCOMES THROUGH EVIDENCE-BASED NURSING PRACTICES IN CRITICAL CARE SETTINGS	Assoc. Prof. Dr. Ayan Mukhtar, Assis. Prof. Dr. Serik Tulegenov,
		2	MATERNAL MENTAL HEALTH: ASSESSING THE IMPACT OF POSTPARTUM SUPPORT PROGRAMS	Prof. Dr. Zharaskan Amanov Ayesha Khan, Prof. Dr. Rizwan Iqbal, lec. Sana Farooq
		3	THE ROLE OF MIDWIVES IN REDUCING INFANT MORTALITY IN RURAL COMMUNITIES	Assoc. Prof. Dr. Aliya Bek, Assis. Prof. Dr. Nurlan Sadykov,
		4	ENHANCING NURSING EDUCATION THROUGH SIMULATION-BASED LEARNING AND VIRTUAL REALITY	Hamza Malik, Sadaf Javed, Dr. Mahnoor Iqbal
		5	INTEGRATING CULTURAL COMPETENCY TRAINING INTO HEALTHCARE PROFESSIONAL DEVELOPMENT	Prof. Dr. Shazia Abbas, Assis. Prof. Dr. Faizan Tariq, lec. Hina Rauf
		6	STRATEGIES FOR PREVENTING HOSPITAL-ACQUIRED INFECTIONS IN INTENSIVE CARE UNITS	Leila Hariri, Prof. Dr. Marwan Haddad, Nadine Karam
		7	ADVANCEMENTS IN NEONATAL CARE: EARLY INTERVENTIONS FOR HIGH-RISK INFANTS	Assoc. Prof. Dr. Samina Iqbal Prof. Dr. Imran Aslam, Noor Baloch
		8	ASSESSING THE EFFECTIVENESS OF TELEHEALTH SERVICES IN CHRONIC DISEASE MANAGEMENT	Assoc. Prof. Dr. Zarina Sadykova, Assis. Prof. Dr. Bauyrzhan Kairat, Prof. Dr. Aigerim Tursynova

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HALL / SALON 2	**Prof. Dr. Rahmatullah Hashemi,**	1	THE IMPACT OF NURSING LEADERSHIP ON EMPLOYEE SATISFACTION AND RETENTION	Assoc. Prof. Dr. Layla Hamidi, Prof. Dr. Farid Wardak,
		2	PROMOTING BREASTFEEDING PRACTICES THROUGH COMMUNITY-BASED INTERVENTIONS	Assoc. Prof. Dr. Amal Hassan, Zeinab Khatib, Rania Touma
		3	EXPLORING THE ROLE OF NURSES IN MANAGING EMERGING INFECTIOUS DISEASE OUTBREAKS	Dr. Nabil Zayani, Assis. Prof. Dr. Samir Belkacem,
		4	PAIN MANAGEMENT IN PALLIATIVE CARE: EVALUATING MULTIDISCIPLINARY APPROACHES	Assoc. Prof. Dr. Jasmin Kabuli, Prof. Dr. Haroon Rahman,
		5	IMPLEMENTING MENTAL HEALTH SCREENING IN PRIMARY HEALTHCARE SETTINGS	Assoc. Prof. Dr. Leila Bennis, Assis. Prof. Dr. Anis Cherif, Prof. Dr. Amira Boudjelal
		6	THE USE OF MOBILE HEALTH APPLICATIONS FOR PATIENT EDUCATION AND SELF-MANAGEMENT	Assoc. Prof. Dr. Maryam Farooqi, Prof. Dr. Tariq Azhar Sana Qureshi
		7	NURSE-LED INTERVENTIONS IN REDUCING HOSPITAL READMISSION RATES AMONG ELDERLY PATIENTS	Assoc. Prof. Dr. Fawzia Rahimi, Prof. Dr. Rahmatullah Hashemi, lec. Sahar Mohammadi
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HALL / SALON 3	**Assis. Prof. Dr. Bilal Benyamina, Prof. Dr. Karima Ouchene**	1	ADDRESSING NUTRITIONAL CHALLENGES IN NEONATAL INTENSIVE CARE UNITS	Assis. Prof. Dr. Bilal Benyamina, Prof. Dr. Karima Ouchene
		2	EVALUATING THE EFFECTIVENESS OF STRESS REDUCTION PROGRAMS FOR NURSES IN HIGH-INTENSITY ENVIRONMENTS	Prof. Dr. Rana Al-Khatib, Prof. Dr. Jad Saade, Nour El Hage
		3	IMPROVING PATIENT SAFETY THROUGH ELECTRONIC HEALTH RECORDS AND DIGITAL MONITORING	Assoc. Prof. Dr. Amirullah Safi, Dr. Hakim Wazir,
		4	DEVELOPING TRAINING MODULES FOR MIDWIVES ON COMPLICATIONS DURING CHILDBIRTH	Dr. Anisa Rahimi, Dr. Abdul Qadir Khan, Samina Mir
		5	IMPROVING PATIENT SAFETY AND QUALITY OF CARE THROUGH ADVANCED NURSING INTERVENTIONS IN ACUTE CARE SETTINGS	Assoc. Prof. Dr. Ayan Bektemir lec. Zhanar Tulegenova Prof. Dr. Yerlan Sadykov Assoc. Prof. Dr. Asma Riaz Hamza Qureshi
		6	MENTAL HEALTH PROMOTION AMONG ADOLESCENTS IN RURAL AREAS: CHALLENGES AND STRATEGIES FOR NURSES AND HEALTH PROFESSIONALS	Prof. Dr. Rustam Kairat Dilshad Akhmed
		7	MATERNAL AND NEWBORN HEALTH OUTCOMES: INTEGRATING EVIDENCE-BASED MIDWIFERY PRACTICES WITH COMMUNITY HEALTH INITIATIVES	Dr. Nadia Al-Khatib Dr. Faridah Mansoor
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HALL / SALON 4	**Prof. Dr. Yacine Bensalem**	1	MARITAL COMMUNICATION AND CONFLICT RESOLUTION STRATEGIES IN LONG-TERM RELATIONSHIPS	Amina Belkacem Prof. Dr. Adewale Okoye Rizky Pratama
		2	IMPACT OF DIFFERENT PARENTING STYLES ON CHILD EMOTIONAL AND SOCIAL DEVELOPMENT	Dr. Fatimah Zainal Prof. Dr. Hassan Alimov Maria Santos
		3	COUPLE THERAPY APPROACHES FOR BLENDED FAMILIES AND STEP-PARENTING CHALLENGES	Nurul Hidayah Dr. Leila Karimova Juan dela Cruz
		4	THE ROLE OF CULTURAL VALUES AND BELIEFS IN FAMILY THERAPY INTERVENTIONS	Assoc. Prof. Dr. Anisa Rahman Timur Beketov Sofia Mammadova
		5	PREMARITAL COUNSELING AND STRATEGIES TO ENHANCE RELATIONSHIP SATISFACTION	Assis. Prof. Dr. Ahmed Benyamina Rizal Fahri Kristine Reyes
		6	FAMILY THERAPY TECHNIQUES FOR SUPPORTING SUBSTANCE ABUSE RECOVERY AND RELAPSE PREVENTION	Halima Olanrewaju Alif Prasetyo Elina Nazarova
		7	THE EFFECTS OF DIGITAL TECHNOLOGY AND SOCIAL MEDIA ON FAMILY DYNAMICS AND INTERACTIONS	Prof. Dr. Yacine Bensalem Farida Iskandarova Miguel Santos
		8	TRAUMA-INFORMED FAMILY THERAPY APPROACHES FOR HEALING EMOTIONAL AND PSYCHOLOGICAL WOUNDS	Dr. Aisha Mohammed Dwi Lestari Rashid Akhmedov
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HALL / SALON 5	**Prof. Dr. Yasmin Al-Hussein**	1	ANCIENT CIVILIZATIONS AND THEIR ECONOMIC, SOCIAL, AND POLITICAL STRUCTURES	Assoc. Prof. Dr. Khalid Al-Mansouri Lec. Ahmed Rezaei
		2	OTTOMAN EMPIRE: SOCIAL HIERARCHIES, GOVERNANCE, AND CULTURAL TRANSFORMATIONS	Dr. Nour Al-Farouq Lec. Fatima Al-Hadid
		3	HISTORY OF ISLAMIC SCIENCE, TECHNOLOGY, AND KNOWLEDGE TRANSMISSION ACROSS REGIONS	Prof. Dr. Omar Al-Khalifa Assoc. Prof. Dr. Rania Saeed
		4	MODERN MIDDLE EAST POLITICAL HISTORY: NATION BUILDING, CONFLICTS, AND INTERNATIONAL RELATIONS	Dr. Layla Abou-Taleb Assoc. Prof. Dr. Tariq Al-Zahra Lec. Samira Al-Basri
		5	TRADE ROUTES, CULTURAL EXCHANGES, AND THE SPREAD OF IDEAS IN HISTORICAL CONTEXTS	Prof. Dr. Salma Shalabi Lec. Mohammad Al-Saadi
		6	HISTORY OF WOMEN IN THE ARAB WORLD: SOCIAL ROLES, RIGHTS, AND HISTORICAL NARRATIVES	Assoc. Prof. Dr. Dina Al-Mahdi Dr. Leila Kanaan Lebanon
		7	COLONIALISM AND POST-COLONIAL TRANSFORMATIONS IN MIDDLE EASTERN SOCIETIES	Prof. Dr. Hana Al-Mutairi Lec. Fadi Al-Karim
		8	HISTORICAL ARCHITECTURE AND URBAN DEVELOPMENT IN MIDDLE EASTERN CITIES	Prof. Dr. Yasmin Al-Hussein
		9	MILITARY STRATEGIES, CONFLICTS, AND WARFARE IN MIDDLE EASTERN HISTORY	Prof. Dr. Samah Al-Saleh Lec. Nabil Al-Ahmad

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HALL / SALON 6	**Prof. Dr. Adeel Khan,**	1	THE IMPACT OF NATURAL FERMENTATION TECHNIQUES ON THE NUTRITIONAL QUALITY OF TRADITIONAL DAIRY PRODUCTS	Yerlan Akhmetov, Assoc. Prof. Dr. Aigul Sadykova, Bauyrzhan Tleugabyl
		2	INNOVATIVE METHODS FOR PRESERVING SHELF-LIFE IN READY-TO-EAT MEALS USING BIOACTIVE PACKAGING	Sana Ullah, Prof. Dr. Adeel Khan, Imran Haider
		3	APPLICATION OF MICROWAVE AND ULTRASOUND TECHNOLOGIES IN FOOD PROCESSING FOR ENERGY EFFICIENCY	Alua Zhumagaliyeva, Daniyar Omarov, Nurlan Sagin
		4	DEVELOPMENT OF FUNCTIONAL FOODS ENRICHED WITH PLANT-BASED PROTEINS AND ANTIOXIDANTS	Amna Tariq, Saima Iqbal, Assis. Prof. Dr. Fahad Rashid
		5	UTILIZATION OF AGRICULTURAL WASTE IN BIOACTIVE FOOD INGREDIENT PRODUCTION	Hina Malik, Prof. Dr. Imran Abbas, Farah Zaman
		6	APPLICATION OF 3D FOOD PRINTING IN CULINARY INNOVATIONS AND PERSONALIZED NUTRITION	Aisha Rahimi, Lec. Dr. Farid Noori, Mohammad Shirzad
		7	EXPLORING THE ROLE OF NANOTECHNOLOGY IN FOOD SAFETY AND QUALITY MONITORING	Zahra Khan, Prof. Dr. Ali Raza, Asma Qureshi
		8	OPTIMIZATION OF ENZYMATIC PROCESSES FOR PLANT-BASED DAIRY ALTERNATIVES	Samira Najafi, Assoc. Prof. Dr. Nadia Karimi, Ramin Farhad

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HALL / SALON 1	**Asst. Prof. Dr. HALIL FEDAI**	1	THE RELATIONSHIP BETWEEN VENTRICULAR EXTRASYSTOLE FREQUENCY AND C-REACTİVE PROTEIN ALBUMIN RATIO	Asst. Prof. Dr. HALIL FEDAI
		2	THE EFFECT OF BODY IMAGE ON SEXUAL QUALİTY OF LİFE İN WOMEN WİTH BREAST CANCER WHO HAVE RECEİVED NEOADJUVANT CHEMOTHERAPY	Dr. Öğretim Üyesi, Mehmet Emin ŞANLI Öğretim Görevlisi Dr. Mahmut DİNÇ Öğretim Görevlisi Dr. Uğur ÖNER Dr. Öğretim Üyesi, Tülay YILDIRIM ÜŞENMEZ
		3	POSTPARTUM ANNE VE YENİDOĞAN GÜVENLİĞİ: PSİKOLOJİK RİSKLER VE ÇEVRİMİÇİ EĞİTİM MÜDAHALELERİNİN ROLÜ	Zekiye SİVRİDAĞ Prof.Dr. Şengül YAMAN SÖZBİR
		4	MENOPAUSE AND SEXUAL LIFE	Ebe Remziye KARADOĞAN KOSANOĞLU Doç. Dr. Ayça ŞOLT KIRCA Doç. Dr. Elif DAĞLI
		5	HEART DISEASES DURING PREGNANCY	Ebe Remziye KARADOĞAN KOSANOĞLU Doç. Dr. Ayça ŞOLT KIRCA Doç. Dr. Elif DAĞLI
		6	SEZARYENİN KLİNİK ETKİLERİ VE POSTOPERATİF YÖNETİM	Arş. Gör., SONGÜL KEKİL
		7	GEBELİKTE KÜLTÜREL İNANIŞLAR VE GELENEKSEL UYGULAMALAR	Arş. Gör., SONGÜL KEKİL
		8	THREE-MONTH FOLLOW-UP OF GREATER OCCIPITAL NERVE BLOCK EFFICACY IN CHRONIC MIGRAINE	Assis. Prof. Dr. Derya Yavuz Demiray

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HALL / SALON 2	**Assist. Prof. Dr. KEZBAN YILDIZ-DALGINLI**	1	MONOAMİN OKSİDAZ A'NIN GENETİK MANİPÜLASYONU: NÖROTRANSMİTTER METABOLİZMASINDAN DAVRANIŞA	Uzman Biyoteknolog/Projeler Koordinatörü, SELİN URAL Araştırmacı Öğrenci, SEDEN TOLUNAY
		2	KEDİLERDE HERPES VİRÜS ENFEKSİYONLARININ OKSİDATİF STRES PARAMETRELERİ ÜZERİNE GÜNCEL RAŞTIRMALARI	ESRA KÖSE Prof. Dr. SENA ÇENESİZ
		3	KÖPEKLERDE PARVOVİRÜS ENFEKSİYONLARININ GÜNCEL TEDAVİ YÖNTEMLERİ	ESRA KÖSE Prof. Dr. SENA ÇENESİZ
		4	CONTRIBUTION OF ARTVİN (TÜRKİYE) BROWN BEAR POPULATIONS TO THE MOLECULAR PHYLOGENY OF Ursus arctos	MSc Student Serap ÖZDEMİR Dr. Gökçe Ali KELEŞ Asst. Prof. Dr. Ahmet Yesari SELÇUK Asst. Prof. Dr. Perinçek Seçkinozan ŞEKER
		5	SHILAJIT ATTENUATES 5-FLUOROURACIL-INDUCED HEPATOTOXICITY THROUGH ANTIOXIDANT AND SIRTUIN PATHWAYS IN RATS	Assist. Prof. Dr. KEZBAN YILDIZ-DALGINLI Lecturer, MELEK OZTURKLER-DUNDAR Prof. Dr. ONUR ATAKISI
		6	INVESTIGATION OF THE CAUSES OF EUCALYPTUS CAMALDULENSIS SEEDLINGS MORTALITY IN TARSUS FOREST NURSERY	Öğr. Gör. Dr. Deniz ÇAKAR Doç. Dr. Ebru DERELLİ TÜFEKÇİ Fatih AYTAR Prof. Dr. Seçil AKILLI ŞİMŞEK

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HALL / SALON 3	**Assoc. Prof. Ceyda GÜRMAN**	1	A PSYCHOLOGICAL EXAMINATION OF THE RELATIONSHIP BETWEEN POSTMODERNISM AND RELIGIOUS FATIGUE	Dr. Öğr. Üyesi, Mehmet Emrullah DURAN
		2	WHAT IS JERUSALEM?	Assistant Professor KENAN KARAGÖZ
		3	İBÂZİYYE'DE HALKU'L-KUR'AN TARTIŞMALARI	Doç. Dr. Metin YILDIZ
		4	İBÂZİYYE'DE EF'ÂL-İ İBÂD	Doç. Dr. Metin YILDIZ
		5	THE GOVERNANCE PRINCIPLES AND EXEMPLARITY OF THE PROPHET	Dr. Öğr. Üyesi, Sadettin GÜRMAN
		6	The Ottoman Ulema's Defence of Ahl al-Sunnah Against the Muʿtazila: Treatises on al-Anʿām 6:158	Assoc. Prof. Ceyda GÜRMAN

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HALL / SALON 4	**Assoc. Prof. Dr., Fatma Köprülü Assoc. Prof. Dr., Şengül Başarı**	1	STUDENTS' VIEWS ON PLACE-BASED EDUCATION PRACTICES IN SCIENCE COURSE	Uzman Öğretmen, Cahide SERDAROĞLU Prof. Dr. Munise Handan GÜNEŞ
		2	TÜRKİYE YÜZYILI MAARİF MODELİNE PROGRAM GELİŞTİRME VE FELSEFİ YAKLAŞIM AÇISINDAN ELEŞTİREL BİR BAKIŞ	Dr. Okan DEDE
		3	Sınıf Öğretmenlerinin Akran Zorbalığına Yönelik Görüşlerinin Yapay Zeka Destekli Eğitim Yaklaşımlarıyla İncelenmesi	Assoc. Prof. Dr., Fatma Köprülü Assoc. Prof. Dr., Şengül Başarı
		4	EVALUATİNG THE PRİMARY SCHOOL CLASSROOM GUİDANCE PROGRAM THROUGH THE CIPP MODEL	Esma TAŞKIRAN ÇAPRAK Dr. Öğr. Üyesi Uğur EPÇAÇAN
		5	EVALUATING SOCIAL PARTICIPATION IN THE CONTEXT OF WOMEN THROUGH THEIR PARTICIPATION IN LIFELONG LEARNING	Dr. Deniz Yalçınkaya Hediye Acar
		6	ORTAOKUL ÖĞRENCİLERİNİN ETKİLİ İLETİŞİM BECERİLERİ İLE DİNLEME MOTİVASYONLARI ARASINDAKİ İLİŞKİNİN İNCELENMESİ	Yüksek Lisans Öğrencisi, Yener Emre SEVGİLİ Doç. Dr. Ata PESEN Dr. Öğr. Üyesi Burhan ÜZÜM
		7	BEDEN EĞİTİMİ VE SPOR ÖĞRETMENLİĞİ ÖĞRENCİLERİNDE ÖĞRETİM PROGRAMI OKURYAZARLIĞI	Dr. RUKİYE AYDOĞAN DENİZ ÖRSAN ÖRS
		8	ORTAOKUL, LİSE VE ÜNİVERSİTE ÖĞRENCİLERİNDE NOMOFOBİ DÜZEYLERİNİN İNCELENMESİ: CİNSİYETİN VE EĞİTİM DÜZEYİNİN ROLÜ	Dr. Öğr. Üyesi Halil ASLAN

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HALL / SALON 1	**Assoc. Prof. Dr. Maria Teresa Santos Prof. Dr. Angelo Villanueva**	1	THE ROLE OF INTERFAITH DIALOGUE IN PROMOTING RELIGIOUS TOLERANCE AND SOCIAL COHESION	Assoc. Prof. Dr. Yerzhan Bektas Prof. Dr. Aisulu Muratova Amanzhol Kairat
		2	RECONCILING TRADITIONAL BELIEFS WITH MODERN CHRISTIAN THEOLOGY IN CONTEMPORARY SOCIETIES	Prof. Dr. Haroon Qureshi Lec. Nadia Khattak
		3	ETHICS AND MORALITY IN ISLAMIC PHILOSOPHY: CHALLENGES AND MODERN INTERPRETATIONS	Dr. Kiran Jameel
		4	THEOLOGICAL PERSPECTIVES ON THE CONCEPT OF DIVINE JUSTICE IN ISLAM	Assoc. Prof. Dr. Faridullah Ahmadi Dr. Hameed Safi Zarina Wardak
		5	MYSTICISM AND SPIRITUAL EXPERIENCE IN SUFI TRADITIONS: HISTORICAL AND CONTEMPORARY VIEWS	Prof. Dr. Layth Al-Khatib Prof. Dr. Rana Daher Fadi Tahan Katrina Delos Reyes
		6	INTERPRETATIONS OF SACRED TEXTS IN MODERN PHILIPPINE CATHOLIC THEOLOGY	Assoc. Prof. Dr. Maria Teresa Santos Prof. Dr. Angelo Villanueva
		7	THE IMPACT OF RELIGIOUS EDUCATION ON ETHICAL DEVELOPMENT IN MUSLIM COMMUNITIES	Iqbal Khan Sana Riaz Bilal Ahmed Youssef Benali
		8	CHRISTIAN THEOLOGY AND ENVIRONMENTAL ETHICS: TOWARDS A SUSTAINABLE WORLDVIEW	Assoc. Prof. Dr. Ahmad Hariri Prof. Dr. Leila Mounir

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HALL / SALON 2	**Dr. Askar Zhaksylyk**	1	THEOLOGICAL REFLECTIONS ON SUFFERING AND THEODICY IN ISLAMIC AND CHRISTIAN TRADITIONS	Assoc. Prof. Dr. Abdul Rahman Azimi Prof. Dr. Laila Hamidi Farid Noorzai Timur Akhmetov
			THE INFLUENCE OF ISLAMIC PHILOSOPHY ON CONTEMPORARY ETHICAL THEORY	Dr. Askar Zhaksylyk Prof. Dr. Gulmira Nurgaliyeva
			THE ROLE OF WOMEN IN RELIGIOUS LEADERSHIP: PERSPECTIVES FROM ISLAMIC AND CHRISTIAN TRADITIONS	Dr. Shazia Malik Hina Tariq
			MODERN CHALLENGES IN THEOLOGICAL EDUCATION AND RELIGIOUS STUDIES	Dr. Ahmad Sultani Lec. Lailuma Rahimi
		2	THE CONCEPT OF MERCY AND FORGIVENESS IN DIFFERENT RELIGIOUS TRADITIONS	Prof. Dr. Sami Najjar
		3	HISTORICAL DEVELOPMENT OF CHRISTIAN MONASTICISM AND ITS SOCIO-CULTURAL IMPACT	Felicity Ramos Assoc. Prof. Dr. Maria Angela Flores Dr. Jonathan Cruz
		4	THE INTERSECTION OF POLITICS AND RELIGION: THEOLOGICAL PERSPECTIVES	Samina Rauf Assoc. Prof. Dr. Iqbal Hussain Dr. Anwar Siddiqui
		5	ISLAMIC ESCHATOLOGY AND THE CONCEPT OF THE AFTERLIFE: A COMPARATIVE STUDY	Assoc. Prof. Dr. Nurlan Seitov Prof. Dr. Aiman Tuleyev Dinara Zhumagaliyeva Rami Mansour Lina Saab
			RELIGION, SCIENCE, AND FAITH: NAVIGATING CONTEMPORARY THEOLOGICAL DEBATES	Assoc. Prof. Dr. Ahmad Al-Khalil
			THE ROLE OF RELIGION IN CONFLICT RESOLUTION AND PEACEBUILDING	Assoc. Prof. Dr. Khalid Benamara Samira Bouaziz Yasmine Cherif
		6	THE IMPACT OF GLOBALIZATION ON TRADITIONAL RELIGIOUS PRACTICES AND BELIEFS	Assoc. Prof. Dr. Maria Lourdes Reyes

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HALL / SALON 3	**Prof. Dr. Asif Javed**	1	THE IMPACT OF DIGITAL LEARNING PLATFORMS ON STUDENT ENGAGEMENT IN HIGHER EDUCATION	Yerlan Tulegenov Assoc. Dr. Danaer Beketova Prof. Dr. Aibek Zhumatayev
		2	EXPLORING INCLUSIVE EDUCATION POLICIES FOR CHILDREN WITH SPECIAL NEEDS	Assoc. Prof. Dr. Saadullah Kabirov lAisulu Altynbekova
		3	THE ROLE OF INTERCULTURAL COMPETENCE IN INTERNATIONAL STUDENT SUCCESS	Gulnara Tulegen Assoc. Prof. Dr. Almas Zharkynov Daniyar Serikbay Madina Sarsenova
		4	EFFECTIVE TEACHING STRATEGIES FOR MULTILINGUAL CLASSROOMS	Dr. Sanaullah Qureshi Dr. Rabia Tariq Hamid Raza
		5	THE USE OF AI-DRIVEN ASSESSMENTS TO IMPROVE LEARNING OUTCOMES	Prof. Dr. Asif Javed Samina Khalid
		6	CHALLENGES AND OPPORTUNITIES IN ONLINE STEM EDUCATION	Dr. Mirza Rahman Prof. Dr. Ayesha Khan
		7	THE INFLUENCE OF EDUCATIONAL GAMIFICATION ON STUDENT MOTIVATION	Fahad Riaz Dr. Lubna Ahmed
		8	IMPLEMENTING SUSTAINABLE EDUCATION PRACTICES IN PRIMARY SCHOOLS	Dr. Farid Gulzhan Dana Yerzhan

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HALL / SALON 4	**Assoc. Prof. Dr. Samir Nasser**	1	THE EFFECTIVENESS OF PROJECT-BASED LEARNING IN ENGINEERING PROGRAMS	Dr. Shahrukh Malik Dr. Saba Javed Imran Hussain
		2	LEADERSHIP DEVELOPMENT PROGRAMS FOR SCHOOL ADMINISTRATORS	Khalid Mehmood Sana Tariq
		3	INTEGRATING CULTURAL HERITAGE INTO MODERN CURRICULA	Dr. Farzana Rahimi Omar Karimi Waliullah Faiz
		4	THE ROLE OF PARENTAL INVOLVEMENT IN EARLY CHILDHOOD EDUCATION	Assoc. Prof. Dr. Jamal Alawi Nadia Khoury Rania Saab Tarek Husseini
		5	ADAPTIVE LEARNING SYSTEMS AND PERSONALIZED EDUCATION	Prof. Dr. Imad Zayed Assoc. Prof. Dr. Layla Maroun
		6	THE INFLUENCE OF SCHOOL ENVIRONMENT ON STUDENT WELL-BEING	Faridah Alawi Assoc. Prof. Dr. Hadi Hassan
		7	MOBILE LEARNING APPLICATIONS AND STUDENT PERFORMANCE	Prof. Dr. Khaled Mansour Yasmin Farah Assoc. Prof. Dr. Samir Nasser

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		2	INNOVATIVE APPROACHES TO LANGUAGE ACQUISITION IN SECONDARY SCHOOLS	Assoc. Prof. Dr. Yasin Qureshi
		3	THE ROLE OF ART EDUCATION IN CHILDREN’S COGNITIVE DEVELOPMENT	Rania Khalil Dr. Sami Youssef Dr. Lina Hariri
		4	E-LEARNING AND DIGITAL DIVIDE IN RURAL COMMUNITIES	Assis. Prof. Dr. Ahmed Belkacem
		5	TEACHER TRAINING PROGRAMS FOR EMERGING EDUCATIONAL TECHNOLOGIES	Prof. Dr. Reynaldo Santos Lorna Cruz Assoc. Prof. Dr. Marco dela Rosa
		6	ASSESSING THE IMPACT OF SOCIAL EMOTIONAL LEARNING ON ACADEMIC PERFORMANCE	Fatima Zohra Assis. Prof. Dr. Ali Amrani
		7	PROMOTING ENVIRONMENTAL EDUCATION THROUGH SCHOOL CURRICULA	Kareem Haddad Maya Khoury Rachid Bensaid
		8	THE EFFECTS OF COOPERATIVE LEARNING ON STUDENT INTERACTION	Assis. Prof. Dr. Farid Noorzai Aisha Habibi
		9	EDUCATIONAL INNOVATIONS IN TEACHING MATHEMATICS THROUGH TECHNOLOGY	Dr. Alvaro Santiago Maria Lopez

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HALL / SALON 6	**Prof. Dr. Rania Kassem,**	1	THE EFFECT OF PROBIOTIC ENRICHMENT ON FERMENTED BEVERAGE MICROBIOLOGY	Layla Haddad, Dr. Karim Saab, Nour El-Hayek
		2	ADVANCEMENTS IN NON-THERMAL FOOD PRESERVATION TECHNOLOGIES	Assoc. Prof. Dr. Yasmine Bensalem, Omar Aloui, Hocine Charef
		3	SUSTAINABLE SOURCING AND PROCESSING OF SEAFOOD PRODUCTS FOR GLOBAL MARKETS	Mary Grace Santos, Bianca Cruz Assoc. Prof. Dr. Miguel Reyes,
		4	INFLUENCE OF PLANT EXTRACTS ON THE PHYSICOCHEMICAL PROPERTIES OF BAKERY PRODUCTS	Prof. Dr. Rania Kassem,
		5	APPLICATION OF BIOINFORMATICS IN FOOD ENGINEERING FOR FLAVOR PREDICTION	Dr. Kamran Shafiei,
		6	DEVELOPMENT OF LOW-CALORIE SWEETENERS WITH ENHANCED FUNCTIONALITY	Abdelkader Benyahia,
		7	EFFECTS OF HIGH-PRESSURE PROCESSING ON VITAMIN AND MINERAL RETENTION IN FRUITS	Prof. Dr. Naveed Shah, Asif Iqbal
		8	INNOVATIVE FERMENTATION STRATEGIES FOR THE PRODUCTION OF VEGAN CHEESE	Sohail Ahmed, Fatima Noor, Assoc. Prof. Dr. Danish Malik Elias Fares
		9	INTEGRATION OF SENSORY SCIENCE AND FOOD ENGINEERING IN CULINARY PRODUCT DESIGN	Jamal Al-Khalil, Dr. Layla Karam,
		10	APPLICATION OF BIOPOLYMERS IN EDIBLE COATING TO IMPROVE FRESH PRODUCE SHELF-LIFE	Ayesha Khan, Assoc. Prof. Dr. Iqra Malik,

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HALL / SALON 7	**Assis. Prof. Dr. Usman Ali**	1	ADVANCED NANOTECHNOLOGY APPLICATIONS IN FOOD PRESERVATION AND SAFETY	Zhandos Akhmetov
		2	SUSTAINABLE FOOD PROCESSING METHODS FOR REDUCING ENVIRONMENTAL IMPACT IN GASTRONOMY	Assoc. Prof. Dr. Rakhim Zhumabayev Dinara Kassenova Prof. Dr. Yerlan Abilkhan
		3	INNOVATIVE FERMENTATION TECHNIQUES FOR DEVELOPING FUNCTIONAL FOODS AND BEVERAGES	Aigerim Bektemir Dr. Saltanat Orazbayeva
		4	DEVELOPMENT OF PLANT-BASED PROTEINS AND THEIR INDUSTRIAL APPLICATIONS IN FOOD ENGINEERING	Sana Riaz Assoc. Prof. Dr. Muhammad Ahsan
		5	APPLICATION OF MICROENCAPSULATION TECHNOLOGY TO ENHANCE FLAVOR AND NUTRITIONAL VALUE	Bilal Khan Assis. Prof. Dr. Hira Javed Assoc. Prof. Dr. Fatima Zahra
		6	ADVANCES IN SMART KITCHEN TECHNOLOGIES FOR GASTRONOMIC INNOVATION AND FOOD SAFETY	Assis. Prof. Dr. Usman Ali
		7	EVALUATION OF TRADITIONAL AND MODERN FOOD PRESERVATION METHODS IN EXTREME CLIMATES	Dr. Rahimullah Wardak Fariba Nazari Ahmad Shah
		8	APPLICATION OF NATURAL FOOD COLORANTS AND FLAVOR ENHANCERS IN MODERN CUISINE	Layla Karam Prof. Dr. Omar Benyamina Assoc. Prof. Dr. Nabil Salim lYoussef Bensalem

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HALL / SALON 1	**Assoc. Prof. Dr. Cagla YIGITBAS**	1	EVALUATING WEBINAR-BASED NURSING EDUCATION DURING THE COVID- 19 PANDEMIC: A RETROSPECTIVE ANALYSIS	MSc, SALİHA KOÇ ASLAN
		2	IMPLEMENTING THE BALANCED SCORECARD IN NURSING SERVICES: DEVELOPMENT OF A NURSE PERFORMANCE SCORECARD FOR STRATEGIC PERFORMANCE MANAGEMENT	MSc, SALİHA KOÇ ASLAN
		3	THE MEDIATING ROLE OF PATIENT TRUST IN THE EFFECT OF PERCEIVED HEALTHCARE SERVICE QUALITY ON OUTPATIENTS' BEHAVIORAL INTENTIONS	Doç. Dr. Fuat YALMAN Prof. Dr., Yalçın KARAGÖZ
		4	THE EFFECT OF SOCIAL SUPPORT ON ORGANIZATIONAL SUPPORT AND ORGANIZATIONAL TRUST: AN APPLICATION ON HEALTHCARE WORKERS	Prof. Dr., Yalçın KARAGÖZ Doç. Dr. Fuat YALMAN
		5	The Global Outlook of Technology-Facilitated Violence Against Girls: A Bibliometric Analysis	Assoc. Prof. Dr. Çağla YİĞİTBAŞ
		6	Nuchal Translucency Measurement and Maternal Stress: A Bibliometric Analysis of Global Research Trends	Assoc. Prof. Dr. Cagla YIGITBAS

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HALL / SALON 2	**Assist. Prof. Dr. Tülay YILDIRIM ÜŞENMEZ**	1	MENOPOZ DÖNEMİNDEKİ KADINLARIN BAŞETME YÖNTEMLERİ	Ebe Meltem YAVUZ Doç. Dr. Ayça ŞOLT KIRCA Doç. Dr. Elif DAĞLI
		2	HPV VE BAĞIŞIKLAMA	Ebe Meltem YAVUZ Doç. Dr. Ayça ŞOLT KIRCA Doç. Dr. Elif DAĞLI
		3	UYKU BOZUKLUKLARINDA TAMAMLAYICI VE ALTERNATİF TIP: PALYATİF BAKIM HASTALARINA YÖNELİK YAKLAŞIMLAR	Dr. Öğr. Üyesi, NURDAN YALÇIN ATAR PhD, MURAT KOÇ
		4	THE IMPACT OF ALCOHOL AND SUBSTANCE ABUSE ON WOMEN’S HEALTH	Uzman Ebe, Nilay GELMEZ Doç. Dr. Ayça ŞOLT KIRCA Doç. Dr. Elif DAĞLI
		5	THE EFFECT OF OBESITY ON PREGNANCY	Uzman Ebe, Nilay GELMEZ Doç. Dr. Ayça ŞOLT KIRCA Doç. Dr. Elif DAĞLI
			THE EFFECT OF MİNDFULNESS ON TREATMENT ADHERENCE İN INDİVİDUALS WİTH MENTAL ILLNESS	Assist. Prof. Dr. Tülay YILDIRIM ÜŞENMEZ Res. Assist. Deniz KURTARAN
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HALL / SALON 3	**Doç. Dr. ABİDİN KEMEÇ**	1	AKILLI KENTLER VE DİJİTAL VATANDAŞLIK: YENİ KATILIM BİÇİMLERİ ÜZERİNE BİR İNCELEME	Doç. Dr. ABİDİN KEMEÇ
		2	AKILLI KENTLERDE ETİK SORUNLAR	Doç. Dr. ABİDİN KEMEÇ
		3	YEREL VE ULUSAL DÜZEYDE SİYASAL LİDERLİK TEMSİLLERİ: KATILIMCI SÖYLEMLER ÜZERİNDEN BİR LİDERLİK TİPOLOJİSİ ANALİZİ	Dr. Öğr. Üyesi, MUSTAFA DEMİRCİ
		4	INSTITUTIONAL MECHANISMS IN DISABLED INDIVIDUALS' ACCESS TO PUBLIC SERVICES: A STUDY ON THE HUMAN RIGHTS AND EQUALITY INSTITUTION OF TÜRKİYE (HREIT)	Asst. Prof., Osman KARACAN
		5	PUBLIC POLICIES OF TURKEY AND GERMANY TOWARDS SURVEILLANCE SOCIETY: A COMPARATIVE ANALYSIS	Arş. Gör. Dr. Çiğdem PANK YILDIRIM Doktorant Duygu AKYÜZ
		6	REGULATIONS ON MUNICIPALITIES IN THE PERIOD BETWEEN THE 1921-1924 CONSTITUTION	Prof. Dr. Bayram COŞKUN Doktorant Duygu AKYÜZ
		7	TÜRKİYE'DE YEREL YÖNETİMLERİN TERS GÖÇE YÖNELİK MEKÂNSAL PLANLAMA YETERLİLİĞİ ÜZERİNE BİR DEĞERLENDİRME	Dr. Öğr. Üyesi Fatma ÖKDE
		8	PUBLIC ADMINISTRATION AND THE TRANSFORMATION OF URBAN AND RURAL AREAS AFTER THE METROPOLITAN LAW: THE CASE OF RURAL NEIGHBORHOODS	Dr. Öğr. Üyesi Hatike KOÇAR UZAN
		9	FROM ELECTRONIC STATE TO SMART STATE: ANALYSIS OF DIGITAL TRANSFORMATION IN PUBLIC ADMINISTRATION	Dr. Öğr. Üyesi Ömer ÇAMUR

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		2	BLOCKCHAIN FOR MEDICAL DATA INTEGRITY AND SHARING IN CLINICAL RESEARCH	Asst. Prof.Dr, İlkay Sibel KERVANCI MD, Urology Specialist., Çagdaş ÇEKİÇ
		3	ULUSLARARASI CEZA HUKUKU BAĞLAMINDA SİLAHLI ÇATIŞMALARDA İNSANSIZ HAVA ARAÇLARININ KULLANIMI	Arş. Gör. Seda Yağmur SÜMER
		4	PROHIBITED AND HIGH-RISK ARTIFICIAL INTELLIGENCE APPLICATIONS WITHIN THE FRAMEWORK OF EUROPEAN UNION LEGISLATION	Assoc. Prof. Dr. Sinan Sami AKKURT
		5	TÜRK MEDYASINDA DEEPFAKE SÖYLEMLERİNİN ÇOK BOYUTLU ANALİZİ	Öğr.Gör.Dr. BAHAR BALCI AYDOĞAN
		6	ÖĞRETMENLERİN TÜBİTAK 4005 PROJESİNE BAŞVURU GEREKÇELERİ VE PROFİLLERİ: “KİMYA KONULARININ ÖĞRETİMİNDE YAPAY ZEKÂ UYGULAMALARI” ÖRNEĞİ	Dr. Öğr. Üyesi, SENEM ÇOLAK YAZICI

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		2	DEEP LEARNING TECHNIQUES FOR REAL-TIME TRAFFIC MANAGEMENT SYSTEMS	Prof. Dr. Talant Usubaliev Meerim Sadykova Assoc. Prof. Dr. Aibek Ergeshev
		3	OPTIMIZATION OF CLOUD COMPUTING INFRASTRUCTURES USING AI ALGORITHMS	Aziza Makhmudova Prof. Dr. Jahongir Rasulov
		4	ENHANCING CYBERSECURITY THROUGH MACHINE LEARNING-BASED INTRUSION DETECTION	Assoc. Prof. Dr. Soronai Kubatbekova Aijan Tursunova Prof. Dr. Bekzat Mamatkulov Aibek Doolmatov
		5	INTELLIGENT DATA MINING METHODS FOR FINANCIAL FRAUD DETECTION	Assoc. Prof. Dr. Ioana Popescu Mihai Radu
		6	AUTONOMOUS ROBOTIC SYSTEMS CONTROLLED BY NEURAL NETWORKS	Assoc. Prof. Dr. Olumide Adesanya
		7	AI-BASED NATURAL LANGUAGE PROCESSING FOR MULTILINGUAL TRANSLATION	Assoc. Prof. Dr. Leyla Mammadova Samir Hasanov
		8	SMART GRID MANAGEMENT USING MACHINE LEARNING AND PREDICTIVE ANALYTICS	Assoc. Prof. Dr. Ioan Marinescu lAndreea Stoica Prof. Dr. Vlad Petrescu Mihnea Popa

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DYNAMICAL BEHAVIOR OF A DISCRETE FRACTIONAL PREDATOR–PREY SYSTEM WITH INTRASPECIFIC COMPETITION

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ABSTRACT

In this study, a continuous-time population model is transformed into a discrete-time population model through fractional-order discretization. The Caputo fractional derivative approach is employed in the mathematical formulation of the model in order to more accurately capture the long-term memory effects and hereditary properties commonly observed in biological and ecological systems. A detailed stability analysis is conducted on the resulting discrete-time predator–prey model to gain a deeper understanding of ecological interactions and to determine how different parameters influence system behavior. Unlike the classical models, this model incorporates the intraspecific competition effects of predators and explores how this influence affects the dynamics of prey and predator populations. The stability analysis focuses on the identification of equilibrium points and their corresponding properties. For this model, four distinct equilibrium points are derived, and stability analysis is performed for those that are positive. Establishing stability conditions is crucial for understanding population dynamics and enhancing insights into the intricate interactions among the system’s variables. Finally, this study contributes to the literature by analyzing the dynamic behavior of a predator-prey model with intraspecific competition using fractional-order discretization.

Keywords : Fractional-order discretization, Predator–prey model, Intraspecific competition, Stability analysis.

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SIGNIFICANT RESULTS ON PSEUDOPARALLEL PARA-KENMOTSU MANIFOLDS WITH RESPECT TO THE W_1-CURVATURE TENSOR

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ABSTRACT

Para-Kenmotsu manifolds, as an important class of almost paracontact metric manifolds, have been extensively investigated in recent years due to their rich curvature structure and their close relationship with both Riemannian and pseudo-Riemannian geometries. Within this framework, the study of submanifolds plays a fundamental role in understanding how the ambient geometry influences lower-dimensional structures embedded within it.

One important notion in submanifold theory is *pseudoparallelism*, which generalizes the concepts of parallel and semiparallel submanifolds by imposing certain curvature conditions involving the second fundamental form. When the pseudoparallel condition is considered with respect to a given curvature tensor, it provides deeper insights into the geometric constraints and possible classifications of the submanifolds.

The aim of this paper is to investigate the geometry of pseudoparallel submanifolds in Para-Kenmotsu manifolds with respect to the W_1-curvature tensor. We establish necessary and sufficient conditions for such submanifolds to satisfy specific geometric properties, and we discuss the implications of these conditions in the broader context of differential geometry. The results obtained extend the current literature on paracontact metric geometry and provide new directions for the classification of special submanifolds.

Key Words: Para-Kenmotsu Manifold, Invariant Submanifold, Pseudoparallel Submanifold.

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1 INTRODUCTION

In the differential geometry of manifolds endowed with Riemannian and more general geometric structures, the theory of submanifolds emerges as a significant field of research. In this context, totally geodesic submanifolds stand out as those most compatible with the geometry of the ambient manifold. A submanifold being totally geodesic means that all of its geodesics are also geodesics of the ambient manifold. This property provides great convenience and simplicity both in the in-depth study of geometric structures and in the development of applied mathematical models.

Totally geodesic submanifolds have significant applications not only from the perspective of abstract differential geometry but also in physics, particularly in the theory of general relativity and theoretical physics. In particular, totally geodesic submanifolds on Lorentzian manifolds are employed to describe the motion of light rays and free particles in spacetime models. Moreover, in applied sciences such as control theory, optimization, and robotics, these structures play a critical role in understanding and simplifying the natural trajectories of systems.

However, the existence and characterization of totally geodesic submanifolds are closely related to the additional structures possessed by the manifold. Therefore, the study of totally geodesic submanifolds under different geometric structures yields rich results from both theoretical and applied perspectives, while also providing profound insights into the global and local properties of the manifold.

Para-Kenmotsu manifolds, as an important class of almost paracontact metric manifolds, have been extensively investigated in recent years due to their rich curvature structure and their close relationship with both Riemannian and pseudo-Riemannian geometries. Within this framework, the study of submanifolds plays a fundamental role in understanding how the ambient geometry influences lower-dimensional structures embedded within it.

One important notion in submanifold theory is pseudoparallelism, which generalizes the concepts of parallel and semiparallel submanifolds by imposing certain curvature conditions involving the second fundamental form. When the pseudoparallel condition is considered with respect to a given curvature tensor, it provides deeper insights into the geometric constraints and possible classifications of the submanifolds.

In this study, the fundamental properties, significance, and behaviors of totally geodesic submanifolds in different geometric contexts will be discussed.

The aim of this paper is to investigate the geometry of pseudoparallel submanifolds in Para-Kenmotsu manifolds with respect to the W_1-curvature tensor. We establish necessary and sufficient conditions for such submanifolds to satisfy specific geometric properties, and we discuss the implications of these conditions in the broader context of differential geometry. The results obtained extend the current literature on paracontact metric geometry and provide new directions for the classification of special submanifolds.

2 PRELIMINARIES

Let $\bar{M}$ be a $n-$dimensional differentiable manifold. If it admits a tensor field ϕ of type (1,1), a vector field ξ, a 1-form η satisfying the following conditions;

$$\phi^2 X = X - \eta(X)\xi, \tag{1}$$

$$\eta(\phi X) = 0, \phi\xi = 0, \eta(\xi) = 1, \tag{2}$$

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for all $X, Y, Z \in \chi(\bar{M})$, then the (ϕ, ξ, η) is called almost paracontact structure and $(\bar{M}, \phi, \xi, \eta)$ is called almost paracontact manifold.

If the semi-Riemannian metric g on $\bar{M}$ satisfies the following conditions

$$g(X, \xi) = \eta(X), \tag{3}$$

$$g(\phi X, \phi Y) = -g(X, Y) + \eta(X)\eta(Y), \tag{4}$$

for all $X, Y \in \chi(\bar{M})$, then the (ϕ, ξ, η, g) is called almost paracontact metric structure and $(\bar{M}, \phi, \xi, \eta, g)$ is called almost paracontact metric manifold.

A n-dimensional almost paracontact metric manifold $\bar{M}$ is called para-Kenmotsu manifold if it satisfies the following condition

$$(\bar{\nabla}_X \phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X, \tag{5}$$

where $\bar{\nabla}$ stands for the Levi-Civita connection of g.

Lemma 1 *For an n-dimensional para-Kenmotsu manifold $\bar{M}$ the following equations are provided.*

$$\bar{\nabla}_X \xi = \phi^2 X = X - \eta(X)\xi, \tag{6}$$

$$(\bar{\nabla}_X \eta)Y = g(X, Y) - \eta(X)\eta(Y), \tag{7}$$

$$\bar{R}(X, Y)\xi = \eta(X)Y - \eta(Y)X, \tag{8}$$

$$\bar{R}(\xi, X)Y = -g(X, Y)\xi + \eta(Y)X, \tag{9}$$

$$S(X, \xi) = -(n-1)\eta(X), \tag{10}$$

$$Q\xi = -(n-1)\xi, \tag{11}$$

for all $X, Y \in \chi(\bar{M})$, where $\bar{R}, Q$ and S are the Riemann curvature tensor, Ricci operator, Ricci curvature tensor of manifold $\bar{M}$, respectively.

Now, let M be an immersed submanifold of a para-Kenmotsu manifold $\bar{M}$. By $\Gamma(TM)$ and $\Gamma(T^{\perp}M)$, we denote the tangent and normal subspaces of M in $\bar{M}$. Then the **Gauss and Weingarten formulae** are, respectively, given by

$$\bar{\nabla}_X Y = \nabla_X Y + \sigma(X, Y), \tag{12}$$

and

$$\bar{\nabla}_X V = -A_V X + \nabla_X^{\perp} V, \tag{13}$$

for all $X, Y \in \Gamma(TM)$ and $V \in \Gamma(T^{\perp}M)$, where ∇ and $\nabla^{\perp}$ are the induced connections on M and $\Gamma(T^{\perp}M)$, σ and A are called **the second fundamental form** and **shape operator** of M, respectively.

They are related by

$$g(A_V X, Y) = g(\sigma(X, Y), V). \tag{14}$$

The covariant derivative of σ is defined by

$$(\bar{\nabla}_X \sigma)(Y, Z) = \nabla_X^{\perp}\sigma(Y, Z) - \sigma(\nabla_X Y, Z) - \sigma(Y, \nabla_X Z), \tag{15}$$

for all $X, Y, Z \in \Gamma(TM)$. If

$$\bar{\nabla}\sigma = 0,$$

then the submanifold M is said to be its second fundamental form is parallel.

By R, we denote the Riemannian curvature tensor of submanifold M, we have the following Gauss equation

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$$\bar{R}(X,Y)Z = R(X,Y)Z + A_{\sigma(X,Z)}Y - A_{\sigma(Y,Z)}X + (\bar{\nabla}_X\sigma)(Y,Z) - (\bar{\nabla}_Y\sigma)(X,Z). \quad (16)$$

Let M be a Riemannian manifold, T is $(0,k)$ −type tensor field and A is $(0,2)$ −type tensor field. In this case, Tachibana tensor field $Q(A,T)$ is defined as

$$Q(A,T)(X,\ldots,X_k;X,Y) = -T\big((X\wedge_A Y)X_1,\ldots,X_k\big) - \ldots - T(X_1,\ldots,X_{k-1},(X\wedge_A Y)X_k), \quad (17)$$

where,

$$(X\wedge_A Y)Z = A(Y,Z)X - A(X,Z)Y, \quad (18)$$

$k \geq 1, X_1, X_2, \ldots, X_k, X, Y \in \Gamma(TM)$.

3 PSEUDOPARALLEL PARA-KENMOTSU MANIFOLDS WITH RESPECT TO THE W_1-CURVATURE TENSOR

Next, we will discuss the types of submanifolds given in the definition for the invariant submanifold M of a para-Kenmotsu manifold $\bar{M}$.

Let M be an immersed submanifold of a para-Kenmotsu manifold $\bar{M}$. If

$$\phi(T_xM) \subseteq T_xM,$$

for each point $x \in M$, then M is said to be an **invariant submanifold**. We clearly know that almost all properties of an invariant submanifold inherit an ambient manifold as well.

In the rest of this paper, we will assume that M is an invariant submanifold of a para-Kenmotsu manifold $\bar{M}$.

Lemma 2 *Let M be an invariant submanifold of a para-Kenmotsu manifold $\bar{M}$. Then the following equalities hold on M.*

$$\sigma(\phi X,Y) = \sigma(X,\phi Y) = \phi\sigma(X,Y), \quad (19)$$

$$\sigma(X,\xi) = 0, \quad (20)$$

$$\nabla_X\xi = X - \eta(X)\xi, \quad (21)$$

$$R(X,Y)\xi = \eta(X)Y - \eta(Y)X, \quad (22)$$

$$R(\xi,X)Y = -g(X,Y)\xi + \eta(Y)X, \quad (23)$$

$$S(X,\xi) = -(n-1)\eta(X), \quad (24)$$

$$Q\xi = -(n-1)\xi, \quad (25)$$

Lemma 3 *On an n-dimensional para-Kenmotsu manifold M, the W_1-curvature tensor satisfies the following properties:*

$$W_1(X,Y)Z = R(X,Y)Z + \frac{1}{n-1}[S(Y,Z)X - S(X,Z)Y], \quad (26)$$

$$W_1(X,Y)\xi = 2[\eta(X)Y - \eta(Y)X], \quad (27)$$

$$W_1(\xi,Y)\xi = 2[Y - \eta(Y)\xi], \quad (28)$$

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$$\eta(W_1(X,Y)Z) = 2g(\eta(Y)X - \eta(X)Y, Z). \tag{29}$$

Definition 1 *Let M be the invariant submanifold of the n −dimensional para-Kenmotsu manifold $\bar{M}$. If $W_1 \cdot \sigma$ and $Q(g,\sigma)$ are linearly dependent, M is called W_1-pseudoparallel submanifold.*

In the same sense, it can be said that there is a function λ_1 on the set $M_1 = \{x \in M | \sigma(x) \neq g(x)\}$ such that

$$W_1 \cdot \sigma = \lambda_1 Q(g,\sigma).$$

If $\lambda_1 = 0$ specifically, M is called a W_1-semiparallel submanifold.

Theorem 1 *Let M be an invariant submanifold of the n −dimensional para-Kenmotsu manifold $\bar{M}$. If M is a W_1-pseudoparallel submanifold, then M is either a total geodesic or $\lambda_1 = -2$.*

Proof. Let's assume that M is a W_1-pseudoparallel submanifold. So, we can write

$$(W_1(X,Y) \cdot \sigma)(U,V) = \lambda_1 Q(g,\sigma)(U,V;X,Y),$$

for all $X, Y, U, V \in \Gamma(T\widetilde{M})$. It is clear that

$$\begin{aligned} &R^{\perp}(X,Y)\sigma(U,V) - \sigma(W_1(X,Y)U,V) \\ &-\sigma(U,W_1(X,Y)V) = -\lambda_1\{g(Y,U)\sigma(X,V) \\ &-g(X,U)\sigma(Y,V) + g(Y,V)\sigma(U,X) \\ &-g(X,V)\sigma(U,Y)\}. \end{aligned} \tag{30}$$

If we choose $X = V = \xi$ in (30) and make use of $(3), (20), (28)$, we get

$$[2 + \lambda_1]\sigma(U,Y) = 0.$$

This completes proof of the theorem.

Thus we have the following corollaries.

Corollary 1 *Let M be the invariant submanifold of the n −dimensional para-Kenmotsu manifold $\bar{M}$. Then M is a W_1-semiparallel submanifold if and only if M is a total geodesic.*

Definition 2 *Let M be the invariant submanifold of the n −dimensional para-Kenmotsu manifold $\bar{M}$. If $W_1 \cdot \sigma$ and $Q(S,\sigma)$ are linearly dependent, M is called W_1- Ricci generalized pseudoparallel submanifold.*

In this case, there is a function λ_2 on the set $M_2 = \{x \in M | \sigma(x) \neq S(x)\}$ such that

$$W_1 \cdot \sigma = \lambda_2 Q(S,\sigma).$$

If $\lambda_2 = 0$ specifically, M is called a W_1-Ricci generalized semiparallel submanifold.

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Theorem 2 *Let M be the invariant submanifold of the n –dimensional para-Kenmotsu manifold $\bar{M}$. If M is W_1-Ricci generalized pseudoparallel submanifold, then M is either a total geodesic or $\lambda_2 = \frac{2}{n-1}$.*

Proof. Let's assume that M is a generalized W_1-Ricci generalized pseudoparallel submanifold. So, we can write

$$(W_1(X,Y)\cdot\sigma)(U,V) = \lambda_2 Q(S,\sigma)(U,V;X,Y),$$

that is

$$\begin{aligned} &R^{\perp}(X,Y)\sigma(U,V) - \sigma(W_1(X,Y)U,V) \\ &-\sigma(U,W_1(X,Y)V) = -\lambda_2\{\sigma\big((X\wedge_S Y)U,V\big) \\ &+\sigma(U,(X\wedge_S Y)V)\}. \end{aligned} \tag{31}$$

for all $X,Y,U,V \in \Gamma(TM)$. If we choose $X = V = \xi$ in (31) and ın view of (3), (20), (24), (28), we get

$$[2-(n-1)\lambda_2]\sigma(U,Y) = 0,$$

This completes the proof of the theorem.

Thus we have the following corollaries.

Corollary 2 *Let M be the invariant submanifold of the n –dimensional para-Kenmotsu manifold $\bar{M}$. Then M is a W_1-Ricci generalized semiparallel submanifold if and only if M is a total geodesic.*

Definition 3 *Let M be the invariant submanifold of the n –dimensional para-Kenmotsu manifold $\bar{M}$. If $W_1\cdot\bar{\nabla}\sigma$ and $Q(g,\bar{\nabla}\sigma)$ are linearly dependent, then M is called generalized W_1-2 pseudoparallel submanifold.*

In this case, it can be said that there is a function λ_3 on the set
$M_3 = \{x \in M | \bar{\nabla}\sigma(x) \neq g(x)\}$ such that

$$W_1\cdot\bar{\nabla}\sigma = \lambda_3 Q(g,\bar{\nabla}\sigma).$$

If $\lambda_3 = 0$ specifically, M is called a generalized W_1- 2 semiparallel submanifold.

Theorem 3 *Let M be the invariant submanifold of the n –dimensional para-Kenmotsu manifold $\bar{M}$. If M is a generalized W_1- 2 pseudoparallel submanifold, then M is either a total geodesic or $\lambda_3 = -2$.*

Proof. Let's assume that M is a generalized W_1-2 pseudoparallel submanifold. So, we can write

$$(W_1(X,Y)\cdot\bar{\nabla}\sigma)(U,V,Z) = \lambda_3 Q(g,\bar{\nabla}\sigma)(U,V,Z;X,Y),$$

for all $X,Y,U,V,Z \in \Gamma(TM)$. Then, we have

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$$R^{\perp}(X,Y)(\overline{\nabla}_U\sigma)(V,Z)-\left(\overline{\nabla}_{W_1(X,Y)U}\sigma\right)(V,Z)$$
$$-(\overline{\nabla}_U\sigma)(W_1(X,Y)V,Z)-(\overline{\nabla}_U\sigma)(V,W_1(X,Y)Z)$$
$$=-\lambda_3\left\{\left(\overline{\nabla}_{(X\wedge_g Y)U}\sigma\right)(V,Z)+(\overline{\nabla}_U\sigma)\left((X\wedge_g Y)V,Z\right)\right. \tag{32}$$
$$\left.+(\overline{\nabla}_U\sigma)\left(V,(X\wedge_g Y)Z\right)\right\}.$$

If we choose $X=Z=\xi$ in (32), we can write

$$R^{\perp}(\xi,Y)(\overline{\nabla}_U\sigma)(V,\xi)-\left(\overline{\nabla}_{W_1(\xi,Y)U}\sigma\right)(V,\xi)$$
$$-(\overline{\nabla}_U\sigma)(W_1(\xi,Y)V,\xi)-(\overline{\nabla}_U\sigma)(V,W_1(\xi,Y)\xi)$$
$$=-\lambda_3\left\{\left(\nabla_{(\xi\wedge_g Y)U}\sigma\right)(V,\xi)+(\nabla_U\sigma)\left((\xi\wedge_g Y)V,\xi\right)\right. \tag{33}$$
$$\left.+(\nabla_U\sigma)\left(V,(\xi\wedge_g Y)\xi\right)\right\}.$$

Let's calculate all the expressions in (33). So, we can write

$$R^{\perp}(\xi,Y)(\overline{\nabla}_U\sigma)(V,\xi)=R^{\perp}(\xi,Y)\{\nabla_U^{\perp}\sigma(V,\xi)$$
$$-\sigma(\nabla_U V,\xi)-\sigma(V,\nabla_U\xi)\} \tag{34}$$
$$=-R^{\perp}(\xi,Y)\sigma(V,U),$$

$$\left(\overline{\nabla}_{W_1(\xi,Y)U}\sigma\right)(V,\xi)=\nabla^{\perp}_{W_1(\xi,Y)U}\sigma(V,\xi)$$
$$-\sigma\left(\nabla_{W_1(\xi,Y)U}V,\xi\right)-\sigma\left(V,\nabla_{W_1(\xi,Y)U}\xi\right)$$
$$=-\sigma(V,W_1(\xi,Y)U-\eta(W_1(\xi,Y)U)\xi) \tag{35}$$
$$=-2\eta(U)\sigma(V,Y),$$

$$(\overline{\nabla}_U\sigma)(W_1(\xi,Y)V,\xi)=\nabla_U^{\perp}\sigma(W_1(\xi,Y)V,\xi)$$
$$-\sigma(\nabla_U W_1(\xi,Y)V,\xi)-\sigma(W_1(\xi,Y)V,\nabla_U\xi) \tag{36}$$
$$=-2\eta(V)\sigma(Y,U),$$

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$$
\begin{aligned}
&(\overline{\nabla}_U\sigma)(V,W_1(\xi,Y)\xi)=(\overline{\nabla}_U\sigma)(V,2[Y-\eta(Y)\xi])\\
&=2(\overline{\nabla}_U\sigma)(V,Y)-2(\overline{\nabla}_U\sigma)(V,\eta(Y)\xi)\\
&=2(\overline{\nabla}_U\sigma)(V,Y)+2\eta(Y)\sigma(V,U),
\end{aligned}
\tag{37}
$$

$$
\begin{aligned}
&\left(\overline{\nabla}_{(\xi\wedge_g Y)U}\sigma\right)(V,\xi)=\nabla^{\perp}_{(\xi\wedge_g Y)U}\sigma(V,\xi)\\
&-\sigma\left(\nabla_{(\xi\wedge_g Y)U}V,\xi\right)-\sigma\left(V,\nabla_{(\xi\wedge_g Y)U}\xi\right)\\
&=\eta(U)\sigma(V,Y),
\end{aligned}
\tag{38}
$$

$$
\begin{aligned}
&(\overline{\nabla}_U\sigma)\left((\xi\wedge_g Y)V,\xi\right)=\nabla^{\perp}_U\sigma\left((\xi\wedge_g Y)V,\xi\right)\\
&-\sigma\left(\nabla_U(\xi\wedge_g Y)V,\xi\right)-\sigma\left((\xi\wedge_g Y)V,\nabla_U\xi\right)\\
&=-\sigma(g(Y,V)\xi-g(\xi,V)Y,U-\eta(U)\xi)\\
&=\eta(V)\sigma(Y,U),
\end{aligned}
\tag{39}
$$

$$
\begin{aligned}
&(\overline{\nabla}_U\sigma)\left(V,(\xi\wedge_g Y)\xi\right)=(\overline{\nabla}_U\sigma)(V,\eta(Y)\xi-Y)\\
&=(\overline{\nabla}_U\sigma)(V,\eta(Y)\xi)-(\overline{\nabla}_U\sigma)(V,Y)\\
&=-\eta(Y)\sigma(V,U)-(\overline{\nabla}_U\sigma)(V,Y).
\end{aligned}
\tag{40}
$$

Hence, we substitute (34)-(40) in (33), we obtain

$$
\begin{aligned}
&-R^{\perp}(\xi,Y)\sigma(V,U)+2\eta(U)\sigma(V,Y)\\
&+2\eta(V)\sigma(Y,U)-2\eta(Y)\sigma(V,U)\\
&-2(\nabla_U\sigma)(V,Y)=-\lambda_3\{\eta(V)\sigma(U,Y)\\
&+\eta(U)\sigma(Y,V)-\eta(Y)\sigma(V,U)\\
&-(\overline{\nabla}_U\sigma)(V,Y)\}.
\end{aligned}
\tag{41}
$$

If we choose $V=\xi$ in (41) and use (2), (20), then we have

$$2\sigma(Y,U)-2(\overline{\nabla}_U\sigma)(\xi,Y)=-\lambda_3[\sigma(Y,U)-(\overline{\nabla}_U\sigma)(\xi,Y)]. \tag{42}$$

On the other hand,

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$$(\overline{\nabla}_U\sigma)(\xi,Y)=-\sigma(U,Y), \tag{43}$$

and if the (43) is written in place of (42), we get

$$[2+\lambda_3]\sigma(Y,U)=0.$$

This completes proof of the theorem.

Thus we have the following corollaries.

Corollary 3 *Let M be the invariant submanifold of the n −dimensional para-Kenmotsu manifold $\bar{M}$. Then M is a W_1-2 semiparallel submanifold if and only if M is a total geodesic.*

Definition 4 *Let M be an invariant pseudoparallel submanifold of the n −dimensional para-Kenmotsu manifold $\bar{M}$. If $W_1\cdot\overline{\nabla}\sigma$ and $Q(S,\overline{\nabla}\sigma)$ are linearly dependent, $\bar{M}$ is called W_1 Ricci generalized 2-pseudoparallel submanifold.*

Then, there is a function λ_4 on the set $M_4=\{x\in M|\overline{\nabla}\sigma(x)\neq S(x)\}$ such that

$$W_1\cdot\overline{\nabla}\sigma=\lambda_4 Q(S,\overline{\nabla}\sigma).$$

If $\lambda_4=0$ specifically, M is called a W_1-Ricci generalized 2-semiparallel submanifold.

Theorem 4 *Let M be an invariant pseudoparallel submanifold of the n-dimensional para-Kenmotsu manifold $\bar{M}$. If M is a generalized W_1-Ricci generalized 2-pseudoparallel submanifold, then M is either a total geodesic or $\lambda_4=\frac{-2}{n-1}$.*

Proof. Let's assume that M is a W_1-Ricci generalized 2-pseudoparallel submanifold. So, we can write

$$(W_1(X,Y)\cdot\overline{\nabla}\sigma)(U,V,Z)=\lambda_4 Q(S,\overline{\nabla}\sigma)(U,V,Z;X,Y), \tag{44}$$

for all $X,Y,U,V,Z\in\Gamma(TM)$. If we choose $X=V=\xi$ in (44), then we can write

$$\begin{aligned}&R^{\perp}(\xi,Y)(\overline{\nabla}_U\sigma)(\xi,Z)-\left(\overline{\nabla}_{W_1(\xi,Y)U}\sigma\right)(\xi,Z)\\&-(\overline{\nabla}_U\sigma)(W_1(\xi,Y)\xi,Z)-(\overline{\nabla}_U\sigma)(\xi,W_1(\xi,Y)Z)\\&=-\lambda_4\{\left(\overline{\nabla}_{(\xi\wedge_S Y)U}\sigma\right)(\xi,Z)+(\overline{\nabla}_U\sigma)\left((\xi\wedge_S Y)\xi,Z\right)\\&+(\overline{\nabla}_U\sigma)(\xi,(\xi\wedge_S Y)Z)\}.\end{aligned} \tag{45}$$

Let's calculate all the expressions in (45). Firstly, we will calculate

$$\begin{aligned}R^{\perp}(\xi,Y)(\overline{\nabla}_U\sigma)(\xi,Z)&=R^{\perp}(\xi,Y)\{\nabla_U^{\perp}\sigma(\xi,Z)\\&-\sigma(\nabla_U Z,\xi)-\sigma(Z,\nabla_U\xi)\}\\&=-R^{\perp}(\xi,Y)\sigma(Z,U),\end{aligned} \tag{46}$$

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$$\begin{aligned}&\left(\overline{\nabla}_{W_1(\xi,Y)U}\sigma\right)(\xi,Z)=\nabla^{\perp}_{W_1(\xi,Y)U}\sigma(\xi,Z)\\&-\sigma\left(\nabla_{W_1(\xi,Y)U}\xi,Z\right)-\sigma\left(\xi,\nabla_{W_1(\xi,Y)U}Z\right)\\&=-2\eta(U)\sigma(Y,Z),\end{aligned}\tag{47}$$

$$\begin{aligned}&(\overline{\nabla}_U\sigma)(W_1(\xi,Y)\xi,Z)=(\overline{\nabla}_U\sigma)(2[Y-\eta(Y)\xi],Z)\\&=2(\overline{\nabla}_U\sigma)(Y,Z)+2\eta(Y)\sigma(U,Z),\end{aligned}\tag{48}$$

$$\begin{aligned}&(\overline{\nabla}_U\sigma)(\xi,W_1(\xi,Y)Z)=\nabla^{\perp}_U\sigma(\xi,W_1(\xi,Y)Z)\\&-\sigma(\nabla_U\xi,W_1(\xi,Y)Z)-\sigma(\xi,\nabla_UW_1(\xi,Y)Z)\\&=-2\eta(Z)\sigma(U,Y)\end{aligned}\tag{49}$$

$$\begin{aligned}&\left(\overline{\nabla}_{(\xi\wedge_SY)U}\sigma\right)(\xi,Z)=\nabla^{\perp}_{(\xi\wedge_SY)U}\sigma(\xi,Z)\\&-\sigma\left(\nabla_{(\xi\wedge_SY)U}\xi,Z\right)-\sigma\left(\xi,\nabla_{(\xi\wedge_SY)U}Z\right)\\&=-(n-1)\eta(U)\sigma(Y,Z),\end{aligned}\tag{50}$$

$$\begin{aligned}&(\overline{\nabla}_U\sigma)\left((\xi\wedge_SY)\xi,Z\right)=(\overline{\nabla}_U\sigma)(S(Y,\xi)\xi-S(\xi,\xi)Y,Z)\\&=(\overline{\nabla}_U\sigma)(-(n-1)\eta(Y)\xi+(n-1)Y,Z)\\&=(n-1)(\overline{\nabla}_U\sigma)(Y,Z)+(n-1)\eta(Y)\sigma(U,Z),\end{aligned}\tag{51}$$

$$\begin{aligned}&(\overline{\nabla}_U\sigma)(\xi,(\xi\wedge_SY)Z)=(\overline{\nabla}_U\sigma)(\xi,S(Y,Z)\xi-S(\xi,Z)Y)\\&=(\overline{\nabla}_U\sigma)(\xi,S(Y,Z)\xi)-(\overline{\nabla}_U\sigma)(\xi,S(\xi,Z)Y)\\&=-(n-1)\eta(Z)\sigma(U,Y).\end{aligned}\tag{52}$$

If we substitute (46) – (52) in (45), we obtain

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$$
\begin{aligned}
&-R^{\perp}(\xi,Y)\sigma(Z,U)+2\eta(U)\sigma(Y,Z)\\
&-2\eta(Y)\sigma(U,Z)+2\eta(Z)\sigma(U,Y)\\
&-2(\overline{\nabla}_U\sigma)(Y,Z)=-\lambda_4\{-(n-1)\eta(U)\sigma(Y,Z)\\
&+(n-1)\eta(Y)\sigma(U,Z)-(n-1)\eta(Z)\sigma(U,Y)\\
&+(n-1)(\overline{\nabla}_U\sigma)(Y,Z)\}.
\end{aligned}
\tag{53}
$$

If we choose $Z=\xi$ in (53) and use $(2),(20)$, we get

$$-2(\overline{\nabla}_U\sigma)(Y,\xi)+2\sigma(U,Y)=-\lambda_4\big[(n-1)\big((\overline{\nabla}_U\sigma)(Y,\xi)+\sigma(U,Y)\big)\big] \tag{54}$$

On the other hand,

$$(\overline{\nabla}_U\sigma)(\xi,Y)=-\sigma(U,Y), \tag{55}$$

and if the (55) is written in place of (54), we have

$$[2+(n-1)\lambda_4]\sigma(U,Y)=0.$$

This completes the proof of the theorem.

Thus we have the following corollaries.

Corollary 4 *Let M be the invariant submanifold of the n –dimensional para-Kenmotsu manifold $\overline{M}$. Then M is a W_1-Ricci generalized 2-semiparallel submanifold if and only if M is a total geodesic.*

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GEOMETRİC PROPERTIES OF PARA-KENMOTSU MANIFOLDS ADMITTING η-RICCI-BOURGUIGNON SOLITONS

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ABSTRACT

Para-Kenmotsu manifolds, as a significant subclass of almost paracontact metric manifolds, have been extensively studied due to their rich curvature properties and deep connections with various geometric structures. The investigation of special curvature tensors, such as the Riemann curvature tensor, the Ricci tensor, and generalized curvature tensors like W_1 and W_2, plays an essential role in understanding the intrinsic and extrinsic geometry of these manifolds.

On the other hand, the concept of Ricci solitons, originally introduced in the context of the Ricci flow, has become an important tool in the study of geometric evolution equations. Among their generalizations, the , η-Ricci Bourguignon soliton has attracted considerable attention, as it unifies and extends several well-known soliton structures by incorporating an additional term depending on the metric and the potential vector field. This generalized soliton structure provides a natural framework for exploring the interplay between curvature and geometric flows on various manifolds.

In the setting of Para-Kenmotsu manifolds, η-Ricci Bourguignon solitons offer a fertile ground for investigating how curvature tensors interact under the influence of the soliton equation. In particular, analyzing the relations between the Riemann, Ricci, W_1 and W_2 curvature tensors in the presence of an , η-Ricci Bourguignon soliton leads to a deeper understanding of the manifold's curvature behavior and symmetry properties.

The aim of this paper is to examine these interactions and to establish necessary and sufficient conditions under which specific geometric properties arise. The obtained results contribute to the growing body of literature on both soliton theory and paracontact geometry, providing new perspectives for further research in differential geometry and geometric analysis

Key Words: Para-Kenmotsu Manifolds, Ricci Soliton, η-Ricci-Bourguignon Soliton.

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1 Introduction

Different types of contact and paracontact geometries have attracted attention as an important research topic in the field of differential geometry. These structures allow the investigation of various geometric properties through certain tensor fields, metric structures, and connections defined on a manifold. In this context, Kenmotsu manifolds were first introduced in 1972 by Kenji Kenmotsu as an alternative contact metric structure to Sasakian manifolds.

Paracontact geometry, which developed in parallel, is related to paracomplex structures and can be regarded as an analogue of classical contact geometry. Studies on paracontact manifolds have gradually paved the way for the investigation of Sasakian and Kenmotsu structures. In this framework, para-Kenmotsu manifolds emerged as an adaptation of the classical Kenmotsu manifold structure to paracontact geometry. Para-Kenmotsu manifolds constitute a special class of manifolds defined by certain covariant derivative conditions and possessing distinctive curvature and symmetry properties.

The interest in para-Kenmotsu manifolds has not been limited to their defining properties, but has been enriched over time by quarter-symmetric and semi-symmetric conditions, metric connections, and curvature properties. In recent years, by combining them with Lorentzian metrics, studies have also been carried out on Lorentzian para-Kenmotsu manifolds, making these structures noteworthy from the perspective of theoretical physics. In particular, in the context of general relativity and cosmology, the applicability of these manifolds, which possess timelike vector fields, to certain physical models is being investigated.

2 PRELIMINARIES

Let M^n be a $n-$dimensional differentiable manifold. If it admits a tensor field ϕ of type (1,1), a vector field ξ, a 1-form η satisfying the following conditions;

$$\phi^2X = X - \eta(X)\xi, \tag{1}$$

$$\eta(\phi X) = 0, \phi\xi = 0, \eta(\xi) = 1, \tag{2}$$

for all $X, Y \in \chi(M)$, then the (ϕ, ξ, η) is called almost paracontact structure and (M^n, ϕ, ξ, η) is also called almost paracontact manifold. If the semi-Riemannian metric g on an almost paracontact manifold M^n satisfies the following conditions

$$g(X, \xi) = \eta(X), \tag{3}$$

$$g(\phi X, \phi Y) = -g(X, Y) + \eta(X)\eta(Y), \tag{4}$$

for all $X, Y \in \chi(M)$, then the (ϕ, ξ, η, g) is called almost paracontact metric structure and $(M^n, \phi, \xi, \eta, g)$ is called almost paracontact metric manifold.

A n-dimensional almost paracontact metric manifold M^n is called para-Kenmotsu manifold if the covariant derivative of ϕ satisfies the following condition

$$(\nabla_X\phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X, \tag{5}$$

where ∇ stands for the Levi-Civita connection of g.

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Lemma 1 *For an n-dimensional para-Kenmotsu manifold M^n the following equations are provided.*

$$\nabla_X \xi = X - \eta(X)\xi, \tag{6}$$

$$(\nabla_X \eta)Y = g(X,Y) - \eta(X)\eta(Y), \tag{7}$$

$$\eta(R(X,Y)Z) = g(\eta(Y)X - \eta(X)Y, Z), \tag{8}$$

$$R(X,Y)\xi = \eta(X)Y - \eta(Y)X, \tag{9}$$

$$R(\xi,X)Y = -g(X,Y)\xi + \eta(Y)X, \tag{10}$$

$$R(X,\xi)Y = g(X,Y)\xi - \eta(Y)X, \tag{11}$$

$$S(X,\xi) = -(n-1)\eta(X), \tag{12}$$

$$Q\xi = -(n-1)\xi, \tag{13}$$

for all $X, Y, Z \in \chi(M)$, where R, Q and S are the Riemann curvature tensor, Ricci operator, Ricci curvature tensor of manifold M^n, respectively.

Let M be a Riemannian manifold, T is $(0,k)-$type tensor field and A is $(0,2)-$type tensor field. In this case, Tachibana tensor field $Q(A,T)$ is defined as

$$Q(A,T)(X,\ldots,X_k;X,Y) = -T\big((X \wedge_A Y)X_1,\ldots,X_k\big) -$$

$$\ldots - T(X_1,\ldots,X_{k-1},(X \wedge_A Y)X_k),$$

where,

$$(X \wedge_A Y)Z = A(Y,Z)X - A(X,Z)Y,$$

$k \geq 1, X_1, X_2, \ldots, X_k, X, Y \in \Gamma(TM)$.

Within the framework of Riemannian and semi-Riemannian geometry, the interaction between differential equations and geometric structures constitutes one of the most significant research areas of modern geometry. In this context, the Ricci flow and its special solutions, namely Ricci solitons, stand out as powerful tools for understanding the geometric and topological properties of manifolds. Ricci solitons are natural generalizations of Einstein metrics and represent solutions to geometric flows.

The Bourguignon metrics, introduced by Bourguignon, were developed as a generalization of the classical concept of Ricci solitons. Subsequently, the Ricci–Bourguignon flow provided the opportunity to study the evolution of metrics on a manifold within a more flexible family of parameters. In this framework, the concept of an η–Ricci–Bourguignon soliton emerged as a more general type of soliton, obtained by considering a Ricci–Bourguignon soliton together with an 1-form η on the manifold. This structure has led to

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significant geometric results, particularly in contact, Kenmotsu, Sasakian manifolds, and their Lorentzian counterparts.

The significance of η–Ricci–Bourguignon solitons arises not only from their theoretical interest within differential geometry, but also from their applications in mathematical physics and general relativity. In particular, η–Ricci–Bourguignon solitons have potential applications in the modeling of Einstein-type field equations, in the stability analysis of geometric flows, in the modeling of expansion or contraction behaviors of space-time, and in thermodynamic formalisms.

In this study, the historical development, fundamental definitions, and various applications of η–Ricci–Bourguignon solitons are examined, and the geometric significance of this structure as well as its potential research directions are discussed. Furthermore, by investigating the existence conditions and characteristic properties of these solitons under certain classes, this work aims to contribute to the existing literature.

Precisely, a Ricci Bourguignon soliton on a (semi) Riemannian manifold (M, g) is defined as a triple (g, V, λ) on M satisfying

$$L_V g + 2S + 2(\lambda - \rho r)g = 0,$$

where L_V is the Lie derivative operator along the vector field V and λ is a real constant. On the other hand, generalization is the notion of η –Ricci Bourguignon soliton defined by Aubin as a quadruple (g, V, λ, μ) satisfying

$$L_V g + 2S + 2(\lambda - \rho r)g + 2\mu\eta \otimes \eta = 0,$$

where λ and μ are real constants and η is the dual of ξ and S denotes the Ricci tensor of M.

In particular, the characterizations of η–Ricci–Bourguignon solitons are classified according to the sign of λ. If $\lambda > 0$, the manifold is called expanding; if $\lambda = 0$, the manifold is called steady; and if $\lambda < 0$, the manifold is called shrinking.

3 ALMOST η –RICCI-BOURGUIGNON SOLITONS ON PARA-KENMOTSU MANIFOLD

Now let (g, ξ, λ, μ) be almost η –Ricci Bourguignon soliton on n-dimensional para-Kenmotsu manifold M^n. Hence, by definition, we have

$$\begin{aligned}&\left(L_\xi g\right)(X, Y) + 2S(X, Y) + 2(\lambda - \rho r)g(X, Y)\\ &+2\mu\eta(X)\eta(Y) = 0.\end{aligned} \tag{14}$$

Now, let us compute the Lie derivative $L_\xi g$ along the vector field ξ. Then we have

$$\begin{aligned}\left(L_\xi g\right)(X, Y) &= L_\xi g(X, Y) - g\left(L_\xi X, Y\right) - g\left(X, L_\xi Y\right)\\ &= \xi[g(X, Y)] - g([\xi, X], Y) - g(X, [\xi, Y])\\ &= g(\nabla_X \xi, Y) + g(X, \nabla_Y \xi),\end{aligned}$$

for all $X, Y \in \Gamma(TM)$. By using (6), we have

$$\left(L_\xi g\right)(X, Y) = 2[g(X, Y) - \eta(X)\eta(Y)]. \tag{15}$$

Thus, in a para-Kenmotsu manifolds, from (14) and (15), we get

$$S(X, Y) = [\rho r - (\lambda + 1)]g(X, Y) + [1 - \mu]\eta(X)\eta(Y). \tag{16}$$

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Thus we can state the following theorem.

Theorem 1 *An n-dimensional para-Kenmotsu manifold admitting almost η-Ricci Bourguignon soliton is an η-Einstein manifold provided $\rho r \neq \lambda + 1$ and $\mu \neq 1$.*

A corollary of this theorem can be stated as follows.

Corollary 1 *An n-dimensional para-Kenmotsu manifold admitting almost η-Ricci Bourguignon soliton is an Einstein manifold provided $\rho r \neq \lambda + 1$ and $\mu = 1$.*

For $Y = \xi$ in (16), this implies that

$$S(\xi, X) = [\rho r - (\lambda + \mu)]\eta(X). \tag{17}$$

Taking into account of (12) and (17), we obtain the following important result.

Corollary 2 *On an n-dimensional para-Kenmotsu manifold admitting a η-Ricci Bourguignon soliton, the relationship between λ and μ can be expressed as*

$$\lambda + \mu = \rho r + (n - 1). \tag{18}$$

Definition 1 *Let M^n be an n-dimensional para-Kenmotsu manifold. If $R \cdot S$ and $Q(g, S)$ are linearly dependent, then the M is said to be* ***Ricci pseudosymmetric****.*

In this case, there exists a function L_R on M^n such that

$$R \cdot S = L_R Q(g, S).$$

In particular, if $L_R = 0$, the manifold M^n is said to be **Ricci semisymmetric**.

Let us now investigate the Ricci pseudosymmetric case of the n-dimensional para-Kenmotsu manifolds.

Theorem 2 *Let M^n be para-Kenmotsu manifold and (g, ξ, λ, μ) be almost η –Ricci Bourguignon soliton on M^n. If M^n is a Ricci pseudosymmetric, then at least one of the following holds:*

i. $L_R = -1$,
ii. $\lambda = \rho r + (n - 2)$ and $\mu = 1$,
iii. M^n is an Einstein manifold,
iv. M^n is an expanding if $\rho r > 2 - n$,
v. M^n is a steady if $\rho r = 2 - n$,
vi. M^n is a shrinking if $\rho r < 2 - n$,
vii. The η–Ricci Bourguignon soliton reduces to a Ricci Bourguignon soliton.

Proof. Let's assume that para-Kenmotsu manifold M^n be Ricci pseudosymmetric and (g, ξ, λ, μ) be almost η –Ricci Bourguignon soliton on para-Kenmotsu manifold M^n. That's mean

$$(R(X, Y) \cdot S)(U, V) = L_R Q(g, S)(U, V; X, Y),$$

for all $X, Y, U, V \in \Gamma(TM)$. From the last equation, we can easily write

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$$S(R(X,Y)U,V)+S(U,R(X,Y)V)$$
$$=L_R\left\{S\left(\left(X\wedge_g Y\right)U,V\right)+S\left(U,\left(X\wedge_g Y\right)V\right)\right\}. \tag{19}$$

If we choose $V=\xi$ in (19), we get

$$S(R(X,Y)U,\xi)+S(U,R(X,Y)\xi)$$
$$=L_R\{S(g(Y,U)X-g(X,U)Y,\xi)$$
$$+S(U,\eta(Y)X-\eta(X)Y)\}.$$

If we make use of $(3),(8),(9)$ and (12) in last equation, we have

$$-(n-1)g(\eta(Y)X-\eta(X)Y,U)+S(U,\eta\eta(X)Y-(Y)X)$$
$$=L_R\{-(n-1)g(\eta(X)Y-\eta(Y)X,U) \tag{20}$$
$$+S(U,\eta(Y)X-\eta(X)Y)\}.$$

If we use (16) in (20), we get

$$[(n-1)+\rho r-(\lambda+1)](1+L_R)g(\eta(X)Y-\eta(Y)X,U)=0. \tag{21}$$

It is clear from (21) that the conditions of the theorem are satisfied. This completes the proof.

We can give the results obtained from this theorem as follows.

Corollary 3 *Let M^n be para-Kenmotsu manifold and (g,ξ,λ,μ) be almost η –Ricci Bourguignon soliton on M^n. If M^n is a Ricci semisymmetric, then the following holds:*

i. M^n is an Einstein manifold,
ii. $\lambda=\rho r+(n-2)$ and $\mu=1$,
iii. M^n is an expanding if $\rho r>2-n$,
iv. M^n is a steady if $\rho r=2-n$,
v. M^n is a shrinking if $\rho r<2-n$.

Lemma 2 *On an n-dimensional para-Kenmotsu manifold M^n, the W_1-curvature tensor satisfies the following properties:*

$$W_1(X,Y)Z=R(X,Y)Z+\frac{1}{n-1}[S(Y,Z)X-S(X,Z)Y], \tag{22}$$

$$W_1(X,Y)\xi=2[\eta(X)Y-\eta(Y)X], \tag{23}$$

$$\eta(W_1(X,Y)Z)=2g(\eta(Y)X-\eta(X)Y,Z). \tag{24}$$

Definition 2 *Let M^n be an n-dimensional para-Kenmotsu manifold. If $W_1\cdot S$ and $Q(g,S)$ are linearly dependent, then the manifold is said to be W_1-**Ricci pseudosymmetric.***

In this case, there exists a function L_{W_1} on M^n such that

$$W_1\cdot S=L_{W_1}Q(g,S).$$

In particular, if $L_{W_1}=0$, the manifold M^n is said to be W_1 –**Ricci semisymmetric.**

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Let us now investigate the W_1-Ricci pseudosymmetric case of the para-Kenmotsu manifold.

Theorem 3 *Let M^n be an n-dimensional para-Kenmotsu manifold and (g,ξ,λ,μ) be almost η −Ricci Bourguignon soliton on M^n. If M^n is a W_1-Ricci pseudosymmetric, then at least one of the following holds:*

i. $L_{W_1} = -2$,
ii. $\lambda = \rho r + (n-2)$ and $\mu = 1$,
iii. M^n is an Einstein manifold,
iv. M^n is an expanding if $\rho r > 2-n$,
v. M^n is a steady if $\rho r = 2-n$,
vi. M^n is a shrinking if $\rho r < 2-n$,
vii. The η–Ricci Bourguignon soliton reduces to a Ricci Bourguignon soliton.

Proof. Let's assume that para-Kenmotsu manifold M^n be W_1-Ricci pseudosymmetric and (g,ξ,λ,μ) be almost η −Ricci Bourguignon soliton on para-Kenmotsu manifold M^n. That's mean

$$(W_1(X,Y)\cdot S)(U,V) = L_{W_1} Q(g,S)(U,V;X,Y),$$

for all $X,Y,U,V \in \Gamma(TM)$. From the last equation, we can easily write

$$S(W_1(X,Y)U,V) + S(U,W_1(X,Y)V)$$
$$= L_{W_1}\left\{S\left((X\wedge_g Y)U,V\right) + S\left(U,(X\wedge_g Y)V\right)\right\}. \tag{25}$$

If we choose $V=\xi$ in (25), we get

$$S(W_1(X,Y)U,\xi) + S(U,W_1(X,Y)\xi)$$
$$= L_{W_1}\{S(g(Y,U)X - g(X,U)Y,\xi)$$
$$+S(U,\eta(Y)X-\eta(X)Y)\}.$$

If we make use of (3), (12), (23) and (24) in the last equation, we have

$$-2(n-1)g(\eta(Y)X-\eta(X)Y,U)$$
$$+2Sg(\eta(X)Y-\eta(Y)X,U)$$
$$= L_{W_1}\{-(n-1)g(\eta(X)Y-\eta(Y)X,U) \tag{26}$$
$$+S(U,\eta(Y)X-\eta(X)Y)\} = 0.$$

If we use (16) in the (26), we get

$$[(n-1)+\rho r-(\lambda+1)]\left(2+L_{W_1}\right)g(\eta(X)Y-\eta(Y)X,U) = 0. \tag{27}$$

It is clear from (27) that the conditions of the theorem are satisfied. This completes the proof.This completes the proof.

We can give the result obtained from this theorem as follow.

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Corollary 4 *Let M^n be n-dimensional para-Kenmotsu manifold and (g,ξ,λ,μ) be almost η –Ricci Bourguignon soliton on M^n. If M^n is a W_1-Ricci semisymmetric, then the following holds:*

i. M^n is an Einstein manifold,
ii. $\lambda = \rho r + (n-2)$ and $\mu = 1$,
iii. M^n is an expanding if $\rho r > 2-n$,
iv. M^n is a steady if $\rho r = 2-n$,
v. M^n is a shrinking if $\rho r < 2-n$.

Lemma 3 *On an n-dimensional para-Kenmotsu manifold M^n, the W_2-curvature tensor satisfies the following properties:*

$$W_2(X,Y)Z = R(X,Y)Z - \frac{1}{n-1}[g(Y,Z)QX - g(X,Z)QY], \tag{28}$$

$$W_2(X,Y)\xi = [\eta(X)Y - \eta(Y)X] - \frac{1}{n-1}[\eta(Y)QX - \eta(X)QY], \tag{29}$$

$$\eta(W_2(X,Y)Z) = g(\eta(Y)X - \eta(X)Y, Z) + \frac{1}{n-1}S(\eta(Y)X - \eta(X)Y, Z). \tag{30}$$

Definition 3 *Let M^n be an n-dimensional para-Kenmotsu manifold. If $W_2 \cdot S$ and $Q(g,S)$ are linearly dependent, then the manifold is said to be W_2-**Ricci pseudosymmetric.***

In this case, there exists a function L_{W_2} on M^n such that

$$W_2 \cdot S = L_{W_2} Q(g,S).$$

In particular, if $L_{W_2} = 0$, the manifold M^n is said to be W_2 –**Ricci semisymmetric.**

Let us now investigate the W_2-Ricci pseudosymmetric case of the para-Kenmotsu manifold.

Theorem 4 *Let M^n be an n-dimensional para-Kenmotsu manifold and (g,ξ,λ,μ) be almost η –Ricci Bourguignon soliton on M^n. If M^n is a W_2-Ricci pseudosymmetric, then at least one of the following holds:*

i. $L_{W_1} = \frac{(\lambda+1)-(n-1)-\rho r}{(n-1)}$,
ii. $\lambda = \rho r + (n-2)$ and $\mu = 1$,
iii. M^n is an Einstein manifold,
iv. M^n is an expanding if $\rho r > 2-n$,
v. M^n is a steady if $\rho r = 2-n$,
vi. M^n is a shrinking if $\rho r < 2-n$,
vii. The η–Ricci Bourguignon soliton reduces to a Ricci Bourguignon soliton.

Proof. Let's assume that para-Kenmotsu manifold M^n be W_2-Ricci pseudosymmetric and (g,ξ,λ,μ) be almost η –Ricci Bourguignon soliton on para-Kenmotsu manifold M^n. That's mean

$$(W_2(X,Y) \cdot S)(U,V) = L_{W_2} Q(g,S)(U,V;X,Y),$$

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for all $X, Y, U, V \in \Gamma(TM)$. From the last equation, we can easily write

$$S(W_2(X,Y)U,V) + S(U, W_2(X,Y)V)$$
$$= L_{W_2}\left\{S\left((X \wedge_g Y)U, V\right) + S\left(U, (X \wedge_g Y)V\right)\right\}. \tag{31}$$

If we choose $V = \xi$ in (31), we get

$$S(W_2(X,Y)U,\xi) + S(U, W_2(X,Y)\xi)$$
$$= L_{W_2}\{S(g(Y,U)X - g(X,U)Y, \xi)$$
$$+S(U, \eta(Y)X - \eta(X)Y)\}.$$

If we make use of (3), (12), (29) and (30) in the last equation, we have

$$\begin{aligned} &-(n-1)g(\eta(Y)X - \eta(X)Y, U) \\ &-2Sg(\eta(Y)X - \eta(X)Y, U) \\ &-\tfrac{1}{n-1}S(U, \eta(Y)QX - \eta(X)QY) \\ &= L_{W_2}\{-(n-1)g(\eta(X)Y - \eta(Y)X, U) \\ &+S(U, \eta(Y)X - \eta(X)Y)\} = 0. \end{aligned} \tag{32}$$

If we use (16) in the (32), we get

$$\begin{aligned} &-(n-1)g(\eta(Y)X - \eta(X)Y, U) \\ &-2[\rho r - (\lambda+1)]g(\eta(Y)X - \eta(X)Y, U) \\ &-\tfrac{1}{n-1}[\rho r - (\lambda+1)]S(\eta(Y)X - \eta(X)Y, U) \\ &= L_{W_2}\{-(n-1)g(\eta(X)Y - \eta(Y)X, U) \\ &+[\rho r - (\lambda+1)]g(\eta(Y)X - \eta(X)Y, U)\}. \end{aligned} \tag{34}$$

If we again use (16) in (34), we have

$$[(n-1) + \rho r - (\lambda+1)]\left[\left(1 + L_{W_2}\right) + \frac{\rho r - (\lambda+1)}{(n-1)}\right] g(\eta(X)Y - \eta(Y)X, U) = 0. \tag{35}$$

It is clear from (35) that the conditions of the theorem are satisfied. This completes the proof.

We can give the result obtained from this theorem as follow.

Corollary 5 *Let M^n be n-dimensional para-Kenmotsu manifold and (g, ξ, λ, μ) be almost η −Ricci Bourguignon soliton on M^n. If M^n is a W_2-Ricci semisymmetric, then the following holds:*

i. M^n is an Einstein manifold,
ii. $\lambda = \rho r + (n-2)$ and $\mu = 1$,
iii. M^n is an expanding if $\rho r > 2 - n$,
iv. M^n is a steady if $\rho r = 2 - n$,
v. M^n is a shrinking if $\rho r < 2 - n$.

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vi. The η–Ricci Bourguignon soliton reduces to a Ricci Bourguignon soliton.

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TOPSIS METHOD BASED ON HYBRID AHP-ENTROPY WEIGHTING IN SPHERICAL FUZZY SETS AND ITS APPLICATION

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ABSTRACT

In this study, it is aimed to enhance the widely used TOPSIS method in decision-making processes by integrating a hybrid AHP-entropy based weighting approach. For this purpose, the spherical fuzzy set theory, which is effective in situations characterized by high uncertainty, will be utilized. In the hybrid weighting approach, the objective weights of the criteria are determined using the entropy method, while subjective weighting based on the decision-makers' preferences is conducted via the AHP method. This approach allows for more accurate and effective incorporation of criteria weights into the solution of decision-making problems. Consequently, the proposed hybrid AHP-entropy based TOPSIS method aims to achieve more reliable and robust results in decision-making processes. Subsequently, the newly developed method will be applied to the performance evaluation of energy storage systems. Furthermore, the results of this problem will be compared with those obtained by some existing methods in the literature to demonstrate the efficiency, accuracy, and reliability of the proposed method.

Keywords: AHP, Entropy, TOPSIS, hybrid weighting

1. INTRODUCTION

Fuzzy set theory, first introduced by Zadeh [1], marked a fundamental shift in the way uncertainty and imprecision are modeled in decision-making processes. Unlike classical sets, where elements have binary membership (either 0 or 1), fuzzy sets allow each element to possess a degree of membership ranging between 0 and 1. This flexibility has enabled the modeling of vague and ambiguous information across numerous fields. To address limitations in fuzzy sets where only membership degrees are considered, intuitionistic fuzzy sets (IFSs) were introduced by Atanassov [2]. IFSs incorporate both membership and non-membership degrees for each element, with the condition that their sum does not exceed one, thereby allowing for a hesitancy degree that accounts for indeterminacy. Expanding further, picture fuzzy sets (PFSs)-introduced by Cuong [3]-consider three degrees for each element:

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membership, non-membership, and neutrality (abstention). This extension is particularly useful in scenarios such as voting systems, where individuals may abstain rather than expressing agreement or disagreement. However, PFSs are still limited in cases where the sum of the three degrees exceeds one. To overcome these constraints, spherical fuzzy sets (SFSs) were developed by Kahraman and Kutlu Gündoğdu [4]. In SFSs, the sum of the squares of the membership, non-membership, and neutral-membership degrees must be less than or equal to one. This condition provides greater modeling flexibility and better reflects human reasoning in complex and uncertain environments. Due to its capability to handle higher degrees of uncertainty, the SFS theory has been applied extensively in multi-criteria decision-making (MCDM) problems. To address the quantification of uncertainty in such environments, entropy measures have been proposed.

Entropy, as introduced by Shannon [5], measures the degree of uncertainty or the expected amount of information in a system. In the fuzzy set context, De Luca and Termşnş [6] defined fuzzy entropy through an axiomatic approach. Hung and Yang [7] extended this concept to IFSs, while Thaoa and Smarandache [8] developed entropy for PFSs. For SFSs, Aydoğdu and Gül [9] proposed a novel entropy measure and demonstrated its compliance with required theoretical properties. They also applied this measure to solve MCGDM problems effectively. An alternative and widely adopted approach for deriving the weights of criteria is the Analytic Hierarchy Process (AHP), first proposed by Saaty [10]. This method allows decision-makers to decompose complex decision problems into a hierarchical structure and to conduct pairwise comparisons of criteria using a standardized scale, commonly ranging from 1 to 9, in order to capture relative significance consistently. After the emergence of fuzzy set theory, traditional AHP was extended to various fuzzy environments to better address uncertainty in human judgments Van and Pedrycz [11] were the first to integrate fuzzy numbers into AHP by using triangular membership functions. Buckley [12] advanced this by applying geometric mean operators with trapezoidal fuzzy numbers Chang [13] further refined the fuzzy AHP method by introducing the extent analysis method using triangular fuzzy numbers. Over the years, numerous studies have applied fuzzy AHP in fields such as risk assessment ([14], [15]), supply chain management ([16], [17]), and technology adoption ([18]). With the development of SFS theory, AHP has also been extended to the spherical fuzzy environment [19]. These extensions allow decision-makers to account for hesitancy and uncertainty more effectively in pairwise comparisons. For instance, Tian et al. [20] applied spherical fuzzy AHP to assess tourism attraction alternatives, while more recent studies integrated spherical fuzzy AHP with other MCDM methods to solve complex decision problems such as supplier selection, transportation, and healthcare logistics ([21], [22]).

Among the wide range of MCDM/MCGDM techniques applied in fuzzy environments, the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) has emerged as one of the most prominent and frequently employed approaches. The fundamental idea of TOPSIS is intuitive: the optimal alternative is the one that exhibits the shortest distance from the positive ideal solution while simultaneously being farthest from the negative ideal solution. This dual consideration allows decision-makers to account for both the most desirable and the least desirable outcomes within a unified framework. The popularity of TOPSIS can largely be

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attributed to its methodological simplicity, its ability to provide a complete ranking of alternatives from best to worst, and its computational efficiency, which makes it highly practical for real-world applications. The adaptation of TOPSIS to fuzzy environments was pioneered by Chen [23], who extended the classical approach to incorporate linguistic evaluations expressed through fuzzy sets. Building on this advancement, Tsaur et al. [24] proposed a hybrid framework by integrating the AHP with fuzzy TOPSIS, and successfully applied it to the evaluation of airline service quality. Subsequent comprehensive reviews by Mardani et al. [25] and Pal czewskia and Salabun [26] highlighted the versatility and effectiveness of TOPSIS across a wide spectrum of applications, including those based on different extensions of fuzzy sets. Within the more recent context of SFS, Kutlu Gündoğdu and Kahraman [27] introduced the spherical fuzzy adaptation of the TOPSIS method and demonstrated its applicability through a hospital location selection problem, where the weights of criteria were determined subjectively. Recognizing the limitations of subjective weighting, Barukab et al. [28] advanced this line of research by developing an enhanced spherical fuzzy TOPSIS model. This improved version allowed for the objective determination of both decision-maker weights and criteria weights, and its effectiveness was validated through a case study involving robot selection. In parallel, Naeem et al. [29] proposed a spherical fuzzy multi-criteria group decision-making (MCGDM) model, which combined entropy-based weighting with aggregation operators. This model was applied to a green supplier selection problem involving two-dimensional fuzzy information, showcasing its capacity to address sustainability-related decision contexts. Collectively, the studies ([23], [30]-[32]) provide strong evidence of the adaptability and robustness of TOPSIS-based models in handling decision problems under spherical fuzzy environments. A notable advantage of many of these extensions is their ability to objectively compute both decision-maker and criteria weights, thereby reducing subjectivity, enhancing transparency, and improving the overall robustness of the decision-making process under uncertainty.

The primary novelty of this study lies in the development of a hybrid AHP-Entropy based TOPSIS method within SFS framework for solving complex MCDM problems. Although both the AHP and entropy methods have individually been used to calculate subjective and objective weights in various decision-making applications, their integration under the spherical fuzzy environment remains a significantly underexplored area in the literature. The hybrid weighting approach proposed in this study provides a more comprehensive weighting mechanism by combining the subjective judgments of decision-makers (via AHP) with objective data dispersion (via entropy). This dual-perspective ensures that the decision-making process accounts for both expert opinions and data-driven insights, thus increasing robustness, transparency, and reliability. Furthermore, while previous studies have implemented AHP or entropy separately under SFS conditions ([33]), this research is among the first to simultaneously apply both AHP and entropy in a hybrid structure under the spherical fuzzy environment. Also, this hybrid weight technique is integrated into the TOPSIS method, and we apply the method to the real-world problem of energy storage system performance evaluation, a critical area in sustainable energy planning. In addition, by comparing the proposed method's results with another established method in the literature, the study highlights the superiority of

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the hybrid approach in terms of accuracy, consistency, and decision quality. These comparisons not only validate the method's effectiveness but also position it as a more reliable tool for MCDM problems characterized by high uncertainty.

2. PRELIMINARIES

In this section, we revisit the concept of spherical fuzzy sets along with their algebraic operations. Throughout the remainder of this paper, the universal set will be denoted by X.

Definition 1. [4] A SFS over the universe X is given by

$$S = \{< x, \mu_S(x), \iota_S(x), \nu_S(x) > | x \in X\} \tag{1}$$

where $\mu_S, \iota_S, \nu_S: X \to [0,1]$ and $\mu_S^2(x) + \iota_S^2(x) + \nu_S^2(x) \leq 1$ for all $x \in X$.

In the framework of SFSs, the values $\mu_S(x)$, $\iota_S(x)$, and $\nu_S(x)$ represent, respectively, the membership degree, the neutral-membership degree, and the non-membership degree of an element x with respect to the set S. In addition to these three parameters, a refusal (or hesitancy) degree is introduced to capture the remaining uncertainty not explicitly expressed by the other components. This is defined as $\pi_S(x) = \sqrt{1 - \mu_S^2(x) - \iota_S^2(x) - \nu_S^2(x)}$ for all $x \in X$. The refusal function ensures that the overall information associated with an element remains bounded within the unit sphere. The collection of all SFSs defined over the universal set X is denoted by $SFS(X)$, which forms the basis for further theoretical developments and applications in decision-making problems.

Also, the triplet $S = (\mu_S, \iota_S, \nu_S)$ is said to be a spherical fuzzy number (SFN) where $\mu_S, \iota_S, \nu_S \in [0,1]$ and $\mu_S^2 + \iota_S^2 + \nu_S^2 \leq 1$.

In what follows, we recall the definitions of the algebraic operations on SFNs. These operations form the mathematical foundation for performing calculations within the spherical fuzzy environment, enabling the combination, comparison, and transformation of SFNs in a consistent manner. The operations are given in the next definition:

Definition 2. [4] Let $k \geq 0$ and $S = (\mu_S, \iota_S, \nu_S)$, $S_1 = (\mu_{S_1}, \iota_{S_1}, \nu_{S_1})$, $S_2 = (\mu_{S_2}, \iota_{S_2}, \nu_{S_2})$ be three SFNs. The algebraic operations on SFNs are subsequently defined as follows:

(i) $S^c = (\nu_S, \iota_S, \mu_S)$,

(ii) $S_1 \leq S_2$ iff $\mu_{S_1} \leq \mu_{S_2}$, $\iota_{S_1} \leq \iota_{S_2}$ and $\nu_{S_1} \geq \nu_{S_2}$,

(iii) $S_1 = S_2$ iff $S_1 \leq S_2$ and $S_2 \leq S_1$,

(iv) $S_1 \oplus S_2 = \left(\sqrt{\mu_{S_1}^2 + \mu_{S_2}^2 - \mu_{S_1}^2\mu_{S_2}^2}, \sqrt{(1-\mu_{S_1}^2)\iota_{S_2}^2 + (1-\mu_{S_2}^2)\iota_{S_1}^2 - \iota_{S_1}^2\iota_{S_2}^2}, \nu_{S_1}\nu_{S_2} \right)$,

(v) $S_1 \otimes S_2 = \left(\mu_{S_1}\mu_{S_2}, \sqrt{(1-\nu_{S_1}^2)\iota_{S_2}^2 + (1-\nu_{S_2}^2)\iota_{S_1}^2 - \iota_{S_1}^2\iota_{S_2}^2}, \sqrt{\nu_{S_1}^2 + \nu_{S_2}^2 - \nu_{S_1}^2\nu_{S_2}^2} \right)$,

(vi) $k \times S = \left(\sqrt{1-(1-\mu_S^2)^k}, \sqrt{(1-\mu_S^2)^k - (1-\mu_S^2-\iota_S^2)^k}, \nu_S^k \right)$,

(vii) $S^k = \left(\mu_S^k, \sqrt{(1-\nu_S^2)^k - (1-\nu_S^2-\iota_S^2)^k}, \sqrt{1-(1-\nu_S^2)^k}\right)$

The following definition gives an aggregation operator to merge the SFNs:

Definition 3. [4] Let $\mathcal{S}$ be a family of the SFNs and $(S_1, S_2, \ldots, S_n) \in \mathcal{S}$ where $S_k = \left(\mu_{S_k}, \iota_{S_k}, \nu_{S_k}\right)$ for all $k = 1, \ldots, n$. Then the spherical fuzzy weighted average (SFWA) operator $SFWA_v: \mathcal{S}^n \to \mathcal{S}$ is defined as

$$SFWA_v(\mathrm{S}_1, \mathrm{S}_2, \ldots, \mathrm{S}_\mathrm{n}) = \bigoplus_{k=1}^{n} v_k \times S_k = (v_1 \times S_1) \oplus \ldots \oplus (v_n \times S_n) \quad (2)$$

where $v_k \geq 0$ for all $k = 1, \ldots, n$ holds $\sum v_k = 1$ and $v = (v_1, v_2, \ldots, v_n)^T$ denotes the weight vector for $(S_k)_{k=1}^n$.

Theorem 1. [4] The value $SFWA_v(\mathrm{S}_1, \mathrm{S}_2, \ldots, \mathrm{S}_\mathrm{n})$ is also a SFN and is obtained by

$$SFWA_v(\mathrm{S}_1, \mathrm{S}_2, \ldots, \mathrm{S}_\mathrm{n}) = \bigoplus_{k=1}^{n} v_k \times S_k = \left(\sqrt{1-\prod_{k=1}^{n}\left(1-\mu_{S_k}^2\right)^{v_k}}, \sqrt{\prod_{k=1}^{n}\left(1-\mu_{S_k}^2\right)^{v_k} - \prod_{k=1}^{n}\left(1-\mu_{S_k}^2-\iota_{S_k}^2\right)^{v_k}}, \prod_{k=1}^{n} \nu_{S_k}^{v_k}\right) \quad (3)$$

where $(S_1, S_2, \ldots, S_n) \in \mathcal{S}^n$, $S_k = \left(\mu_{S_k}, \iota_{S_k}, \nu_{S_k}\right)$ and $v = (v_1, v_2, \ldots, v_n)^T$ is the weight vector for $(S_k)_{k=1}^n$ holds $v_k \geq 0$ for all $k = 1, \ldots, n$ and $\sum v_k = 1$.

Definition 4. [4] Let $\mathcal{S}$ be the collection of the SFNs and $S, S_1, S_2 \in \mathcal{S}$ where $S = (\mu_S, \iota_S, \nu_S)$ and $S_k = \left(\mu_{S_k}, \iota_{S_k}, \nu_{S_k}\right)$ for $k = 1,2$.

(1) A score function on $\mathcal{S}$ ($SF: \mathcal{S} \to [-1,1]$) and an accuracy function on $\mathcal{S}$ ($AF: \mathcal{S} \to [0,1]$) are given as $SF(S) = \left(\mu_{S_k} - \iota_{S_k}\right)^2 - \left(\nu_{S_k} - \iota_{S_k}\right)^2$, $AF(S) = \mu_{S_k}^2 + \iota_{S_k}^2 + \nu_{S_k}^2$, respectively.

(2) The ranking procedure (comparison approach) based on score/accuracy functions is defined as follows:

(i) If $SF(S_1) < SF(S_1)$, then $S_1 \prec S_2$,

(ii) If $SF(S_1) > SF(S_2)$, then $S_1 \succ S_2$,

(iii) $SF(S_1) = SF(S_2)$, then

(a) If $AF(S_1) < AF(S_1)$, then $S_1 \prec S_2$,

(b) If $AF(S_1) > AF(S_2)$, then $S_1 \succ S_2$,

(c) $AF(S_1) = AF(S_2)$, then $S_1 = S_2$.

3. METHOD

In the introduced method, we combine the AHP method and entropy to determine hybrid criterion weights, and then apply the TOPSIS method to obtain the optimal solution for MCDM problems. Suppose that $A = \{A_1, A_2, \ldots, A_k\}$ is the set of k options and $C = \{C_1, C_2, \ldots, C_m\}$ is

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the set of criterion. We denote the sub-criterion related to the main criterion by $c_{i i_j}$ for all $i = 1, \ldots, m$ where i_j means that the i-th main criterion has j sub-criterion.

The steps of the proposed method are explained as follows:

Step 1: The decision-maker constructs the pairwise comparison matrices for the criterion and sub-criterion. At this stage, the DMs express the relative significance of the i-th criterion, drawing on their individual knowledge and practical experience, by using the spherical fuzzy linguistic terms presented in Table 1 ([19]).

Linguistic terms	$(\boldsymbol{\mu_S, \iota_S, \nu_S})$	**SI**
Absolutely more significance (AMS)	(.9, .0, .1)	9
Very high significance (VHS)	(.8, .1, .2)	7
High significance (HS)	(.7, .2, .3)	5
Slightly more significance (SMS)	(.6, .3, .4)	3
Equally significance (ES)	(.5, .4, .4)	1
Slightly low significance (SLS)	(.4, .3, .6)	1/3
Low significance (LS)	(.3, .2, .7)	1/5
Very low significance (VLS)	(.2, .1, .8)	1/7
Absolutely low significance (ALS)	(.1, .0, .9)	1/9

Table 1: Linguistic terms along with their corresponding SFNs and Score Indices (SI) [19]

Let us write the comparison matrix by $M = (l_{ij})_{m \times m}$ where $l_{ij} = \left(\mu_{l_{ij}}, \iota_{l_{ij}}, \nu_{l_{ij}}\right)$ for all $i, j = 1, .., m$ and $r = 1, \ldots, n$. Here, l_{ij} shows the significance of i-th criterion relative to the j-th criterion satisfying $SI(l_{ij}).SI(l_{ji}) = 1$ where SI denotes the score index (given in [19])

$$SI(S) = \sqrt{|100((\mu_s - \iota_s)^2 - (\nu_s - \iota_s)^2)\ |} \tag{4}$$

for a SFN $S = (\mu_S, \iota_S, \nu_S)$ whenever S equals to AMS, VHS, HS, SMS and ES;

$$\frac{1}{SI(S)} = \frac{1}{\sqrt{|100((\mu_s - \iota_s)^2 - (\nu_s - \iota_s)^2)\ |}} \tag{5}$$

whenever S equals to and ALS, VLS, LS, SLS and ES. If $SI(l_{ij})$> 1, the i-th attribute is considered more important than the j-th attribute; if $SI(l_{ij})$< 1, the i-th attribute is regarded as less important; and if $SI(l_{ij})$= 1, both attributes are equally important. This procedure is repeated for all sub-criterion.

Step 2: The consistency ratio (CR) of all pairwise comparison matrices is verified using the classical consistency check. To begin, the comparison matrix is transformed into a real matrix based on the score indices of its elements. Next, the maximum eigenvalue of this real matrix (denoted by δ_{max}) is computed, and the consistency ratio is obtained using the formula $CR = \frac{CI}{RI}$, where RI is the random index (given in Table 2) and CI, the consistency index, is calculated as $CI = \frac{\delta_{max} - m}{m-1}$. Finally, the CR value is assessed against the 10\% threshold: if

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$CR < 0.1$, the data is considered consistent; otherwise, a new pairwise comparison matrix must be reconstructed to achieve consistency.

n	1	2	3	4	5	6	7	8	9	10
RI	.00	.00	.52	.89	1.11	1.25	1.35	1.40	1.45	1.49

Table 2. Random index (RI)

Step 3: Determine the local weights of each criterion and sub-criterion.

At this stage, the elements contained in each row of the matrix M are combined by applying SFWA operator. Through this aggregation process, the local spherical fuzzy number weights corresponding to each main criterion, denoted by w_{C_i}, as well as those corresponding to the sub-criterion, denoted by $w_{c_{ij}}$, are derived. The computation of these local weights is carried out using the following expression:

$$w_{C_i} = SFWA_{\frac{1}{m}}(l_{i1}, l_{i2}, \ldots, l_{im})$$

$$= \left(\sqrt{1 - \prod_{k=1}^{n} \left(1 - \mu_{l_{ik}}^2\right)^{\frac{1}{m}}}, \sqrt{\prod_{k=1}^{n} \left(1 - \mu_{l_{ik}}^2\right)^{\frac{1}{m}} - \prod_{k=1}^{n} \left(1 - \mu_{l_{ik}}^2 - \iota_{l_{ik}}^2\right)^{\frac{1}{m}}}, \prod_{k=1}^{n} \nu_{l_{ik}}^{\frac{1}{m}} \right) \qquad (6)$$

Then, the weights $w_{C_i} = \left(\mu_{w_{C_i}}, \iota_{w_{C_i}}, \nu_{w_{C_i}}\right)$ calculated as a SFN, are transformed into crisp values using the following formula:

$$S(w_{C_i}) = \sqrt{100 \left| \left(3\mu_{w_{C_i}} - \frac{\iota_{w_{C_i}}}{2}\right)^2 - \left(\frac{\nu_{w_{C_i}}}{2} - \iota_{w_{C_i}}\right)^2 \right|}. \qquad (7)$$

Then the normalized local weights $\left(s(w_{C_i})\right)$ are obtained by using $s(w_{C_i}) = \frac{s(w_{C_i})}{\sum_{i=1}^{n} s(w_{C_i})}$. The same procedures are applied for sub-criterion to obtain the local weights of sub-criterion.

Step 4: Compute the global weights corresponding to each sub-criterion.

The global weights of the sub-criterion are determined by integrating both the local weight of each sub-criterion and the weight of the corresponding main criterion. In other words, each sub-criterion inherits the significance of its associated main criterion, and its local contribution is proportionally adjusted by multiplying these two values. This ensures that the global weight not only reflects the relative significance of the sub-criterion within its group but also accounts for the overall priority of the main criterion in the decision-making hierarchy. Specifically, the formula $\widetilde{w_{c_{ij}}} = \frac{s(w_{C_{ij}}) s(w_{C_i})}{\sum_{i,j} s(w_{C_{ij}}) s(w_{C_i})}$ is applied to compute the global weight of each sub-criterion. Next, the sub-criterion are arranged as though they were main criterion, and the subjective criterion weights are denoted by w_q. For instance, we assign $\widetilde{w_{c_{11}}} = w_1$, $\widetilde{w_{c_{12}}} = w_2$ and so on. In this way, the subjective weights of the criterion are determined.

Step 5: At this stage, the decision-maker assesses the alternatives A_p with respect to the attribute C_q, taking into account the influence of C_q on A_p. Based on this evaluation, the

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spherical fuzzy decision matrix (SFDM) $D=\left(d_{pq}\right)_{k\times m}$ is constructed, where each element is represented as $d_{pq}=\left(\mu_{D_{pq}},\iota_{D_{pq}},\nu_{D_{pq}}\right)$ for all $p=1,\ldots,k$ and $q=1,\ldots,m$. The SFDM D can be expressed as follows:

$$D=\left(d_{pq}\right)_{k\times m}=\begin{pmatrix}\left(\mu_{D_{11}},\iota_{D_{11}},\nu_{D_{11}}\right) & \left(\mu_{D_{12}},\iota_{D_{12}},\nu_{D_{12}}\right) & \ldots & \left(\mu_{D_{1m}},\iota_{D_{1m}},\nu_{D_{1m}}\right)\\ \left(\mu_{D_{21}},\iota_{D_{21}},\nu_{D_{21}}\right) & \left(\mu_{D_{22}},\iota_{D_{22}},\nu_{D_{22}}\right) & \ldots & \left(\mu_{D_{2m}},\iota_{D_{2m}},\nu_{D_{2m}}\right)\\ \ldots & \ldots & \ldots & \ldots\\ \left(\mu_{D_{k1}},\iota_{D_{k1}},\nu_{D_{k1}}\right) & \left(\mu_{D_{k2}},\iota_{D_{k2}},\nu_{D_{k2}}\right) & \ldots & \left(\mu_{D_{km}},\iota_{D_{km}},\nu_{D_{km}}\right)\end{pmatrix}.$$

Also, in this step, it is important to account for the fact that the criterion in a given MCDM problem can be classified as either benefit-type or non-benefit type. Therefore, the evaluations provided by the decision-makers are normalized using the following procedure:

$$s_{pq}=\begin{cases}d_{pq}, & for\ benefit\ criterion\ C_q\\ d_{pq}^{c}, & for\ non-benefit\ criterion\ C_q\end{cases} \tag{8}$$

for all $p=1,..,k$, and $q=1,\ldots,m$ where d_{pq}^{c} is the complement of d_{pq}. Hence, the normalized spherical fuzzy decision matrix (NSFDM) $D_N=\left(s_{pq}\right)_{k\times m}$ where $s_{pq}=\left(\mu_{pq},\iota_{pq},\nu_{pq}\right)$ for all $p=1,..,k$, and $q=1,\ldots,m$, are shown as follows:

$$D_N=\left(s_{pq}\right)_{k\times m}=\begin{pmatrix}\left(\mu_{11},\iota_{11},\nu_{11}\right) & \left(\mu_{12},\iota_{12},\nu_{12}\right) & \ldots & \left(\mu_{1m},\iota_{1m},\nu_{1m}\right)\\ \left(\mu_{21},\iota_{21},\nu_{21}\right) & \left(\mu_{22},\iota_{22},\nu_{22}\right) & \ldots & \left(\mu_{2m},\iota_{2m},\nu_{2m}\right)\\ \ldots & \ldots & \ldots & \ldots\\ \left(\mu_{k1},\iota_{k1},\nu_{k1}\right) & \left(\mu_{k2},\iota_{k2},\nu_{k2}\right) & \ldots & \left(\mu_{km},\iota_{km},\nu_{km}\right)\end{pmatrix}.$$

Step 6: This step calculates the criterion weight objectively by using entropy function. To do this, the entropies of the normalized decision matrix are calculates according to the criterion. That is, objective criterion weights are found by using the following equation:

$$v_q=\frac{1-E\left(c_q\right)}{\sum_{q=1}^{m}1-E\left(c_q\right)},\forall\, q=1,\ldots,m \tag{9}$$

where $E\left(c_q\right)=\frac{1}{k}\sum_{p=1}^{k}1-\frac{4}{5}\left(\left|f_{A_p}\left(c_q\right)^2-h_{A_p}\left(c_q\right)^2\right|+\left|g_{A_p}\left(c_q\right)^2-.25\right|\right)$.

Step 7: Now, we merge the subjective and objective weights by using the following formula:

$$\xi_q=a.v_q+(1-a).w_q \tag{10}$$

$q=1,\ldots,m$. Here, a depends on the subjective and objective weights' significance and a should be in the interval [0,1].

Step 8: Construct the aggregated weighted spherical fuzzy decision matrix (AWSFDM) $D'=\left(s_{pq}{}'\right)_{k\times m}$ by multiplying the NSFDM with the weight value ξ_q found in the last step. That is, the element $s_{pq}{}'$ of D' is calculated as follows:

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$$s'_{pq} = \left(\sqrt{1-\left(1-\mu_{pq}^2\right)^{\xi_q}}, \sqrt{\left(1-\mu_{pq}^2\right)^{\xi_q}-\left(1-\mu_{pq}^2-\iota_{pq}^2\right)^{\xi_q}}, \nu_{pq}^{\xi_q}\right)$$

If we write $s_{pq}{}' = \left(\mu_{pq}{}', \iota_{pq}{}', \nu_{pq}{}'\right)$ for all $p = 1,..,k$, and $q = 1,\dots,m$, then the AWSFDM is constructed as:

$$D' = \begin{pmatrix} s'_{11} & s'_{12} & \dots & s'_{1m} \\ s'_{21} & s'_{22} & \dots & s'_{2m} \\ \vdots & \vdots & \dots & \vdots \\ s'_{k1} & s'_{k2} & \dots & s'_{km} \end{pmatrix}$$

Step 9: Since the elements of AWSFDM D' are SFN, the score matrix D^* must be constructed by using the score function. Thus, the score matrix $D^* = \left(s_{pq}^*\right)_{k\times m}$ is constructed as follows:

$$D^* = \begin{pmatrix} s^*_{11} & s^*_{12} & \dots & s^*_{1m} \\ s^*_{21} & s^*_{22} & \dots & s^*_{2m} \\ \vdots & \vdots & \dots & \vdots \\ s^*_{k1} & s^*_{k2} & \dots & s^*_{km} \end{pmatrix}$$

where $s_{pq}^* = \left(\mu'_{pq} - \iota_{pq}{}'\right)^2 - \left(\nu'_{pq} - \iota_{pq}{}'\right)^2$ for all $p = 1,..,k$, and $q = 1,\dots,m$.

Step 10: Calculate the positive ideal solution P^+, negative ideal solution N^- and find the distances of the positive ideal solution and the negative ideal solution from each alternative A_p. Let $\mathcal{C}_B$ and $\mathcal{C}_C$ denote the set of benefit type and cost type criterion, respectively. Then, the positive ideal solution P^+ and the negative ideal solution N^- are found as follows:

$$P^+ = \{P^+(c_1), P^+(c_2), \dots P^+(c_m)\}, \quad N^- = \{N^-(c_1), N^-(c_2), \dots, N^-(c_m)\}$$

where $P^+(c_q)$ and $N^-(c_q)$ are evaluated for all $q = 1,2,\dots,m$ as:

$$P^+(c_q) = \left\{\left(max_p s_{pq}^* \middle| c_q \in \mathcal{C}_B\right), \left(min_p s_{pq}^* \middle| c_q \in \mathcal{C}_c\right) \middle| 1 \le p \le k\right\},$$

$$N^-(c_q) = \left\{\left(min_p s_{pq}^* \middle| c_q \in \mathcal{C}_B\right), \left(max_p s_{pq}^* \middle| c_q \in \mathcal{C}_c\right) \middle| 1 \le p \le k\right\},$$

To obtain the distances between alternatives and positive/negative ideal solutions, we apply Euclidean distance as:

$$d(A_p, P^+) = \sqrt{\textstyle\sum_{q=1}^m \left(A_p(c_q) - P^+(c_q)\right)^2}, \; d(A_p, N^-) = \sqrt{\textstyle\sum_{q=1}^m \left(A_p(c_q) - N^-(c_q)\right)^2}$$

Step 11: To find the ranking, we measure the closeness to the positive ideal solution and furtherness to the negative ideal solution. Then, we obtain the relative closeness index of an alternative A_p by considering these values:

$$R(A_p) = \frac{d(A_p,N^-)}{d(A_p,N^-)+d(A_p,P^+)} \quad \text{for all } p = 1,2,\dots,k.$$

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Finally, the alternatives are evaluated by ranking them in descending order according to the relative closeness index $R(A_p)$.

4. ILLUSTRATIVE EXAMPLE

In this section, we solve a problem related to choosing the energy storage systems (ESSs). ESSs are crucial for integrating variable renewable energy sources into national power grids, especially in managing intermittency and supply-demand mismatches. Despite renewable energy being cheaper than fossil fuels, challenges like rationing and grid modernization persist. In the considered numerical example, Egypt, facing an electricity surplus due to overgeneration, is considering various ESS technologies to store excess power. Seven ESS types—Hydrogen (A_1), Pumped Hydro (A_2), Compressed Air (A_3), Flywheel (A_4), Supercapacitors (A_5), Superconducting Magnetic (A_6), and Lithium-ion Batteries (A_7)—have been identified as potential solutions. In the context of selecting the most appropriate and sustainable Energy Storage System (ESS) for Egypt, four principal dimensions have been identified. These dimensions represent the key factors that exert a direct influence on the decision-making process, shaping both the evaluation of alternatives and the final choice of the optimal ESS solution. By considering these dimensions, the analysis ensures that the selected system not only meets technical requirements but also aligns with economic, environmental, and contextual sustainability goals specific to Egypt. Additionally, several sub-criterion must be considered in solving the problem. These criterion are shown in Table 3.

Criterion	Sub-criterion
C_1: Technology dimension	c_{11}: Energy efficiency c_{12}: Adaptability for mobile systems c_{13}: Energy intensity c_{14}: Technological maturity c_{15}: Material intensity c_{16}: Storage capacity
C_2: Environmental dimension	c_{21}: Resource utilization c_{22}: Air and water contamination c_{23}: Land disturbance c_{24}: CO2 intensity
C_3: Economic dimension	c_{31}: Capital cost c_{32}: Operating and maintenance cost c_{33}: Lifetime
C_4: Social-political dimension	c_{41}: Social acceptance c_{42}: Job creation c_{43}: Health and safety c_{44}: Governmental inducement c_{45}: Political acceptability

Table 3: The criterion and sub-criterion considered in the problem

We now apply the proposed method to the given problem to demonstrate its practical implementation. The solution process is carried out step by step, with each stage of the method explained in detail to illustrate how the approach can be systematically employed to obtain the final results.

Step 1: In the first step, the decision-maker is responsible for constructing the pairwise comparison matrices corresponding to both the main criterion and the associated sub-criterion. These matrices serve as the foundation for evaluating the relative significance of each element within the decision hierarchy, as they capture the preferences and judgments of the decision-

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maker in a structured format. The resulting pairwise comparison matrices are presented in Table 4-8. These evaluations are taken from the paper [34].

Table 4: Pairwise comparison matrix of main criterion

	C_1	C_2	C_3	C_4
C_1	(.5, .5, .5)	(.1, .9, .1)	(.1, .9, .1)	(.3, .7, .3)
C_2	(.9, .1, .1)	(.5, .5, .5)	(.4, .6, .4)	(.8, .2, .2)
C_3	(.9, .1, .1)	(.6, .4, .4)	(.5, .5, .5)	(.3, .2, .7)
C_4	(.7, .3, .3)	(.2, .8, .2)	(.9, .1, .1)	(.5, .5, .5)

Table 5: Pairwise comparison matrix of subcriterion C_1

C1	c11	c12	c13	c14	c15	c16
c11	(.5, .5, .5)	(.8, .2, .2)	(.9, .1, .1)	(.7, .3, .3)	(.8, .2, .2)	(.1, .9, .1)
c12	(.2, .8, .2)	(.5, .5, .5)	(.3, .7, .3)	(.8, .2, .2)	(.9, .1, .1)	(.8, .2, .2)
c13	(.1, .9, .1)	(.7, .3, .3)	(.5, .5, .5)	(.1, .9, .1)	(.7, .3, .1)	(.8, .2, .2)
c14	(.3, .7, .3)	(.2, .8, .2)	(.9, .1, .1)	(.5, .5, .5)	(.4, .6, .1)	(.9, .1, .1)
c15	(.2, .8, .2)	(.1, .9, .1)	(.3, .7, .3)	(.6, .4, .4)	(.5, .5, .5)	(.9, .1, .1)
c16	(.9, .1, .1)	(.2, .8, .2)	(.2, .8, .2)	(.1, .9, .1)	(.1, .9, .1)	(.5, .5, .5)

Table 6: Pairwise comparison matrix of subcriterion C_2

C2	c21	c22	c23	c24	c25
c21	(.5, .5, .5)	(.7, .3, .3)	(.9, .1, .1)	(.7, .3, .3)	(.2, .8, .2)
c22	(.3, .7, .3)	(.5, .5, .5)	(.3, .7, .3)	(.8, .2, .2)	(.9, .1, .1)
c23	(.1, .9, .1)	(.7, .3, .3)	(.5, .5, .5)	(.1, .9, .1)	(.7, .3, .3)
c24	(.3, .7, .3)	(.2, .8, .2)	(.9, .1, .1)	(.5, .5, .5)	(.4, .6, .4)
c25	(.8, .2, .2)	(.1, .9, .1)	(.3, .7, .3)	(.6, .4, .4)	(.5, .5, .5)

Table 7: Pairwise comparison matrix of subcriterion C_3

C3	c31	c32	c33
c31	(.5, .5, .5)	(.2, .8, .2)	(.9, .1, .1)
c32	(.8, .2, .2)	(.5, .5, .5)	(.3, .7, .3)
c33	(.1, .9, .1)	(.7, .3, .3)	(.5, .5, .5)

Table 8: Pairwise comparison matrix of subcriterion C_4

C4	c41	c42	c43	c44	c45
c41	(.5, .5, .5)	(.6, .4, .4)	(.9, .1, .1)	(.7, .3, .3)	(.1, .9, .1)
c42	(.4, .6, .4)	(.5, .5, .5)	(.2, .8, .2)	(.8, .2, .2)	(.9, .1, .1)
c43	(.1, .9, .1)	(.8, .2, .2)	(.5, .5, .5)	(.1, .9, .1)	(.7, .3, .3)
c44	(.3, .7, .3)	(.2, .8, .2)	(.9, .1, .1)	(.5, .5, .5)	(.9, .1, .1)
c45	(.9, .1, .1)	(.1, .9, .1)	(.3, .7, .3)	(.1, .9, .1)	(.5, .5, .5)

Step 2 is skipped as the consistency ratio has been calculated in the paper [34].

Step 3: The local weights of each criterion are calculated by using Eq. 6-7 and the weights are found as $s(w_{C_1}) = 4.3135$, $s(w_{C_2})$ =20.3479, $s(w_{C_3})$ =18.8155, $s(w_{C_4})$ =18.5229 for main criterion and $s(w_{c_{11}})$ =19.8347, $s(w_{c_{12}})$ =18.8157, $s(w_{c_{13}})$ =14.3306, $s(w_{c_{14}})$ =18.1735, $s(w_{c_{15}})$ =13.9582, $s(w_{c_{16}})$ =11.3346, $s(w_{c_{21}})$ =18.7305, $s(w_{c_{22}})$ =18.1811, $s(w_{c_{23}})$ =11.7019, $s(w_{c_{24}})$ =15.0014, $s(w_{c_{25}})$ =13.2065, $s(w_{c_{31}})$ =18.2719, $s(w_{c_{32}})$ =15.6845, $s(w_{c_{33}})$ = 11.6350, $s(w_{c_{41}})$ =17,6247, $s(w_{c_{42}})$ =18,1346, $s(w_{c_{43}})$ =13,1986, $s(w_{c_{44}})$ =19,3922, $s(w_{c_{45}})$ =13,1904 for sub-criterion.

Step 4: The global weights of each criterion are found by using the values given in previous step and these values are $\widetilde{w}_{c_{11}} = w_1$ =.0150, $\widetilde{w}_{c_{12}} = w_2$ =.0143, $\widetilde{w}_{13} = w_3$=.0109, $\widetilde{w}_{c_{14}} = w_4 =$ .0138, $\widetilde{w}_{c_{15}} = w_5 =$ .0106, $\widetilde{w}_{c_{16}} = w_6 =$ .0086, $\widetilde{w}_{c_{21}} = w_7 =$ .0897, $\widetilde{w}_{c_{22}} = w_8 =$ .0871, $\widetilde{w}_{c_{23}} = w_9$ =.0560, $\widetilde{w}_{c_{24}} = w_{10}$ =.0718, $\widetilde{w}_{c_{25}} = w_{11} =$ .0632, $\widetilde{w}_{c_{31}} = w_{12} =$ .1129, $\widetilde{w}_{c_{32}} = w_{13} =$ .0969, $\widetilde{w}_{c_{33}} = w_{14} =$ .0719, $\widetilde{w}_{c_{41}} = w_{15} =$ .0599, $\widetilde{w}_{c_{42}} = w_{16} =$ .0617, $\widetilde{w}_{c_{43}} = w_{17}$ =.0449, $\widetilde{w}_{c_{44}} = w_{18}$ .0659, $\widetilde{w}_{c_{45}} = w_{19}$ =.0449.

Step 5: The decision matrix established by the decision-maker [34] is given as follows:

Table 3. Decision matrix D

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D	c11	c12	c13	c14	c15	c16	c21
A1	(.2,.8,.2)	(.9,.1,.1)	(.1,.9,.1)	(.7,.3,.3)	(.7,.3,.3)	(.7,.3,.3)	(.2,.8,.2)
A2	(.9,.1,.1)	(.8,.2,.2)	(.9,.1,.1)	(.7,.3,.3)	(.8,.2,.2)	(.9,.1,.1)	(.8,.2,.2)
A3	(.9,.1,.1)	(.9,.1,.1)	(.6,.4,.4)	(.9,.1,.1)	(.7,.3,.3)	(.6,.4,.4)	(.6,.4,.4)
A4	(.8,.2,.2)	(.9,.1,.1)	(.8,.2,.2)	(.1,.9,.1)	(.9,.1,.1)	(.7,.3,.3)	(.9,.1,.1)
A5	(.5,.5,.5)	(.1,.9,.1)	(.1,.9,.1)	(.5,.5,.5)	(.1,.9,.1)	(.4,.6,.4)	(.5,.5,.5)
A6	(.2,.8,.2)	(.6,.4,.4)	(.2,.8,.2)	(.2,.8,.2)	(.5,.5,.5)	(.3,.7,.3)	(.2,.8,.2)
A7	(.3,.7,.3)	(.1,.9,.1)	(.4,.6,.4)	(.4,.6,.4)	(.3,.7,.3)	(.2,.8,.2)	(.1,.9,.1)

D	c22	c23	c24	c25	c31	c32	c33
A1	(.9,.1,.1)	(.6,.4,.4)	(.6,.4,.4)	(.9,.1,.1)	(.6,.4,.4)	(.6,.4,.4)	(.9,.1,.1)
A2	(.8,.2,.2)	(.7,.3,.3)	(.8,.2,.2)	(.7,.3,.3)	(.8,.2,.2)	(.9,.1,.1)	(.8,.2,.2)
A3	(.9,.1,.1)	(.1,.9,.1)	(.9,.1,.1)	(.9,.1,.1)	(.7,.3,.3)	(.2,.8,.2)	(.9,.1,.1)
A4	(.4,.6,.4)	(.8,.2,.2)	(.2,.8,.2)	(.7,.3,.3)	(.2,.8,.2)	(.9,.1,.1)	(.9,.1,.1)
A5	(.4,.6,.4)	(.1,.9,.1)	(.7,.3,.3)	(.5,.5,.5)	(.5,.5,.5)	(.2,.8,.2)	(.2,.8,.2)
A6	(.4,.6,.4)	(.8,.2,.2)	(.1,.9,.1)	(.1,.9,.1)	(.8,.2,.2)	(.2,.8,.2)	(.4,.6,.4)
A7	(.1,.9,.1)	(.8,.2,.2)	(.2,.8,.2)	(.8,.2,.2)	(.5,.5,.5)	(.2,.8,.2)	(.5,.5,.5)

D	c41	c42	c43	c44	c45
A1	(.8,.2,.2)	(.7,.3,.3)	(.7,.3,.3)	(.7,.3,.3)	(.9,.1,.1)
A2	(.9,.1,.1)	(.8,.2,.2)	(.7,.3,.3)	(.9,.1,.1)	(.8,.2,.2)
A3	(.7,.3,.3)	(.9,.1,.1)	(.9,.1,.1)	(.6,.4,.4)	(.9,.1,.1)
A4	(.8,.2,.2)	(.2,.8,.2)	(.2,.8,.2)	(.8,.2,.2)	(.2,.8,.2)
A5	(.2,.8,.2)	(.8,.2,.2)	(.8,.2,.2)	(.2,.8,.2)	(.5,.5,.5)
A6	(.4,.6,.4)	(.1,.9,.1)	(.2,.8,.2)	(.3,.7,.3)	(.3,.7,.3)
A7	(.5,.5,.5)	(.2,.8,.2)	(.1,.9,.1)	(.1,.9,.1)	(.1,.9,.1)

If we consider the sub-criterion under the main criterion C_3, we need to normalize the decision matrix.

Step 6: In this step, we calculate the subjective weights by using entropy measure function as: $v_1 = .0521$, $v_2 = .0712$, $v_3 = .0501$, $v_4 = .0429$, $v_5 = .0503$, $v_6 = .0425$, $v_7 = .0464$, $v_8 = .0471$, $v_9 = .0620$, $v_{10} = .0505$, $v_{11} = .0643$, $v_{12} = .0412$, $v_{13} = .0493$, $v_{14} = .0557$, $v_{15} = .0496$, $v_{16} = .0608$, $v_{17} = .0575$, $v_{18} = .0519$, $v_{19} = .0544$.

Step 7: Now we find the weight of the hybrid criterion by taking $a = .5$ in the Eq. 10 and these values are ξ_1 =.0336, $\xi_2 = .0427$, $\xi_3 = .0305$, $\xi_4 = .0284$, $\xi_5 = .0304$, $\xi_6 = .0256$, $\xi_7 = .0680$, $\xi_8 = .0671$, $\xi_9 = .0590$, $\xi_{10} = .0612$, $\xi_{11} = .0638$, $\xi_{12} = .0770$, $\xi_{13} = .0731$, $\xi_{14} = .0638$, $\xi_{15} = .0548$, $\xi_{16} = 0612$, $\xi_{17} = .0512$, $\xi_{18} = .0589$, $\xi_{19} = .0496$.

Step 8: We then obtain the aggregated weighted spherical fuzzy decision matrix.

Step 9: We calculate the score values of each element in the aggregated weighted spherical fuzzy decision matrix to construct the score matrix (D^*) and this matrix is obtained as follows:

$$D^* = \begin{pmatrix} -0.91, -0.69, -0.96, -0.79, -0.78, -0.81, -0.87, -0.57, -0.67, -0.66, -0.58, 0.62, -0.63, -0.58, -0.68, -0.67, -0.70, -0.68, -0.66 \\ -0.75, -0.73, -0.76, -0.79, -0.78, -0.79, -0.63, -0.65, -0.66, -0.67, -0.64, -0.52, -0.61, -0.66, -0.63, -0.65, -0.70, -0.61, -0.70 \\ -0.75, -0.69, -0.77, -0.78, -0.78, -0.79, -0.64, -0.57, -0.94, -0.60, -0.58, -0.62, -0.86, -0.58, -0.69, -0.60, -0.65, -0.67, -0.66 \\ -0.77, -0.69, -0.78, -0.96, -0.76, -0.81, -0.56, -0.72, -0.66, -0.88, -0.66, -0.86, -0.54, -0.58, -0.68, -0.88, -0.89, -0.66, -0.89 \\ -0.74, -0.95, -0.96, -0.76, -0.96, -0.83, -0.62, -0.72, -0.94, -0.67, -0.64, -0.60, -0.86, -0.87, -0.88, -0.65, -0.69, -0.88, -0.68 \\ -0.91, -0.73, -0.92, -0.92, -0.75, -0.88, -0.87, -0.72, -0.66, -0.94, -0.94, -0.59, -0.86, -0.73, -0.75, -0.94, -0.89, -0.81, -0.83 \\ -0.86, -0.95, -0.82, -0.83, -0.87, -0.92, -0.93, -0.93, -0.66, -0.88, -0.64, -0.60, -0.86, -0.64, -0.66, -0.88, -0.94, -0.94, -0.95 \end{pmatrix}$$

Step 10-11: Now, we find the positive ideal solution and negative ideal solution as follows:

$$P^+ = \{-.74, -.69, -.76, -.76, -.75, -.92, -.93, -.93, -.94, -.94, \\ -.94, -.86, -.86, -.87, -.88, -.94, -.94, -.94, -.95\ \}$$

$$N^- = \{-.91, -.95, -.96, -.96, -.96, -.79, -.56, -.57, -.66, -.60, \\ -.58, -.52, -.54, -.58, -.63, -.60, -.65, -.61, -.66\ \}$$

According to these values, we first calculate the distances of the positive ideal solution and the negative ideal solution from each alternative and then we compute the relative closeness index. All these values are shown in Table 11.

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	$d(A_p, N^-)$	$d(A_p, P^+)$	Relative closeness index	Ranking
A_1	0.5029	1.0087	0.10	6
A_2	0.4586	1.0164	0.00	7
A_3	0.6299	1.0086	0.38	5
A_4	0.7537	0.8445	0.81	3
A_5	0.7190	0.8630	0.72	4
A_6	0.9214	0.5911	1.43	1
A_7	0.9437	0.6656	1.40	2

Table 11: Distances, relative closeness index and ranking

As a result of the analysis, the best alternative for this problem is identified as A_6. Furthermore, when the ranking outcome is compared with the results reported in [34], it is observed that both studies yield the same optimal alternative. This consistency demonstrates that the proposed method is capable of producing reliable and robust solutions, thereby validating its applicability and effectiveness in addressing the decision-making problem under consideration.

CONCLUSION

In this study, a novel hybrid decision-making approach has been proposed by integrating AHP and the entropy method for weighting criterion within the framework of the spherical fuzzy TOPSIS method. The hybrid AHP-entropy approach enables the combination of both subjective expert evaluations and objective data-driven insights, leading to a more balanced and realistic representation of criterion weights in complex decision environments. By employing the SFS theory, the proposed method effectively handles uncertainty, hesitancy, and imprecision that are frequently encountered in real-world MCDM problems. The application of the method to the performance evaluation of energy storage systems demonstrated its practical applicability and effectiveness. While the current study provides a solid foundation, several potential avenues for future research can be listed. The proposed hybrid AHP-entropy based TOPSIS method can be extended to dynamic or time-dependent decision-making problems, especially where the significance of criterion may change over time. Future studies can explore the integration of other aggregation operators (e.g., Dombi, Einstein, Hamacher) within the spherical fuzzy framework to further enhance flexibility in information fusion. The method can also be adapted for group decision-making scenarios where multiple decision-makers with differing significance weights are involved, possibly by developing new techniques for objective determination of decision-maker weights. Additionally, the method can be applied and validated in different domains such as healthcare, supply chain management, sustainable energy planning, and risk assessment, to test its scalability and domain-specific relevance.

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MATEMATİK ALAN BECERİLERİNİN GENEL MATEMATİK BAŞARISINI AÇIKLAMADAKİ ROLÜ: MTK VE REGRESYON TEMELLİ BİR İNCELEME

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ÖZET

Bu araştırma, ortaokul yedinci sınıf öğrencilerinin matematik başarısında rol oynayan alt becerilerin görece önemini incelemeyi amaçlamaktadır. Nicel araştırma yöntemleri çerçevesinde ilişkisel tarama desenine dayalı olarak yürütülen çalışmada, madde tepki kuramı (MTK) ve regresyon birlikte kullanılmıştır. Veri toplama aracı, TÜBİTAK 120K850 numaralı proje kapsamında geliştirilen, dikotomik puanlanan 16 maddelik üst düzey düşünme becerilerini ölçen özgün bir matematik testidir. Test, alan uzmanlarının görüşleri doğrultusunda sayı–işlem, görsel–uzamsal düşünme ve bilgi–veri okuryazarlığı olmak üzere üç alt alan becerisini ölçmek üzere yapılandırılmıştır. Örneklem, 2019–2020 eğitim öğretim yılında Türkiye genelinde 7. sınıfta öğrenim gören 3138 öğrenciden oluşmaktadır. Veri analizinde öğrencilerin genel matematik yetenek puanları ve alt becerilere ait yetenek puanları MTK'nın iki parametreli (2PL) modeli kullanılarak Bayese dayalı bir tahminleyici olan EAP (Expected A Posteriori) yöntemiyle kestirilmiştir. Genel matematik başarısını en iyi hangi alt becerilerin açıkladığını belirlemek amacıyla çoklu doğrusal regresyon ve LMG (Lindeman–Merenda–Gold) göreli önem analizi uygulanmıştır. Varsayım kontrollerinde normallikten sapma ve heteroskedastisite gözlenmiş, bu nedenle katsayıların anlamlılık testlerinde HC3 heteroskedastisiteye dayanıklı standart hatalar kullanılmıştır. Bulgular, üç alt becerinin de genel matematik başarısını anlamlı ve pozitif biçimde yordadığını göstermiştir. En güçlü yordayıcı sayı–işlem becerisi olurken ($\beta = 0.57$; partial $R^2 = .96$), bunu görsel–uzamsal beceri ($\beta = 0.41$; partial $R^2 = .92$) ve bilgi–veri okuryazarlığı becerisi ($\beta = 0.29$; partial $R^2 = .86$) izlemiştir. LMG analizi de bu sıralamayı desteklemiş; toplam varyansın %45.7'si sayı–işlem, %31.7'si görsel–uzamsal ve %21.5'i bilgi–veri okuryazarlığı becerileriyle açıklanmıştır.

Anahtar Kelimeler: Üst düzey düşünme becerileri, madde tepki kuramı, çoklu regresyon

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1. GİRİŞ

Matematik dersine yönelik hazırlanmış öğretim programları ve yapılan uluslararası değerlendirmeler, matematik başarısını birden fazla alt alandaki becerilerin bütünlüğü olarak tanımlamaktadır. Örneğin, Ulusal Matematik Öğretmenleri Konseyi (NCTM) matematik dersine ait temel boyutları; sayı ve işlemler, geometri (görsel-uzamsal) ve veri analizi-olasılık gibi alt alanlar olarak almıştır (NCTM, 2000). Benzer şekilde, uluslararası olarak uygulanan PISA'da matematik okuryazarlığı çerçevesi de nicelik (sayı), uzay ve şekil (geometrik/uzamsal düşünme) ile belirsizlik ve veri olarak alt bileşenlere ayrılmıştır (OECD, 2019). Bu kapsamda, çalışmada ele alınan 7. sınıf düzeyi için geliştirilen matematik testinde yer alan sayı-işlem, görsel-uzamsal düşünme ve bilgi-veri okuryazarlığı alt becerileri, hem ulusal müfredat hem de uluslararası değerlendirme standartlarıyla uyumlu bir biçimde genel matematik yeteneğini tanımlayan kritik bileşenlerdir. Alanda yapılan çalışmalar incelendiğinde özellikle sayı algısı ve aritmetik işlem becerileri, öğrencilerin matematik performansının temel belirleyicileri olarak görülmektedir. Erken yaşlardan itibaren güçlü bir sayı hissine sahip olmak, ileriki matematik öğrenmelerini kolaylaştırmakta ve genel başarıyı artırmaktadır (Harç, 2010; Kayhan Altay, 2010). Hawes ve arkadaşlarının (2019) 4–11 yaş aralığında yaptığı yapısal eşitlik modellemesine dayalı çalışmada, sayısal ve uzamsal becerilerin öğrencilerin matematik başarısındaki varyansın %84'ünü açıkladığını göstermiştir. Bu çalışmada her ne kadar yürütücü işlevler de modele dahil edilmişse de, yalnızca sayısal ve görsel-uzamsal performansın matematik başarısının anlamlı yordayıcıları olduğu bulunmuştur (Hawes et al., 2019). Diğer bir deyişle, sayısal işlemlerde yetkin olan öğrenciler genellikle matematik testlerinde başarılı olurken, bu etkinin önemli bir kısmı aynı zamanda görsel ve uzamsal düşünme becerileri ile bağlantılıdır. Bununla beraber, bilgi-veri okuryazarlığı ve istatistiksel düşünme becerisi de günümüz matematik eğitiminde giderek daha fazla önem kazanmaktadır. Uluslararası yapılan değerlendirmeler neticesinde de, öğrencilerin istatistiksel bilgileri anlama ve yorumlama becerisinin, günlük yaşam durumlarına ait matematiksel düşünme becerileri için gerekli bir alan becerisi olduğu ortaya koyulmuştur (OECD, 2019). Dolayısıyla, bilgi-veri okuryazarlığı güçlü olan öğrencilerin, problem çözme ve veriye dayalı karar verme gerektiren matematik sorularında daha başarılı performans sergilemeleri beklenir.

Bu çalışmada tercih edilen metodolojik yaklaşım, MTK tabanlı yetenek kestirimini ve çoklu doğrusal regresyon analizini bir arada kullanarak genel yetenek ile alt becerilerdeki yetenek kestirimleri arasındaki ilişkiyi ortaya koymaktır. MTK, öğrencilerin yetenek düzeylerini örtük değişken (θ) olarak modele dahil edip test maddelerinin özelliklerini de hesaba katarak, klasik test puanlarından daha hassas bir yetenek tahmini yapmaya olanak tanır. Öyle ki, geniş ölçekli sınavlarda (ör. PISA, TIMSS) da öğrencilerin matematik okuryazarlığı düzeylerini belirlemek için MTK'ya dayalı modeller tercih edilmektedir (OECD, 2019). Dahası, MTK'nın çok boyutlu uzantıları (MIRT, bifaktör IRT gibi) bir testin birincil yetenek boyutunun yanı sıra ikincil içerik boyutlarını da (alt becerileri) aynı anda modelleyebilmeyi mümkün kılar (Erdemir & Atar, 2020). Örneğin TIMSS 2015 matematik testinin maddeleri sayı, cebir, geometri ve veri-olasılık şeklinde dört içerik alanına ayrılmış ve bu alt alanlara ilişkin puanlar, toplam yetenek puanıyla birlikte aynı anda kestirilebilmiştir (Erdemir & Atar, 2020). Yapılan karşılaştırmalar, bu tür eşzamanlı genel ve alt becerilere ait kestiriminin güvenilir bir şekilde

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yapılabildiğini ve özellikle tanıma yönelik geribildirim sağlamada faydalı olduğunu ortaya koymaktadır (Haberman & Sinharay, 2010).

Sonuç olarak, ilgili araştırmalar hem Türkiye'de hem de uluslararası alanda, ortaokul öğrencilerinin matematik başarısının bu üç temel alt beceriyle yakından ilişkili olduğunu ortaya koymaktadır. Matematiksel düşünme becerilerini, tek boyutlu bir toplam puan yerine çok boyutlu bir yapı olarak ele almak, öğrenci başarısını daha derinlemesine anlamamızı sağlayabilir. Bu bağlamda, MTK tabanlı yetenek kestirimi ile çoklu doğrusal regresyon analizinin bir arada kullanılması, 7. sınıf öğrencilerinin genel matematik yeteneği (θ) ile sayı-işlem, görsel-uzamsal düşünme ve bilgi-veri okuryazarlığı alt becerileri arasındaki ilişkinin incelenmesinde güçlü bir kuramsal ve yöntemsel zemin sunmaktadır.

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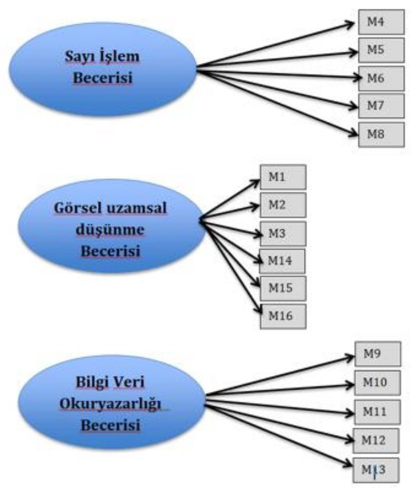
2. UYGULAMALAR

2.1. Araştırmanın Türü

Bu araştırma, nicel araştırma yöntemleri çerçevesinde yürütülmüş genel matematik başarısındaki varyansın en iyi hangi alt beceriler tarafından açıklandığını tespit etmeye yönelik olarak yapılmıştır. Bu çerçevede nicel araştırma türlerinden ilişkisel tarama desenine dayalı, psikometrik analiz (MTK) ve yordayıcı regresyon yöntemlerinin birlikte kullanıldığı bir eğitim araştırmasıdır (Creswell, 2012). Araştırmanın amacı, matematik alan becerilerini ölçen ikili (1-0) puanlanan 16 maddeden oluşan bir testten elde edilen 3138 öğrenciye ait yetenek puanlarını genel matematik başarısı olarak alıp hangi alt becerilerin genel başarıyı en iyi yordadığını ortaya koymaktır.

2.2. Veri Toplama Aracı ve Örneklem

120K850 No.lu 'Çevrimiçi Bilişsel Tanıya Dayalı İzleme Modelinin Üst Düzey Düşünme Becerilerine Etkisi' isimli TÜBİTAK projesi kapsamında 7. Sınıflar için hazırlanan ve matematik alan becerilerini ölçen ikili puanlanan 16 maddelik bir test ölçme aracı olarak kullanılmıştır. Matematik testindeki alan becerileri sayı işlem, görsel uzamsal düşünme ve bilgi veri okuryazarlığı becerisi olarak belirlenmiştir. Becerilerin ölçüldüğü maddeler alan uzmanlarının görüşleri doğrultusunda düzenlenmiş olup Görsel 1.'de verilmiştir. Araştırma evrenini Temel Eğitim Genel Müdürlüğü 2019-2020 verilerine göre 16 917 resmi ortaokulda öğrenim gören 5 354 069 öğrenci oluşturmaktadır. Araştırma örneklemini seçkisiz tabakalı örnekleme yöntemiyle seçilen ve 7. sınıfa devam eden 3138 öğrenci oluşturmaktadır.



Görsel 1. Becerilerin ölçüldüğü maddeler

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2.3. Veri Analizi

Veri analizleri, R (sürüm 4.5.0; R Core Team, 2025) ve RStudio (sürüm 2025.05.1+513 "Mariposa Orchid"; RStudio Team, 2025) kullanılarak gerçekleştirilmiştir. Yetenek kestirimi için kullanılan MTK analizleri 2 parametreli (2PL) modele göre gerçekleştirilmiştir. Öğrenci yetenekleri Bayesci bir tahminleyici olan "EAP", (Expected A Posteriori) tahminleyicisi kullanılarak elde edilmiştir. 2019-2020 eğitim öğretim döneminde 7. sınıfa devam eden 3138 öğrencinin 16 maddelik matematik testine verdikleri yanıtlardan oluşan veri seti kullanılarak öğrencilerin genel matematik yetenek kestirimleri elde edilmiştir. Daha sonra Sayı işlem becerisini ölçen 5 maddelik, görsel uzamsal düşünme becerisini ölçen 6 maddelik ve bilgi-veri okuryazarlığı becerisini ölçen 5 maddelik alt testlerden öğrenci yetenekleri kestirilmiştir. Genel matematik yetenek puanları bağımlı değişken, alt becerilerden elde edilen yetenek puanları bağımsız değişken olarak kabul edilerek çoklu doğrusal regresyon analizi yapılmıştır. Ayrıca LMG (Lindeman–Merenda–Gold) çoklu doğrusal regresyonda her bir yordayıcının modelin toplam R^2'sine yaptığı katkıyı elde etmek için LMG göreli önem analizi yapılmıştır.

3. SONUÇLAR VE DEĞERLENDİRME

Öğrencilerin genel matematik yetenek puanları (θ_G) ve alt becerilere ait yetenek puanları (Sayı işlem becerisi:θ_F1, Görsel uzamsal düşünme becerisi: θ_F2, Bilgi veri okuryazarlığı becerisi: θ_F3) 2 parametreli MTK modeli ile hesaplanmıştır. Yetenek hesaplamalarında EAP **(**Expected A Posteriori) tahminleyicisi kullanılmıştır. Öğrencilerin genel yetenek puanlarını en iyi hangi becerinin yordadığını ortaya koymak için çoklu doğrusal regresyon analizi yapılmıştır.

3.1. Varsayımların Kontrolü

Regresyon analizine geçmeden önce veri setinin varsayımları sağlayıp sağlamadığı kontrol edilmiştir. Varsayım kontrollerinde artıkların normalliği Shapiro–Wilk testiyle sınanmış ve normallikten istatistiksel olarak sapma görülmüştür, $W = 0.99$, $p < .001$ (Shapiro & Wilk, 1965). Bununla birlikte varyansın değişmezliği varsayımı test edilmiş ve heteroskedastisiteye işaret eden bir bulgu elde edilmiştir, $BP(3) = 85.859$, $p < .001$ (Breusch & Pagan, 1979). Bu nedenle tahmin hatalarındaki olası varyans değişimine karşı duyarlı olmayan bir çıkarım elde etmek amacıyla, katsayıların anlamlılık testleri ve HC3 tipi heteroskedastisiteye dayanıklı standart hatalar ile raporlanmıştır (MacKinnon & White, 1985). HC3 (Heteroskedasticity-Consistent Standard Errors, Type 3), özellikle küçük ve orta büyüklükte örneklemlerde tip I hata oranını daha iyi kontrol etmesiyle önerilmektedir ancak büyük örneklemlerde ise tutucu bir seçenektir (Long & Ervin, 2000). Çoklu bağlantılılık varsayımı VIF (Variance Inflation Factor) ile değerlendirilmiş ve Çizelge 1'deki değerler elde edilmiştir.

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Çizelge 1: Yordayıcılara ait VIF Değerleri

Yordayıcı	VIF
θ_F1	1.34
θ_F2	1.29
θ_F3	1.23

Çizelge 1'deki değerlerin 1.23–1.34 aralığında olduğu görülmektedir (*VIF*_θF1 = 1.34; *VIF*_θF2 = 1.29; *VIF*_θF3 = 1.23). Bu değerler düşük çoklu bağlantılılık olarak yorumlanır (Fox & Weisberg, 2018). Etkili gözlemler Cook uzaklığı ile incelenmiş ve hiçbir gözlem için $D > 1$ eşiği aşılmadığı görülmüştür. Gözlenen en büyük değer yaklaşık olarak 0.006 olarak elde edilmiştir. Bu bulgu, tekil gözlemlerin model uyumunu ve katsayıları önemli biçimde yönlendirmediğini düşündürmektedir (R. Cook, 1982; R. D. Cook, 1977).

Özet olarak normallikten sapma ve heteroskedastisite varlığı, HC3 sağlam standart hatalar ile telafi edilmiştir. Çoklu bağlantı düşüktür ve etkili gözlemler model sonuçlarını anlamlı ölçüde bozmamaktadır. Bu nedenle katsayıların istatistiksel anlamlılık değerlendirmeleri HC3'e göre raporlanmıştır (Long & Ervin, 2000; MacKinnon & White, 1985; White, 1980).

3.2. Regresyon Analizi

Çoklu doğrusal regresyon analizi, genel matematik başarısının (θ_G) üç alt beceriye ait yetenek kestirimiyle (θ_F1: Sayı-işlem becerisi, θ_F2: Görsel-uzamsal beceri, θ_F3: Bilgi-veri okuryazarlığı becerisi) anlamlı biçimde açıklandığını göstermiştir. Modelin uyumu yüksektir, $R^2 = 0.99$ (Düzeltilmiş $R^2 = 0.99$); artıkların standart hatası = 0.09. HC3 sağlam standart hatalarla elde edilen katsayılara göre üç alt becerinin her biri genel matematik yetenek puanları (θ_G) üzerinde anlamlı ve pozitif etkiye sahiptir. Elde edilen sonuçlar Çizelge 2'de verilmiştir.

Çizelge 2: Çoklu Doğrusal Regresyon Sonuçları

Yordayıcı	**b**	**SH**	**t**	**β (std.)**	**Partial R²**
Sabit	0.00	0.00156	0.01		
θ_F1	0.63	0.00240	264.45*	0.57	0.96
θ_F2	0.49	0.00236	207.93*	0.41	0.92
θ_F3	0.42	0.00274	152.97*	0.29	0.86
*p<0.01					

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Çizelge 2 incelendiğinde model sabitinin 0'dan anlamlı biçimde farklı olmadığı görülmektedir (**b = 0.00, SH = 0.00156, t(3134) = 0.01, p = .995**). Bu bulgu MTK yetenek puanlarının 0 ortalamaya ölçeklenmesiyle tutarlıdır. Sayı-işlem becerisi için $b_{\theta\ F1}$= 0.63, SH = 0.00240, t(3134) = 264.45, p < .001; görsel-uzamsal düşünme becerisi için $b_{\theta\ F2}$ = 0.49, SH = 0.00236, t(3134) = 207.93, p < .001; bilgi-veri okuryazarlığı becerisi için $b_{\theta\ F3}$ = 0.41943, SH = 0.00274, t(3134) = 152.97, p < .001 olarak elde edilmiştir. Standartlaştırılmış katsayılar (β) ve partial R^2 değerleri, göreli büyüklük bakımından büyükten küçüğe doğru sayı-işlem becerisi (β = 0.57, partial R^2 = 0.96), Görsel-uzamsal düşünme becerisi (β = 0.41, partial R^2 = 0.92) ve bilgi-veri okuryazarlığı becerisi (β = 0.29, partial R^2 = 0.86) olarak sıralanmaktadır.

3.2. LMG Göreli Önem Analizi

LMG (Lindeman–Merenda–Gold) çoklu doğrusal regresyonda her bir yordayıcının modelin toplam R^2'sine yaptığı katkıyı verir. Bunu yaparken değişken sırasına duyarlı olmamak için tüm olası eklenme sıralarını dener ve her değişken için ek artış R^2 değerlerini ortalar**.** Böylece çoklu bağlantılılık sorunu olsa bile katkılar toplamsal ve yorumlanabilir olmaktadır. Çizelge 3'te LMG analizi sonuçları verilmiştir.

Çizelge 3: LMG Göreli Önem Analizi Sonuçları

Beceri	LMG (R^2 payı)	%	R^2 içindeki göreli pay (%)
F1	0.46	45.70	46.22
F2	0.32	31.65	32.02
F3	0.21	21.51	21.76
Toplam	0.99	98.87	100.00

LMG göreli önem analizine göre genel matematik yetenek puanlarındaki toplam açıklanan varyansın yaklaşık %45.70'inin sayı-işlem becerisi, %31.65'inin görsel-uzamsal beceri ve %21.51'inin bilgi-veri okuryazarlığı becerisi tarafından açıklandığını göstermektedir. Genel matematik yeteneğinin açıklanmasında en büyük katkı sayı-işlem (F1) becerisindedir. (LMG=0.46; %46.22). Bunu Görsel-uzamsal beceri (F2) (LMG=0.32; %32.02) ve Bilgi-veri okuryazarlığı becerisi (F3) (LMG=0.215; %21.76) izlemektedir.

4. GENEL DEĞERLENDİRME VE SONUÇLAR

Bu çalışmada, 7. sınıf düzeyinde geliştirilen 16 maddelik matematik testinden MTK-2PL ile elde edilen yetenek kestirimleri kullanılarak genel matematik yeteneği (θ_G) ile üç alt beceri (Sayı-işlem, Görsel-uzamsal düşünme, Bilgi-veri okuryazarlığı) arasındaki ilişkiyi çoklu doğrusal regresyon ve LMG göreli önem analiziyle incelenmiştir. Varsayımlar, Shapiro–Wilk

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ile normallik, Breusch–Pagan ile varyansın eşitliği, VIF ile çoklu bağlantılılık, Cook uzaklığı bakılarak test edilmiştir. Ayrıca heteroskedastisiteye karşı HC3 hesaplanmıştır.

Analiz bulguları üç temel sonuca işaret etmektedir: Bunlardan ilki; üç alt becerinin tamamı genel matematik başarısını anlamlı ve pozitif biçimde yordamaktadır. İkincisi hem standartlaştırılmış katsayılara (β_Sayı-işlem=0.57; β_Görsel-uzamsal=0.41; β_Bilgi-veri=0.29) hem de partial R^2 değerlerine (sırasıyla .96, .92, .86) göre en güçlü yordayıcı Sayı-işlem becerisidir. Son olarak, LMG sonuçları da aynı sıralamayı desteklemektedir. Bu sonuçlara göre toplam açıklanan varyansın yaklaşık %45.7'sini Sayı-işlem, %31.7'sini Görsel-uzamsal, %21.5'ini ise Bilgi-veri okuryazarlığı becerisinin açıkladığını göstermektedir. Bu sıralama, literatürde erken dönem sayısal yeterliklerin merkezi rolünü vurgulayan bulgularla tutarlıdır. Ayrıca görsel-uzamsal ve veri-okuryazarlığı bileşenlerinin anlamlı katkıları, matematiksel yeterliğin çok boyutlu doğasını desteklemektedir.

Kuramsal açıdan, sonuçlar önemli çıkarımlar sunar. Örneğin tek boyutlu toplam puan yerine çok-boyutlu yetenek yapısının modellenmesi, öğrencilerin matematik performansını açıklamada daha ayrıntılı bir çerçeve sağlayabilir. Bir başka çıkarım, aynı testten elde edilen genel ve alt-beceri yeteneklerinin yüksek düzeyde ilişkili olması beklenir. Ancak LMG ile göreceli önem analizi, alt alanların özgün katkısını ayrıştırmaya olanak tanımıştır. Bu, eğitimsel karar verme süreçlerinde hangi becerilerin önceliklendirilmesi gerektiğine dair kanıt sağlayabilir. Özellikle matematik alanında öğrenmenin hiyerarşik doğasına da bir gösterge olabilir.

Bununla birlikte, çalışma belirli sınırlılıkları da barındırmaktadır. Örneğin; nedensel yorumlar dikkatle yapılmalıdır. Çünkü yetenek kestirimleri 2PL ve EAP çerçevesiyle, dikotomik puanlar üzerinden elde edilmiştir; politomik puanlama ya da MIRT/bifaktör yaklaşımlar dağılım varsayımları ve boyutsallık açısından farklı sonuçlar üretebilir. Genel ve alt-beceri kestirimleri aynı testten elde edildiği için yapısal örtüşme ve ölçme hatası ortaklığı kaçınılmazdır. Bu durum da R^2 değerlerini kısmen yükseltebilir. Başka bir sınırlılık da üç alt becerinin seçimi kuramsal ve müfredat temelli olsa da, matematik başarısının başka boyutları (akıl yürütme süreçleri, strateji kullanımı, yürütücü işlevler vb.) modele dahil edilebilir.

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AN ENHANCED MEREC-COPRAS METHOD BASED ON COMPLEX SPHERICAL FUZZY SETS AND ITS APPLICATION IN HEALTHCARE

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ABSTRACT

In today's rapidly changing global environment, decision-making processes are becoming increasingly complex, particularly in sectors such as healthcare. Various decision-making methods have been developed to address such problems. This study introduces the MEREC and COPRAS methods integrated with complex spherical fuzzy sets for solving multi-criteria decision-making problems. The theory of complex spherical fuzzy sets is a generalization of fuzzy set theory that allows consideration of both one-dimensional and two-dimensional data, offering significant advantages in managing decision-making processes characterized by uncertainty and complexity. In this study, the MEREC method is employed to determine the weights of the criteria involved in the decision-making problem, thereby eliminating the influence of subjective opinions and enabling an objective evaluation. The COPRAS method is used to rank alternatives based on the established criteria. The proposed method is applied to a decision-making problem in the healthcare sector, contributing to the optimization of healthcare services through the selection of the best alternative. The results demonstrate that the MEREC-COPRAS methodology provides an effective solution for complex decision-making processes and can be utilized as a strategic decision support tool in the healthcare domain.

Keywords: MEREC, COPRAS, objective weighting, healtcare

1. INTRODUCTION

In today's rapidly evolving global environment, the healthcare sector is under increasing pressure to respond to growing patient demands, technological advancements, and resource constraints, all while maintaining high standards of service delivery. The complexity of healthcare systems requires robust decision-making frameworks that can accommodate both tangible and intangible factors, as well as uncertainty inherent in human judgment. Healthcare decision-making problems often involve multiple, conflicting criteria—such as cost, quality of care, patient satisfaction, safety, and technological capabilities—which must be evaluated simultaneously. These types of problems are well-suited to Multi-Criteria Decision-Making (MCDM) approaches, which have been extensively used in the healthcare domain to support decisions such as hospital site selection, medical device evaluation, treatment planning, resource allocation, and performance assessment of healthcare providers. In situations involving multiple stakeholders or experts, the problem evolves into a Multi-Criteria Group Decision-Making (MCGDM) context.

To handle the ambiguity and vagueness present in healthcare data and expert evaluations, fuzzy set theory-originally proposed by Zadeh [1]-has been widely applied in MCDM models. However, classical fuzzy sets, which only account for the degree of membership, may not be sufficient to capture the full spectrum of uncertainty in complex healthcare decisions. To overcome this limitation, various extensions of fuzzy set theory have been developed. To address this limitation, intuitionistic fuzzy sets (IFSs) were proposed by Atanassov [2], incorporating both membership and non-membership degrees, thus enabling a richer representation of uncertainty. Building upon IFSs, picture fuzzy sets (PFSs) introduced by Cuong [3] added a third dimension—neutral membership—allowing for a more nuanced handling of neutral or hesitant opinions in decision-making processes. Among these, Spherical Fuzzy Sets (SFSs) given by Kahraman and Kutlu Gündoğdu [4] stand out due to their ability to represent three-dimensional uncertainty—membership, non-membership, and hesitancy degrees—under the constraint that the sum of their squares does not exceed one. This makes SFSs particularly effective in complex environments like healthcare, where uncertainty, vagueness, and hesitation are prevalent. Various studies ([5], [6]) have demonstrated the superiority of SFSs in providing more flexible and accurate representations of decision-makers' evaluations.

The mentioned fuzzy set theory and its extensions are highly proficient in handling imprecise information, their capability remains constrained when it comes to modeling two-dimensional data, which frequently occurs in practical MCDM scenarios. To address this shortcoming, complex fuzzy sets (CFSs) were introduced by Ramot et al. [7], incorporating both amplitude and phase components to capture two-dimensional information. Building upon this, complex intuitionistic fuzzy sets (CIFSs) were proposed by Alkouri and Salleh [8] to include positive and negative membership degrees. However, CIFSs suffer from structural limitations when strict constraints on amplitude and phase are violated. In response, complex Pythagorean fuzzy sets (CPyFSs) were developed by Ullah et al. [9] to relax these constraints, offering a broader framework—but still falling short in representing neutral or abstention preferences. To overcome these limitations, the concept of complex picture fuzzy sets (CPFSs) was proposed, integrating positive, neutral, and negative membership degrees in the complex domain, thereby enhancing the ability to model uncertainty and hesitation more accurately. However, CPFSs still could not incorporate the concept of refusal, which is often present in human decision-

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making. To fill this gap, Ali et al. [10] and Akram et al. [11] introduced the complex spherical fuzzy set (CSFS) theory, which combines the strengths of complex fuzzy models with the spherical constraint on membership degrees. CSFSs are defined by complex-valued positive, neutral, and negative membership degrees such that the sum of the squares of their amplitudes does not exceed one. This model is capable of expressing all four dimensions of human judgment: acceptance (yes), rejection (no), hesitation (abstention), and lack of opinion (refusal), while preserving two-dimensional geometric integrity through its complex structure. Thanks to this flexible and expressive structure, CSFSs have been successfully applied to a range of MCGDM problems under uncertainty. Their enhanced modeling capacity makes them particularly suitable for use in healthcare decision-making, supplier selection, technology evaluation, and other domains where subjectivity and multidimensional data coexist.

One of the most important steps in MCDM/MCGDM methods is the determination of criteria weights, as the relative importance assigned to each criterion directly influences the ranking of alternatives. While many subjective and objective methods exist for this purpose, objective methods have gained attention for their data-driven and unbiased nature. Among these, the Method based on the Removal Effects of Criteria (MEREC), introduced by Keshavarz-Ghorabaee et al. [12], has emerged as a powerful tool. MEREC evaluates the impact of removing each criterion on the overall performance of the alternatives, thereby objectively quantifying its importance. The core advantage of MEREC lies in its ability to identify the most influential criteria without requiring input from decision-makers, which is particularly valuable in sensitive domains like healthcare, where subjective bias can significantly affect outcomes.

Among the various MCDM techniques, the COmplex PRoportional ASsessment (COPRAS) method, introduced by Zavadskas et al. [13] has proven to be a robust and efficient tool for solving complex evaluation problems. The distinguishing feature of COPRAS lies in its ability to simultaneously consider both beneficial (maximizing) and non-beneficial (minimizing) criteria, which allows for a comprehensive analysis of alternatives. COPRAS operates under a compensatory decision logic, meaning that strong performance in one criterion can compensate for weaker performance in another, making it particularly suitable for real-world problems where trade-offs are inevitable. Additionally, the method converts qualitative assessments into quantitative measures, enhancing its applicability in scenarios where expert opinions are based on linguistic terms or subjective evaluations. In the healthcare domain, COPRAS has been effectively applied to a range of problems such as hospital site selection, medical equipment evaluation, healthcare service quality assessment, and treatment method selection ([14], [15]). Its transparent calculation process, ease of implementation, and adaptability to different fuzzy set environments have made COPRAS a preferred choice among researchers and practitioners for multi-criteria evaluations in uncertain and complex decision contexts. In literature, there are lots of important studies related to the COPRAS method ([16]-[21]).

The novelty of this study is grounded in the integration of the MEREC-based objective weighting method with the COPRAS decision-making technique within the CSFS environment, a framework capable of representing multidimensional uncertainty more effectively than traditional fuzzy set models. CSFSs provide a rich and nuanced mathematical tool to express positive, neutral, negative, and refusal opinions simultaneously, making them highly suitable

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for real-world problems characterized by ambiguity and imprecision—particularly in healthcare decision-making contexts. What further distinguishes this study is its application to a critical healthcare problem that emerged during the COVID-19 pandemic: identifying individuals who are at the highest risk of contracting the virus. In a time of widespread uncertainty and limited resources, the ability to objectively determine criteria weights (via MEREC) and rank individuals based on multi-criteria risk factors (via COPRAS) has immense practical value. Traditional models often rely heavily on subjective judgments, which may lead to inconsistencies in such high-stakes contexts. By eliminating subjective bias and enabling the inclusion of complex two-dimensional data, the proposed CSFS–MEREC–COPRAS framework offers a novel, systematic, and data-driven approach to support strategic healthcare decisions.

2. PRELIMINARIES

This section reviews several basic definitions that will serve as the foundation for the subsequent sections. Throughout the paper, X represents the universal set.

Definition 1. [10, 11] Let $f, g, h: X \to [0,1]$, $\alpha, \beta, \gamma: X \to [0,2\pi]$ and $i = \sqrt{-1}$. A complex spherical fuzzy set (CSFS) over X is of the form

$$C = \{x, f(x)e^{i\alpha(x)}, g(x)e^{i\beta(x)}, h(x)e^{i\gamma(x)} |\, x \in X\}$$

if the conditions $f^2(x) + g^2(x) + h^2(x) \leq 1$ and $\left(\frac{\alpha(x)}{2\pi}\right)^2 + \left(\frac{\beta(x)}{2\pi}\right)^2 + \left(\frac{\gamma(x)}{2\pi}\right)^2 \leq 1$ are satisfied for all $x \in X$. The functions $\mathscr{f}(x) = f(x)e^{i\alpha(x)}$, $\mathscr{g}(x) = g(x)e^{i\beta(x)}$ and $\mathscr{h}(x) = h(x)e^{i\gamma(x)}$ denote the positive-membership, neutral-membership and negative-membership of x to C, which are restricted to the unit circle and consists of two terms such as amplitude term and phase term. The refusal function is given by

$$t(x) = \sqrt{1 - f^2(x) - g^2(x) - h^2(x)}e^{i2\pi\sqrt{1-\left(\frac{\alpha(x)}{2\pi}\right)^2-\left(\frac{\beta(x)}{2\pi}\right)^2-\left(\frac{\gamma(x)}{2\pi}\right)^2}}$$

for all $x \in X$. We denote the family of CSFSs over X with CSFS(X).

The triplet $C = (\mathscr{f}, \mathscr{g}, \mathscr{h})$ is called a complex spherical fuzzy number (CSFN) where $\mathscr{f} = fe^{i\alpha}$, $\mathscr{g} = ge^{i\beta}$ and $\mathscr{h} = he^{i\gamma}$ for $f, g, h \in [0,1]$, $\alpha, \beta, \gamma \in [0,2\pi]$ satisfying $f^2 + g^2 + h^2 \leq 1$ and $\left(\frac{\alpha}{2\pi}\right)^2 + \left(\frac{\beta}{2\pi}\right)^2 + \left(\frac{\gamma}{2\pi}\right)^2 \leq 1,$

Definition 2. [11] The complement of the CSFS C is denoted by C^c and given as follows

$$C^c = \{x, h(x)e^{i\gamma(x)}, g(x)e^{i\beta(x)}, f(x)e^{i\alpha(x)} |\, x \in X\}$$

Definition 3. [11] Let $C = (fe^{i\alpha}, ge^{i\beta}, he^{i\gamma})$, $C_1 = (f_1e^{i\alpha_1}, g_1e^{i\beta_1}, h_1e^{i\gamma_1})$, $C_2 = (f_2e^{i\alpha_2}, g_2e^{i\beta_2}, h_2e^{i\gamma_2})$ be three CSFNs and $a \geq 0$. Then the operations between CSFNs are defined as follows:

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(i) $$C_1 \oplus C_2 = \begin{pmatrix} \sqrt{f_1^2+f_2^2-f_1^2f_2^2}\, e^{i2\pi\sqrt{\left(\frac{\alpha_1}{2\pi}\right)^2+\left(\frac{\alpha_2}{2\pi}\right)^2-\left(\frac{\alpha_1}{2\pi}\right)^2\left(\frac{\alpha_2}{2\pi}\right)^2}}, \\ \sqrt{(1-f_1^2)g_2^2+(1-f_2^2)g_1^2-g_1^2g_2^2}\, e^{i2\pi\sqrt{\left(1-\left(\frac{\alpha_1}{2\pi}\right)^2\right)\left(\frac{\beta_2}{2\pi}\right)^2+\left(1-\left(\frac{\alpha_2}{2\pi}\right)^2\right)\left(\frac{\alpha_1}{2\pi}\right)^2-\left(\frac{\beta_1}{2\pi}\right)^2\left(\frac{\beta_2}{2\pi}\right)^2}}, \\ h_1h_2e^{i2\pi\left(\frac{\gamma_1}{2\pi}\right)\left(\frac{\gamma_2}{2\pi}\right)} \end{pmatrix}$$

(ii) $$C_1 \oplus C_2 = \begin{pmatrix} f_1f_2e^{i2\pi\left(\frac{\alpha_1}{2\pi}\right)\left(\frac{\alpha_1}{2\pi}\right)}, \\ \sqrt{(1-h_1^2)g_2^2+(1-h_2^2)g_1^2-g_1^2g_2^2}\, e^{i2\pi\sqrt{\left(1-\left(\frac{\gamma_1}{2\pi}\right)^2\right)\left(\frac{\beta_2}{2\pi}\right)^2+\left(1-\left(\frac{\alpha_2}{2\pi}\right)^2\right)\left(\frac{\gamma_1}{2\pi}\right)^2-\left(\frac{\beta_1}{2\pi}\right)^2\left(\frac{\beta_2}{2\pi}\right)^2}}, \\ \sqrt{h_1^2+h_2^2-h_1^2h_2^2}\, e^{i2\pi\sqrt{\left(\frac{\gamma_1}{2\pi}\right)^2+\left(\frac{\gamma_2}{2\pi}\right)^2-\left(\frac{\gamma_1}{2\pi}\right)^2\left(\frac{\gamma_2}{2\pi}\right)^2}} \end{pmatrix}$$

(iii) $$aC = \begin{pmatrix} \sqrt{1-(1-f^2)^a}\, e^{2\pi\sqrt{1-\left(1-\left(\frac{\alpha}{2\pi}\right)^2\right)^a}}, \\ \sqrt{\left(1-f^2\right)^a-\left(1-f^2-g^2\right)^a}\, e^{2\pi\sqrt{\left(1-\left(\frac{\alpha}{2\pi}\right)^2\right)^a-\left(1-\left(\frac{\alpha}{2\pi}\right)^2-\left(\frac{\beta}{2\pi}\right)^2\right)^a}}, \\ h^ae^{i2\pi\left(\frac{\gamma}{2\pi}\right)^a} \end{pmatrix}$$

(iv) $$C^a = \begin{pmatrix} f^ae^{i2\pi\left(\frac{\alpha}{2\pi}\right)^a}, \\ \sqrt{\left(1-h^2\right)^a-\left(1-h^2-g^2\right)^a}\, e^{2\pi\sqrt{\left(1-\left(\frac{\gamma}{2\pi}\right)^2\right)^a-\left(1-\left(\frac{\gamma}{2\pi}\right)^2-\left(\frac{\beta}{2\pi}\right)^2\right)^a}}, \\ \sqrt{1-(1-h^2)^a}\, e^{2\pi\sqrt{1-\left(1-\left(\frac{\gamma}{2\pi}\right)^2\right)^a}} \end{pmatrix}$$

Definition 4. [11] Let $\mathcal{C}$ be a family of the CSFNs and $(C_1, C_2, \ldots, C_n) \in \mathcal{C}^n$ where $C_k = \left(f_ke^{i\alpha_k}, g_ke^{i\beta_k}, h_ke^{i\gamma_k}\right)$ for all $k = 1,2,\ldots,n$ and $w = (w_1, w_2, \ldots, w_n)^T$ be the weight vector corresponding to $(C_k)_{k=1}^n$ such that $w_k \geq 0$ for all k and $\sum_{k=1}^n w_k = 1$. A mapping $CSFWA_w: \mathcal{C}^n \to \mathcal{C}$ is said to be a complex spherical fuzzy weighted average (CSFWA) operator and is defined by

$$CSFWA_w(C_1, C_2, \ldots, C_n) = w_1C_1 \oplus w_2C_2 \oplus \ldots \oplus w_nC_n = \oplus_{k=1}^n w_kC_k. \tag{1}$$

Theorem 1. [11] Let $(C_1, C_2, \ldots, C_n) \in \mathcal{C}^n$. Then the aggregated value $CSFWA_w(C_1, C_2, \ldots, C_n)$ is also a CSFN and is calculated by

$$CSFWA_w(C_1, C_2, \ldots, C_n) = \oplus_{k=1}^n w_kC_k$$

$$= \begin{pmatrix} \sqrt{1-\prod_{k=1}^n(1-f_k^2)^{w_k}}\, e^{2\pi\sqrt{1-\prod_{k=1}^n\left(1-\left(\frac{\alpha_k}{2\pi}\right)^2\right)^{w_k}}}, \\ \sqrt{\prod_{k=1}^n\left(1-f_k^2\right)^{w_k}-\prod_{k=1}^n\left(1-f_k^2-g_k^2\right)^{w_k}}\, e^{2\pi\sqrt{\prod_{k=1}^n\left(1-\left(\frac{\alpha_k}{2\pi}\right)^2\right)^{w_k}-\prod_{k=1}^n\left(1-\left(\frac{\alpha_k}{2\pi}\right)^2-\left(\frac{\beta_k}{2\pi}\right)^2\right)^{w_k}}}, \\ \prod_{k=1}^n h_k^{w_k}\, e^{i2\pi\prod_{k=1}^n\left(\frac{\gamma_k}{2\pi}\right)^{w_k}} \end{pmatrix} \tag{2}$$

where $w = (w_1, w_2, \ldots, w_n)^T$ is the weight vector corresponding to $(C_k)_{k=1}^n$ such that $w_k \geq 0$ for all k and $\sum_{k=1}^n w_k = 1$.

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Definition 5. [11] Let $\mathcal{C}$ be the family of the SFNs and $C, C_1, C_2 \in \mathcal{C}$ where $C = (fe^{i\alpha}, ge^{i\beta}, he^{i\gamma})$ and $C_k = \left(f_k e^{i\alpha_k}, g_k e^{i\beta_k}, h_k e^{i\gamma_k}\right)$ for $k = 1,2$.

(1) A score function on $\mathcal{C}$ ($SF: \mathcal{C} \to [0,2]$) and an accuracy function on $\mathcal{C}$ ($AF: \mathcal{C} \to [0,2]$) are given as

$$SF(C) = \frac{1}{3}\left(4 + f^2 - g^2 - h^2 + \left(\frac{\alpha}{2\pi}\right)^2 - \left(\frac{\beta}{2\pi}\right)^2 - \left(\frac{\gamma}{2\pi}\right)^2\right),$$

$$AF(C) = \frac{1}{3}\left(4 + f^2 + g^2 + h^2 + \left(\frac{\alpha}{2\pi}\right)^2 + \left(\frac{\beta}{2\pi}\right)^2 + \left(\frac{\gamma}{2\pi}\right)^2\right).$$

(2) The ranking procedure (comparison approach) based on score/accuracy functions is defined as follows:

(i) If $SF(C_1) < SF(C_1)$, then $C_1 \prec C_2$,

(ii) If $SF(C_1) > SF(C_2)$, then $C_1 \succ C_2$,

(iii) $SF(C_1) = SF(C_2)$, then

(a) If $AF(C_1) < AF(C_1)$, then $C_1 \prec C_2$,

(b) If $AF(C_1) > AF(C_2)$, then $C_1 \succ C_2$,

(c) $AF(C_1) = AF(C_2)$, then $C_1 = C_2$.

3. METHOD

In this section, we introduce the MEREC-COPRAS method for solving MCGDM problems in a complex spherical fuzzy environment. Let $A = \{A_1, A_2, \dots, A_k\}$ be the set of k alternatives, $C = \{C_1, C_2, \dots, C_m\}$ be the set of m criteria and $E = \{E_1, E_2, \dots, E_n\}$ be set of n experts (DMs) hired for decision-making. Denote the weight of experts by ε_r where $\varepsilon_r \geq 0$ for all $r = 1,2,..,n$ and $\sum_{r=1}^{n} \varepsilon_r = 1$. Each expert E_r evaluate the alternatives A_p with respect to C_q by considering the influence of C_q on the alternatives A_p and by using the linguistic table given in Table 1.

Linguistic terms	**CSFNs**
Very anomalous effectiveness (VAE)	$(.94e^{i2\pi(.93)}, .26e^{i2\pi(.24)}, .19e^{i2\pi(.12)})$
Anomalous effectiveness (AE)	$(.81e^{i2\pi(.78)}, .36e^{i2\pi(.23)}, .25e^{i2\pi(.21)})$
Normal effectiveness (NE)	$(.58e^{i2\pi(.61)}, .44e^{i2\pi(.39)}, .48e^{i2\pi(.37)})$
Moderate effectiveness (ME)	$(.45e^{i2\pi(.41)}, .48e^{i2\pi(.43)}, .67e^{i2\pi(.61)})$
Poor effectiveness (PE)	$(.36e^{i2\pi(.35)}, .46e^{i2\pi(.39)}, .66e^{i2\pi(.62)})$
Very poor effectiveness (VPE)	$(.24e^{i2\pi(.18)}, .34e^{i2\pi(.31)}, .79e^{i2\pi(.89)})$

Table 1: Linguistic terms to evaluate the alternatives via criteria [22]

Then these values establish the complex spherical fuzzy decision matrix (CSFDM) $D^{(r)} = \left(d_{pq}^{(r)}\right)_{k\times m}$ where $d_{pq}^{(r)} = \left(f_{D_{pq}}^{(r)} e^{i\alpha_{D_{pq}}^{(r)}}, g_{D_{pq}}^{(r)} e^{i\beta_{D_{pq}}^{(r)}}, h_{D_{pq}}^{(r)} e^{i\gamma_{D_{pq}}^{(r)}}\right)$ for all $r \in \{1,2,\dots,n\}$. The CSFDM built by expert E_r is represented as follows:

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$$D^{(r)}=\begin{pmatrix}\left(\mathrm{f}_{\mathrm{D}_{11}}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\alpha_{\mathrm{D}_{11}}^{(r)}},\mathrm{g}_{\mathrm{D}_{11}}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\beta_{\mathrm{D}_{11}}^{(r)}},\mathrm{h}_{\mathrm{D}_{11}}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\gamma_{\mathrm{D}_{11}}^{(r)}}\right) & \ldots\left(\mathrm{f}_{\mathrm{D}_{1\mathrm{m}}}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\alpha_{\mathrm{D}_{1\mathrm{m}}}^{(r)}},\mathrm{g}_{\mathrm{D}_{1\mathrm{m}}}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\beta_{\mathrm{D}_{1\mathrm{m}}}^{(r)}},\mathrm{h}_{D_{1m}}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\gamma_{\mathrm{D}_{1\mathrm{m}}}^{(r)}}\right)\\ \left(\mathrm{f}_{\mathrm{D}_{21}}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\alpha_{\mathrm{D}_{21}}^{(r)}},\mathrm{g}_{\mathrm{D}_{21}}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\beta_{\mathrm{D}_{21}}^{(r)}},\mathrm{h}_{\mathrm{D}_{21}}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\gamma_{\mathrm{D}_{21}}^{(r)}}\right) & \ldots\left(\mathrm{f}_{\mathrm{D}_{2\mathrm{m}}}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\alpha_{\mathrm{D}_{2\mathrm{m}}}^{(r)}},\mathrm{g}_{\mathrm{D}_{2\mathrm{m}}}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\beta_{\mathrm{D}_{2\mathrm{m}}}^{(r)}},\mathrm{h}_{D_{2m}}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\gamma_{\mathrm{D}_{2\mathrm{m}}}^{(r)}}\right)\\ \ldots & \ldots \qquad \ldots\\ \left(\mathrm{f}_{\mathrm{D}_{\mathrm{k}1}}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\alpha_{\mathrm{D}_{\mathrm{k}1}}^{(r)}},\mathrm{g}_{\mathrm{D}_{\mathrm{k}1}}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\beta_{\mathrm{D}_{\mathrm{k}1}}^{(r)}},\mathrm{h}_{\mathrm{D}_{\mathrm{k}1}}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\gamma_{\mathrm{D}_{\mathrm{k}1}}^{(r)}}\right) & \ldots\left(\mathrm{f}_{\mathrm{D}_{\mathrm{km}}}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\alpha_{\mathrm{D}_{\mathrm{km}}}^{(r)}},\mathrm{g}_{\mathrm{D}_{\mathrm{km}}}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\beta_{\mathrm{D}_{\mathrm{km}}}^{(r)}},\mathrm{h}_{D_{km}}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\gamma_{\mathrm{D}_{\mathrm{km}}}^{(r)}}\right)\end{pmatrix}$$

The procedure of the new MEREC-COPRAS method consists of the following steps:

Step I: Given that the CSFDM framework can involve both benefit-type and cost-type criteria, it is essential to ensure that the evaluations provided by experts are expressed on a comparable scale. Therefore, as an initial step, the raw information collected from the experts undergoes a normalization process, which is carried out as follows:

$$s_{pq}^{(r)}=\begin{cases}d_{pq}^{(r)}, & for\ benefit\ criterion\ C_q\\ \left(d_{pq}^{(r)}\right)^c, & for\ non-benefit\ criterion\ C_q\end{cases} \tag{3}$$

for all $p=1,..,k,\ \ q=1,...,m$ and $\ r=1,...,n$ where $\left(d_{pq}^{(r)}\right)^c$ is the complement of $d_{pq}^{(r)}$. Hence, the normalized complex spherical fuzzy decision matrix (NCSFDM) $D_N^{(r)}=\left(s_{pq}^{(r)}\right)_{k\times m}$ where $s_{pq}^{(r)}=\left(f_{pq}^{(r)}e^{i\alpha_{pq}^{(r)}},g_{pq}^{(r)}e^{i\beta_{pq}^{(r)}},h_{pq}^{(r)}e^{i\gamma_{pq}^{(r)}}\right)$ for all $p=1,..,k,\ \ \ q=1,...,m$ and $\ r=1,...,n$, is written as follows:

$$D_N^{(r)}=\left(s_{pq}^{(r)}\right)_{k\times m}=\begin{pmatrix}\left(\mathrm{f}_{11}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\alpha_{11}^{(\mathrm{r})}},\mathrm{g}_{11}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\beta_{11}^{(r)}},\mathrm{h}_{11}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\gamma_{11}^{(r)}}\right) & \ldots\left(\mathrm{f}_{1\mathrm{m}}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\alpha_{1\mathrm{m}}^{(\mathrm{r})}},\mathrm{g}_{1\mathrm{m}}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\beta_{1\mathrm{m}}^{(r)}},\mathrm{h}_{1m}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\gamma_{\mathrm{D}_{1\mathrm{m}}}^{(r)}}\right)\\ \left(\mathrm{f}_{21}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\alpha_{21}^{(\mathrm{r})}},\mathrm{g}_{21}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\beta_{21}^{(r)}},\mathrm{h}_{21}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\gamma_{21}^{(r)}}\right) & \ldots\left(\mathrm{f}_{2\mathrm{m}}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\alpha_{2\mathrm{m}}^{(\mathrm{r})}},\mathrm{g}_{2\mathrm{m}}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\beta_{2\mathrm{m}}^{(r)}},\mathrm{h}_{2m}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\gamma_{2\mathrm{m}}^{(r)}}\right)\\ \ldots & \ldots \qquad \ldots\\ \left(\mathrm{f}_{\mathrm{k}1}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\alpha_{\mathrm{k}1}^{(\mathrm{r})}},\mathrm{g}_{\mathrm{k}1}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\beta_{\mathrm{k}1}^{(r)}},\mathrm{h}_{\mathrm{k}1}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\gamma_{\mathrm{D}_{\mathrm{k}1}}^{(r)}}\right) & \ldots\left(\mathrm{f}_{\mathrm{D}_{\mathrm{km}}}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\alpha_{\mathrm{km}}^{(\mathrm{r})}},\mathrm{g}_{\mathrm{km}}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\beta_{\mathrm{km}}^{(r)}},\mathrm{h}_{km}^{(\mathrm{r})}\mathrm{e}^{\mathrm{i}\gamma_{\mathrm{km}}^{(r)}}\right)\end{pmatrix}$$

Step II: To derive the collective judgment of the group, the individual assessments of all experts are integrated by incorporating their associated weights through the CSFWA operator. As a result of this aggregation process, the complex spherical fuzzy evaluations are combined into a single decision framework, leading to the construction of the aggregated complex spherical fuzzy decision matrix (ACSFDM), denoted by $D=\left(s_{pq}\right)_{k\times m}$, which is formulated as follows:

$$s_{pq}=CSFWA_{\varepsilon}\left(s_{pq}^{(1)},s_{pq}^{(2)},\ldots,s_{pq}^{(r)}\right)=\bigoplus_{r=1}^{n}\varepsilon_r s_{pq}^{(r)}$$

$$=\begin{pmatrix}\sqrt{1-\prod_{r=1}^{n}\left(1-\left(f_{pq}^{(r)}\right)^2\right)^{\varepsilon_r}}\,e^{2\pi\sqrt{1-\prod_{k=1}^{n}\left(1-\left(\frac{\alpha_{pq}^{(r)}}{2\pi}\right)^2\right)^{\varepsilon_r}}},\\ \sqrt{\prod_{k=1}^{n}\left(1-\left(f_{pq}^{(r)}\right)^2\right)^{\varepsilon_r}-\prod_{k=1}^{n}\left(1-\left(f_{pq}^{(r)}\right)^2-\left(g_{pq}^{(r)}\right)^2\right)^{\varepsilon_r}}\,e^{2\pi\sqrt{\prod_{k=1}^{n}\left(1-\left(\frac{\alpha_{pq}^{(r)}}{2\pi}\right)^2\right)^{\varepsilon_r}-\prod_{k=1}^{n}\left(1-\left(\frac{\alpha_{pq}^{(r)}}{2\pi}\right)^2-\left(\frac{\beta_{pq}^{(r)}}{2\pi}\right)^2\right)^{\varepsilon_r}}},\\ \prod_{k=1}^{n}\left(h_{pq}^{(r)}\right)^{\varepsilon_r}e^{i2\pi\prod_{k=1}^{n}\left(\frac{\gamma_{pq}^{(r)}}{2\pi}\right)^{\varepsilon_r}}\end{pmatrix} \tag{4}$$

If we denote $s_{pq}=\left(f_{A_p}(c_q)e^{i\alpha_{A_p}(c_q)},g_{A_p}(c_q)e^{i\beta_{A_p}(c_q)},h_{A_p}(c_q)e^{i\gamma_{A_p}(c_q)}\right)$ for all $p=1,2,\ldots,k$ and $q=1,2,\ldots,m$, then $D=\left(s_{pq}\right)_{k\times m}$ is represented as follows:

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$$D = (s_{pq})_{k\times m} = \begin{pmatrix} f_{A_1}(c_1)e^{i\alpha_{A_1}(c_1)}, g_{A_1}(c_1)e^{i\beta_{A_1}(c_1)}, h_{A_1}(c_1)e^{i\gamma_{A_1}(c_1)} \dots f_{A_1}(c_m)e^{i\alpha_{A_1}(c_m)}, g_{A_1}(c_m)e^{i\beta_{A_1}(c_m)}, h_{A_1}(c_m)e^{i\gamma_{A_1}(c_m)} \\ f_{A_2}(c_1)e^{i\alpha_{A_2}(c_1)}, g_{A_2}(c_1)e^{i\beta_{A_2}(c_1)}, h_{A_2}(c_1)e^{i\gamma_{A_2}(c_1)} \dots f_{A_2}(c_m)e^{i\alpha_{A_2}(c_m)}, g_{A_2}(c_m)e^{i\beta_{A_2}(c_m)}, h_{A_2}(c_m)e^{i\gamma_{A_2}(c_m)} \\ \dots \quad \dots . \\ f_{A_k}(c_1)e^{i\alpha_{A_k}(c_1)}, g_{A_k}(c_1)e^{i\beta_{A_k}(c_1)}, h_{A_k}(c_1)e^{i\gamma_{A_k}(c_1)} \dots f_{A_k}(c_m)e^{i\alpha_{A_k}(c_m)}, g_{A_k}(c_m)e^{i\beta_{A_k}(c_m)}, h_{A_k}(c_m)e^{i\gamma_{A_k}(c_m)} \end{pmatrix}.$$

Step III: The criteria weights are calculated via MEREC method as follows:

(I) First, score values of each element of the aggregated complex spherical fuzzy decision matrix are calculated by using Eq. 5. Hence, the matrix $S = \left(SF(s_{pq})\right)_{k\times m}$ is obtained.

(II) Second, the overall performance is computed by using the following formula:

$$G_p = ln\left(1 + \frac{1}{m}\sum_q \left|SF(s_{pq})\right|\right) \tag{5}$$

for all $p = 1,2,..,k$.

(III) Next, the performance of the criteria is analyzed by removing every criterion separately:

$$G_{pq} = ln\left(1 + \frac{1}{m}\sum_{t,t\neq q}\left|SF(s_{pt})\right|\right) \tag{6}$$

for all $p = 1,2,..,k$ and $q = 1,2,\dots,m$.

(IV) Summation of absolute deviations is found as:

$$K_q = \sum_p \left|G_{pq} - G_p\right| \tag{7}$$

for all $q = 1,2,\dots,m$. Finally, the normalized criteria weights are obtained:

$$w_q = \frac{K_q}{\sum_{q=1}^{n} K_q} \tag{8}$$

for all $q = 1,2,\dots,m$.

Step IV: Construct the aggregated weighted complex spherical fuzzy decision matrix (AWCSFDM) $D' = (s'_{pq})_{k\times m} = (w_q s_{pq})_{k\times m}$ by considering the ACSFDM and the weight matrix

$\Omega = (w_1, w_2, \dots, w_m)$ for criteria.

The $s'_{pq} = \left(f'_{A_p}(c_q)e^{i\alpha'_{A_p}(c_q)}, g'_{A_p}(c_q)e^{i\beta'_{A_p}(c_q)}, h'_{A_p}(c_q)e^{i\gamma'_{A_p}(c_q)}\right)$ is calculated as follows:

$$s'_{pq} = \begin{pmatrix} \sqrt{1-\left(1-\left(f'_{A_p}(c_q)\right)^2\right)^a}\, e^{2\pi\sqrt{1-\left(1-\left(\frac{\alpha'_{A_p}(c_q)}{2\pi}\right)^2\right)^a}}, \\ \sqrt{\left(1-\left(f'_{A_p}(c_q)\right)^2\right)^a - \left(1-\left(f'_{A_p}(c_q)\right)^2 - \left(g'_{A_p}(c_q)\right)^2\right)^a}\, e^{2\pi\sqrt{\left(1-\left(\frac{\alpha'_{A_p}(c_q)}{2\pi}\right)^2\right)^a - \left(1-\left(\frac{\alpha'_{A_p}(c_q)}{2\pi}\right)^2 - \left(\frac{\beta'_{A_p}(c_q)}{2\pi}\right)^2\right)^a}}, \\ \left(h'_{A_p}(c_q)\right)^a e^{i2\pi\left(\frac{\gamma'_{A_p}(c_q)}{2\pi}\right)^a} \end{pmatrix}$$

Hence, the AWCSFDM are constructed as:

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$$D' = \left(s'_{pq}\right)_{k\times m} = \begin{pmatrix} f'_{A_1}(c_1)e^{i\alpha'_{A_1}(c_1)}, g'_{A_1}(c_1)e^{i\beta'_{A_1}(c_1)}, h'_{A_1}(c_1)e^{i\gamma'_{A_1}(c_1)} \dots f'_{A_1}(c_m)e^{i\alpha'_{A_1}(c_m)}, g'_{A_1}(c_m)e^{i\beta'_{A_1}(c_m)}, h'_{A_1}(c_m)e^{i\gamma'_{A_1}(c_m)} \\ f'_{A_2}(c_1)e^{i\alpha'_{A_2}(c_1)}, g'_{A_2}(c_1)e^{i\beta'_{A_2}(c_1)}, h'_{A_2}(c_1)e^{i\gamma'_{A_2}(c_1)} \dots f'_{A_2}(c_m)e^{i\alpha'_{A_2}(c_m)}, g'_{A_2}(c_m)e^{i\beta'_{A_2}(c_m)}, h'_{A_2}(c_m)e^{i\gamma'_{A_2}(c_m)} \\ \dots \qquad \dots. \\ f'_{A_k}(c_1)e^{i\alpha'_{A_k}(c_1)}, g'_{A_k}(c_1)e^{i\beta_{A_k}{'}(c_1)}, h'_{A_k}(c_1)e^{i\gamma'_{A_k}(c_1)} \dots f'_{A_k}(c_m)e^{i\alpha'_{A_k}(c_m)}, g'_{A_k}(c_m)e^{i\beta'_{A_k}(c_m)}, h'_{A_k}(c_m)e^{i\gamma'_{A_k}(c_m)} \end{pmatrix}.$$

Step V: Since the elements of the AWCSFDM D' are CSFN, the score matrix D^* have to be constructed by using score function. The score matrix $D^* = \left(s^*_{pq}\right)_{k\times m}$ is constructed as follows:

$$D^* = \begin{pmatrix} s^*_{11} & s^*_{12} & \dots & s^*_{1m} \\ s^*_{21} & s^*_{22} & \dots & s^*_{2m} \\ \vdots & \vdots & \dots & \vdots \\ s^*_{k1} & s^*_{k2} & \dots & s^*_{km} \end{pmatrix}$$

$$s^*_{pq} = \frac{1}{3}\left(4 + f'_{A_p}\left(c_q\right)^2 - g'_{A_p}\left(c_q\right)^2 - h'_{A_p}\left(c_q\right)^2 + \left(\frac{\alpha'_{A_p}(c_q)}{2\pi}\right)^2 - \left(\frac{\beta'_{A_p}(c_q)}{2\pi}\right)^2 - \left(\frac{\gamma'_{A_p}(c_q)}{2\pi}\right)^2\right) \tag{9}$$

for all $p = 1,2,..,k$ and $q = 1,2,\dots,m$.

Step VI: Let $\mathcal{C}_B$ and $\mathcal{C}_C$ denote the set of benefit type and cost type criteria,respectively. Maximizing index $s(P_p)$ and minimizing index $s\left(R_p\right)$ are obtained as follows:

$s\left(P_p\right) = \frac{1}{|\mathcal{C}_B|}\sum_{q\in\mathcal{C}_B} s^*_{pq}$, $s\left(R_p\right) = \frac{1}{|\mathcal{C}_C|}\sum_{q\in\mathcal{C}_C} s^*_{pq}$ for all $p = 1,2,\dots,k$.

Step VII: Calculate the relative weight of each alternative Q_p as:

$$Q_p = s(P_p) + \frac{\sum_{p=1}^{k} s(R_p)}{s\left(R_p\right)\sum_{p=1}^{k}\frac{1}{s\left(R_p\right)}}$$

for all $p = 1,2,\dots,k$.

Step VIII: Determine the priority order Pr_p by using the formula

$$Pr_p = \frac{Q_p}{maxQ_p} * 100$$

for all $p = 1,2,\dots,k$.

Step IX: If $Pr_p \geq Pr_t$, then the ranking alternatives $A_p \geq A_t$ for all $p,t = 1,2,\dots,k$. Hence the alternative with the highest rank is the best solution for the problem.

4. ILLUSTRATIVE EXAMPLE

In this section, we address an MCGDM problem concerning the prioritization of vulnerable groups during the COVID-19 pandemic. In this context, certain populations are considered more susceptible to infection, including the elderly, individuals with disabilities, women, people with pre-existing health conditions, and those with mental impairments. Those informations are studied through the literary survey and we set them up as alternatives: physically challenged (A_1), elder (A_2), women (A_3), persons with comorbidities (A_4), mentally challenged (A_5). The common problem regard vaccine response among them is featured as criteria that are immune response (c_1), side effects (c_2), characteristic changes (c_3), environmental adaptation (c_4), physical appearance (c_5). Because which is more complex to conclude with the consensus, and we may not stop them as they are.

In particular, during the emergence of a vaccine, it is not straightforward to assess the exact safety measures required for vulnerable groups. So, there is a need to formalize a team of health professionals to safeguard them. Neurologist (D_1), psychologist (D_2), immunologist (D_3), family physician (D_4), dermatologist (D_5) and gynecologist (D_6) are grouped to suggest who are all in the complex situation. These groups are regarded as being in a highly critical condition during the pandemic; therefore, it is essential to prioritize them according to healthcare considerations. The data set constructed by each health professionals are taken from the paper [22] and shown in Table 2-7.

D1	c1	c2	c3	c4	c5
A1	(.58e^{i2π(.61)}, .44e^{i2π(.39)}, .48e^{i2π(.37)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.58e^{i2π(.61)}, .44e^{i2π(.39)}, .48e^{i2π(.37)})	(.94e^{i2π(.93)}, .26e^{i2π(.24)}, .19e^{i2π(.12)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})
A2	(.94e^{i2π(.93)}, .26e^{i2π(.24)}, .19e^{i2π(.12)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.94e^{i2π(.93)}, .26e^{i2π(.24)}, .19e^{i2π(.12)})
A3	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.58e^{i2π(.61)}, .44e^{i2π(.39)}, .48e^{i2π(.37)})	(.36e^{i2π(.35)}, .46e^{i2π(.39)}, .66e^{i2π(.62)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.58e^{i2π(.61)}, .44e^{i2π(.39)}, .48e^{i2π(.37)})
A4	(.36e^{i2π(.35)}, .46e^{i2π(.39)}, .66e^{i2π(.62)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})
A5	(.36e^{i2π(.35)}, .46e^{i2π(.39)}, .66e^{i2π(.62)})	(.94e^{i2π(.93)}, .26e^{i2π(.24)}, .19e^{i2π(.12)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})

Table 2: Decision matrix constructed by D_1

D_2	c_1	c_2	c_3	c_4	c_5
A_1	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.58e^i2π(.61), .44e^i2π(.39), .48e^i2π(.37))	(.81e^i2π(.78), .36e^i2π(.23), .25e^i2π(.21))	(.45e^i2π(.41), .48e^i2π(.43), .67e^i2π(.61))	(.81e^i2π(.78), .36e^i2π(.23), .25e^i2π(.21))
A_2	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.94e^i2π(.93), .26e^i2π(.24), .19e^i2π(.12))	(.94e^i2π(.93), .26e^i2π(.24), .19e^i2π(.12))	(.94e^i2π(.93), .26e^i2π(.24), .19e^i2π(.12))	(.81e^i2π(.78), .36e^i2π(.23), .25e^i2π(.21))
A_3	(.58e^i2π(.61), .44e^i2π(.39), .48e^i2π(.37))	(.45e^i2π(.41), .48e^i2π(.43), .67e^i2π(.61))	(.81e^i2π(.78), .36e^i2π(.23), .25e^i2π(.21))	(.45e^i2π(.41), .48e^i2π(.43), .67e^i2π(.61))	(.94e^i2π(.93), .26e^i2π(.24), .19e^i2π(.12))
A_4	(.94e^i2π(.93), .26e^i2π(.24), .19e^i2π(.12))	(.81e^i2π(.78), .36e^i2π(.23), .25e^i2π(.21))	(.94e^i2π(.93), .26e^i2π(.24), .19e^i2π(.12))	(.94e^i2π(.93), .26e^i2π(.24), .19e^i2π(.12))	(.24e^i2π(.18), .34e^i2π(.31), .79e^i2π(.89))
A_5	(.81e^i2π(.78), .36e^i2π(.23), .25e^i2π(.21))	(.94e^i2π(.93), .26e^i2π(.24), .19e^i2π(.12))	(.94e^i2π(.93), .26e^i2π(.24), .19e^i2π(.12))	(.81e^i2π(.78), .36e^i2π(.23), .25e^i2π(.21))	(.94e^i2π(.93), .26e^i2π(.24), .19e^i2π(.12))

Table 3: Decision matrix constructed by D_2

D3	c1	c2	c3	c4	c5
A_1	(.58e^{i2π(.61)}, .44e^{i2π(.39)}, .48e^{i2π(.37)})	(.58e^{i2π(.61)}, .44e^{i2π(.39)}, .48e^{i2π(.37)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})
A_2	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.94e^{i2π(.93)}, .26e^{i2π(.24)}, .19e^{i2π(.12)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})
A_3	(.58e^{i2π(.61)}, .44e^{i2π(.39)}, .48e^{i2π(.37)})	(.36e^{i2π(.35)}, .46e^{i2π(.39)}, .66e^{i2π(.62)})	(.24e^{i2π(.18)}, .34e^{i2π(.31)}, .79e^{i2π(.89)})	(.58e^{i2π(.61)}, .44e^{i2π(.39)}, .48e^{i2π(.37)})	(.36e^{i2π(.35)}, .46e^{i2π(.39)}, .66e^{i2π(.62)})
A_4	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.24e^{i2π(.18)}, .34e^{i2π(.31)}, .79e^{i2π(.89)})

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A5	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.94e^{i2π(.93)}, .26e^{i2π(.24)}, .19e^{i2π(.12)})	(.36e^{i2π(.35)}, .46e^{i2π(.39)}, .66e^{i2π(.62)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})

Table 4: Decision matrix constructed by D_3

D4	c1	c2	c3	c4	c5
A1	(.58e^{i2π(.61)}, .44e^{i2π(.39)}, .48e^{i2π(.37)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.58e^{i2π(.61)}, .44e^{i2π(.39)}, .48e^{i2π(.37)})	(.36e^{i2π(.35)}, .46e^{i2π(.39)}, .66e^{i2π(.62)})	(.58e^{i2π(.61)}, .44e^{i2π(.39)}, .48e^{i2π(.37)})
A2	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})
A3	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.24e^{i2π(.18)}, .34e^{i2π(.31)}, .79e^{i2π(.89)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.24e^{i2π(.18)}, .34e^{i2π(.31)}, .79e^{i2π(.89)})	(.58e^{i2π(.61)}, .44e^{i2π(.39)}, .48e^{i2π(.37)})
A4	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.24e^{i2π(.18)}, .34e^{i2π(.31)}, .79e^{i2π(.89)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})
A5	(.94e^{i2π(.93)}, .26e^{i2π(.24)}, .19e^{i2π(.12)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.58e^{i2π(.61)}, .44e^{i2π(.39)}, .48e^{i2π(.37)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})

Table 5: Decision matrix constructed by D_4

D5	c1	c2	c3	c4	c5
A1	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.36e^{i2π(.35)}, .46e^{i2π(.39)}, .66e^{i2π(.62)})	(.36e^{i2π(.35)}, .46e^{i2π(.39)}, .66e^{i2π(.62)})	(.24e^{i2π(.18)}, .34e^{i2π(.31)}, .79e^{i2π(.89)})	(.58e^{i2π(.61)}, .44e^{i2π(.39)}, .48e^{i2π(.37)})
A2	(.94e^{i2π(.93)}, .26e^{i2π(.24)}, .19e^{i2π(.12)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})
A3	(.58e^{i2π(.61)}, .44e^{i2π(.39)}, .48e^{i2π(.37)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})
A4	(.24e^{i2π(.18)}, .34e^{i2π(.31)}, .79e^{i2π(.89)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.58e^{i2π(.61)}, .44e^{i2π(.39)}, .48e^{i2π(.37)})
A5	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})

Table 6: Decision matrix constructed by D_5

D6	c1	c2	c3	c4	c5
A1	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.36e^{i2π(.35)}, .46e^{i2π(.39)}, .66e^{i2π(.62)})	(.36e^{i2π(.35)}, .46e^{i2π(.39)}, .66e^{i2π(.62)})	(.24e^{i2π(.18)}, .34e^{i2π(.31)}, .79e^{i2π(.89)})	(.58e^{i2π(.61)}, .44e^{i2π(.39)}, .48e^{i2π(.37)})
A2	(.94e^{i2π(.93)}, .26e^{i2π(.24)}, .19e^{i2π(.12)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})
A3	(.58e^{i2π(.61)}, .44e^{i2π(.39)}, .48e^{i2π(.37)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})
A4	(.24e^{i2π(.18)}, .34e^{i2π(.31)}, .79e^{i2π(.89)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.58e^{i2π(.61)}, .44e^{i2π(.39)}, .48e^{i2π(.37)})
A5	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.81e^{i2π(.78)}, .36e^{i2π(.23)}, .25e^{i2π(.21)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})	(.45e^{i2π(.41)}, .48e^{i2π(.43)}, .67e^{i2π(.61)})

Table 7: Decision matrix constructed by D_6

We now apply established MEREC-COPRAS method to solve this problem.

Since each criterion is a benefit type, Step I is skipped.

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Step II: We merge the decision matrices by using CSFWA operator to find the collective decision matrix and this matrix is shown in Table 8.

ACSFDM	c1	c2	c3	c4	c5
A1	(.52e^{i2π(.54)}, .40e^{i2π(.34)}, .56e^{i2π(.47)})	(.58e^{i2π(.58)}, .41e^{i2π(.34)}, .51e^{i2π(.43)})	(.56e^{i2π(.55)}, .40e^{i2π(.31)}, .54e^{i2π(.46)})	(.69e^{i2π(.67)}, .31e^{i2π(.21)}, .46e^{i2π(.44)})	(.64e^{i2π(.62)}, .35e^{i2π(.26)}, .46e^{i2π(.40)})
A2	(.83e^{i2π(.80)}, .30e^{i2π(.19)}, .30e^{i2π(.24)})	(.88e^{i2π(.86)}, .30e^{i2π(.23)}, .25e^{i2π(.19)})	(.73e^{i2π(.70)}, .36e^{i2π(.27)}, .42e^{i2π(.36)})	(.76e^{i2π(.74)}, .34e^{i2π(.25)}, .37e^{i2π(.26)})	(.78e^{i2π(.75)}, .31e^{i2π(.18)}, .35e^{i2π(.29)})
A3	(.58e^{i2π(.58)}, .41e^{i2π(.34)}, .52e^{i2π(.43)})	(.66e^{i2π(.65)}, .38e^{i2π(.30)}, .50e^{i2π(.43)})	(.51e^{i2π(.47)}, .41e^{i2π(.33)}, .61e^{i2π(.58)})	(.51e^{i2π(.49)}, .40e^{i2π(.33)}, .59e^{i2π(.54)})	(.78e^{i2π(.77)}, .36e^{i2π(.30)}, .38e^{i2π(.29)})
A4	(.67e^{i2π(.64)}, .40e^{i2π(.33)}, .50e^{i2π(.44)})	(.62e^{i2π(.59)}, .34e^{i2π(.24)}, .50e^{i2π(.45)})	(.74e^{i2π(.72)}, .38e^{i2π(.30)}, .40e^{i2π(.33)})	(.77e^{i2π(.74)}, .34e^{i2π(.24)}, .36e^{i2π(.33)})	(.51e^{i2π(.48)}, .38e^{i2π(.31)}, .61e^{i2π(.58)})
A5	(.67e^{i2π(.65)}, .41e^{i2π(.34)}, .49e^{i2π(.43)})	(.82e^{i2π(.80)}, .31e^{i2π(.22)}, .33e^{i2π(.26)})	(.81e^{i2π(.79)}, .34e^{i2π(.27)}, .34e^{i2π(.27)})	(.54e^{i2π(.52)}, .41e^{i2π(.34)}, .57e^{i2π(.50)})	(.74e^{i2π(.72)}, .35e^{i2π(.30)}, .47e^{i2π(.39)})

Table 8: Aggregated decision matrix

Step III: We now calculate the criteria weights according to the MEREC method. To find this weight, we first calculate the score values of the aggregated decision matrix and this values are shown in Table 9.

Table 9: Score value of aggregated decision matrix

	c_1	c_2	c_3	c_4	c_5
A_1	1.25	1.31	1.29	1.46	1.41
A_2	1.69	1.76	1.51	1.58	1.61
A_3	1.31	1.40	1.16	1.20	1.58
A_4	1.38	1.37	1.52	1.57	1.18
A_5	1.39	1.66	1.64	1.24	1.49

Next, we compute overall performance as G_1=.8526, G_2= .9661, G_3=.8458, G_4= .8778, G_5=.9094. After, we analyze the criteria effect by removing every criterion separately and we show this values in Table 10.

Table 10: G_{pq} values

.74	.73	.74	.72	.72
.83	.82	.84	.84	.84
.73	.72	.74	.74	.70
.76	.76	.74	.74	.77
.79	.77	.77	.80	.78

Then, we obtain the summation of absolute deviations as K_1=.6098, K_2=.6549, K_3=.6207, K_4=.6149, K_5=.6356. As a conseqeunce, we have the criteria weight w_1=.1945, w_2=.2088, w_3=.1979, w_4=.1961, w_5=.2027.

Step IV: We now compute the aggregated weighted complex spherical fuzzy decision matrix by multiplying the aggregated decision matrix by the criteria weight found in Step III. This matrix is shown in Table 11.

AWCSFDM	c1	c2	c3	c4	c5
A1	(.25e^{i2π(.25)}, .21e^{i2π(.18)}, .89e^{i2π(.86)})	(.29e^{i2π(.28)}, .23e^{i2π(.19)}, .87e^{i2π(.84)})	(.26e^{i2π(.27)}, .22e^{i2π(.16)}, .88e^{i2π(.86)})	(.69e^{i2π(.67)}, .31e^{i2π(.21)}, .46e^{i2π(.44)})	(.64e^{i2π(.62)}, .35e^{i2π(.26)}, .46e^{i2π(.40)})

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A2	(.45e^{i2π(.43)}, .22e^{i2π(.13)}, .79e^{i2π(.76)})	(.52e^{i2π(.50)}, .27e^{i2π(.19)}, .75e^{i2π(.70)})	(.38e^{i2π(.36)}, .23e^{i2π(.16)}, .84e^{i2π(.81)})	(.76e^{i2π(.74)}, .34e^{i2π(.25)}, .37e^{i2π(.26)})	(.78e^{i2π(.75)}, .31e^{i2π(.18)}, .35e^{i2π(.29)})
A3	(.28e^{i2π(.28)}, .23e^{i2π(.18)}, .88e^{i2π(.85)})	(.34e^{i2π(.33)}, .23e^{i2π(.17)}, .86e^{i2π(.84)})	(.24e^{i2π(.22)}, .22e^{i2π(.17)}, .91e^{i2π(.90)})	(.51e^{i2π(.49)}, .40e^{i2π(.33)}, .59e^{i2π(.54)})	(.78e^{i2π(.77)}, .36e^{i2π(.30)}, .38e^{i2π(.29)})
A4	(.33e^{i2π(.31)}, .24e^{i2π(.33)}, .50e^{i2π(.44)})	(.62e^{i2π(.59)}, .34e^{i2π(.24)}, .50e^{i2π(.45)})	(.74e^{i2π(.72)}, .38e^{i2π(.30)}, .40e^{i2π(.33)})	(.77e^{i2π(.74)}, .34e^{i2π(.24)}, .36e^{i2π(.33)})	(.51e^{i2π(.48)}, .38e^{i2π(.31)}, .61e^{i2π(.58)})
A5	(.67e^{i2π(.65)}, .41e^{i2π(.34)}, .49e^{i2π(.43)})	(.82e^{i2π(.80)}, .31e^{i2π(.22)}, .33e^{i2π(.26)})	(.81e^{i2π(.79)}, .34e^{i2π(.27)}, .34e^{i2π(.27)})	(.54e^{i2π(.52)}, .41e^{i2π(.34)}, .57e^{i2π(.50)})	(.74e^{i2π(.72)}, .35e^{i2π(.30)}, .47e^{i2π(.39)})

Table 11: Aggregated weighted decision matrix

Step V: We calculate the score values of each element of the aggregated weighted decision matrix and this score matrix is shown in Table 12.

	c_1	c_2	c_3	c_4	c_5
A_1	.8352	.8718	.8503	.9069	.9040
A_2	1.0381	1.1169	.9385	.9846	.9995
A_3	.8569	.8957	.8007	.8125	.9829
A_4	.8732	.8884	.9493	.9693	.8079
A_5	.8780	1.0404	1.0063	.8275	.9256

Table 12: Score degree of aggregated weighted decision matrix

Step VI-IX: We now apply the steps of the COPRAS method to rank the alternatives. The values computed in the related steps are shown in Table 13.

	$s(P_p)$	Q_p	P_{r_p}	Ranking
A1	.8737	.8737	86,0311	4
A2	1.0155	1.0155	100	1
A3	.8697	.8697	85,6433	5
A4	.8976	.8976	88,3917	3
A5	.9356	.9356	92,1256	2

Table 13: Maximizing index, relative weight, priority order and ranking

As a result, we obtain the ranking as $A_2 > A_5 > A_4 > A_1 > A_3$. This means that elderly people seem to be the most vulnerable people under this data. When we compare the result of the same problem under the solution given in the source paper [22], we realize that we conclude the same best alternative result. Also, the rankings are almost the same under the two methods. Thus, this result shows the reliability and applicability of the presented MEREC-TOPSIS method.

CONCLUSION

This study presents a novel integration of the MEREC objective weighting method with the COPRAS multi-criteria decision-making approach under the framework of CSFS. This combination enables the effective handling of two-dimensional uncertain data and eliminates subjective bias in determining criteria weights, resulting in a robust and systematic decision support tool. The application of this hybrid method to the healthcare domain, particularly for prioritizing individuals at high risk of COVID-19 infection, demonstrates its practical relevance

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and adaptability to critical real-world problems characterized by uncertainty and complexity. The findings confirm that the CSFS-MEREC-COPRAS methodology can enhance decision accuracy and provide reliable rankings in complex environments, making it valuable for strategic healthcare management during pandemics and similar crises.

Future research should focus on developing hybrid models that integrate qualitative and quantitative data, enhance computational efficiency, and expand applicability across various domains such as healthcare, energy, and supply chain management. Ultimately, improving decision-making methods will continue to empower decision-makers to make more informed, transparent, and reliable choices.

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APPROXIMATION AND DERIVATIVE PROPERTIES OF A GENERALIZED BERNSTEIN OPERATOR: ANALYTICAL AND COMPUTATIONAL INVESTIGATIONS

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ABSTRACT

In this study, the derivative properties of a generalized class of Bernstein-type operators are thoroughly investigated. The primary objective of the research is to adapt known identities for the classical Bernstein operator to the generalized operator and to analytically evaluate the newly obtained equalities and inequalities. The rate of approximation is analyzed through moments, with particular focus on the behavior of derivative function classes. The derivatives of Bernstein-type operators are explicitly represented using finite difference techniques, and concrete expressions are derived for both the first and second derivatives. These derivative-based formulations provide significant applications in approximation theory and offer valuable insights for determining error bounds of the operators. Additionally, certain algebraic identities are employed to establish new structural relations among the polynomial families associated with these operators, and these relations are analytically examined. The study not only extends the known properties of classical Bernstein polynomials but also provides new tools for derivative-based approximation and analytical applications. The obtained results demonstrate the applicability of the generalized operators to various function classes and highlight their potential use in derivative analysis. In future research, these methods and derived relations can be applied to different types of generalized Bernstein operators, potentially yielding broader classes of equalities and inequalities. Overall, the study presents a comprehensive framework for examining the derivative properties of operators, offering both theoretical insights and practical tools for further developments in approximation theory and related applications.

Key words: Bernstein Operators, Approximation Theory, Polynomial Identities.

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Introduction

Approximation theory, as a fundamental branch of analysis, serves a crucial role in investigating the convergence properties of functions and operators (DeVore and Lorentz, 1993). Bernstein polynomials were first given by Sergei N. Bernstein in 1912 to provide a constructive proof of the Weierstrass theorem (Bernstein, 1912). In this context, Bernstein polynomials are of central importance due to their ability to guarantee uniform convergence for continuous functions, their positivity, and their structural simplicity (Bernstein, 1912; Lorentz, 1986). These polynomials serve a fundamental role in uniform approximation of continuous functions and find wide applications across various areas of analysis and approximation theory (Lorentz, 1986).

Research on Bernstein polynomials has not only advanced theoretical approximation theory but has also contributed significantly to numerical analysis, functional analysis, and applied mathematics. One of the most notable features of Bernstein polynomials is that they form a family of linear and positive operators (Altomare and Campiti, 1994).

Investigations into Bernstein polynomials have also led to the development of various generalizations (Phillips, 1997). Numerous studies have focused on these generalizations, introducing different classes of operators (Szasz, 1950).

For instance, q-Bernstein polynomials were introduced as a q-analytical extension of the classical operators (Lupaş, 1987, Phillips, 2003). Similarly, Chlodowsky-type Bernstein operators allow approximation of functions defined on unbounded intervals (Chlodovsky, 1937, Büyükyazici and Ibikli, 2006). In a study by Çilo et al. (2012), a generalization of the Bernstein operator was defined, and various approximation properties of this operator were investigated.

The study of derivatives of such operators is particularly significant, as it reveals connections between finite differences and differential expressions (Micchelli, 1969). These connections are also effectively used in estimating errors, obtaining saturation results, and determining various convergence bounds (Gadjiev and Orhan, 2002). For example, the derivative-based approximation properties of Bernstein–Durrmeyer type operators have been studied in detail (Ditzian and Ivanov, 1989).

Investigations into the derivatives of linear positive operators are important not only for approximating the functions themselves but also their derivatives (Butzer, 1982). Such studies provide powerful tools for both analytical and numerical approaches and make significant contributions to approximate derivative analysis (Ditzian and Totik, 1987).

In this study, various identities originally established for the classical Bernstein operator (Bustamante, 2017) were adapted to the operator defined by Çilo et al. (2012), and the resulting new identities and inequalities were analyzed. Additionally, the rate of approximation was investigated in terms of moments, with particular emphasis on the identities provided by the derivatives of the operator. The derivative properties of Bernstein-type operators were addressed, explicit relations were derived using finite difference representations, and applications of these relations in approximation theory were examined. Explicit representations

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for the first and second derivatives of these operators were obtained using finite difference techniques.

Definition 1.1

For each $x \in [-1,1]$ and $f \in C[-1,1]$, the operator denoted by

$$C_n(f;x) = \frac{1}{2^n}\sum_{k=0}^{n}\binom{n}{k}(1-x)^{n-k}(x+1)^k f\left(\frac{2k}{n}-1\right) \tag{1}$$

is called a modified Bernstein type operators. This operators is demonstrated by $C_n(f;x)$. (Cilo, 2012).

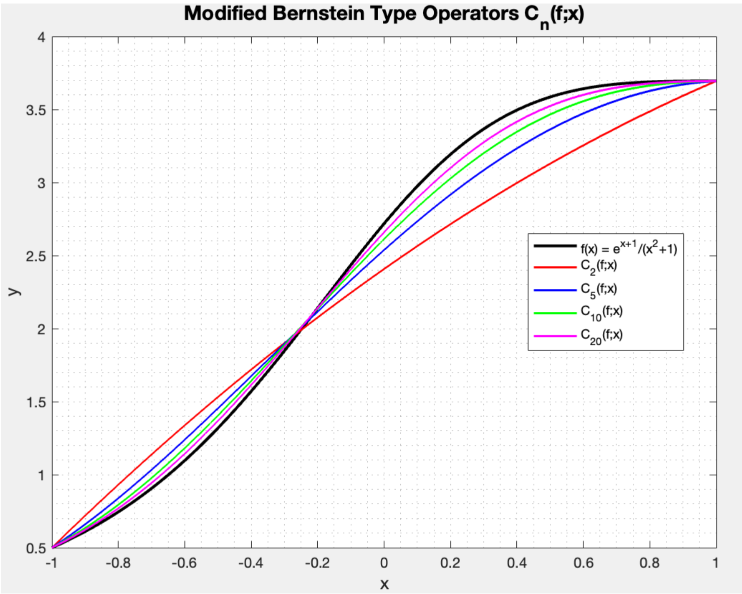

Figure 1: As n increases, the operator converges to the function $f(x) = \frac{e^{x+1}}{x^2+1}$.

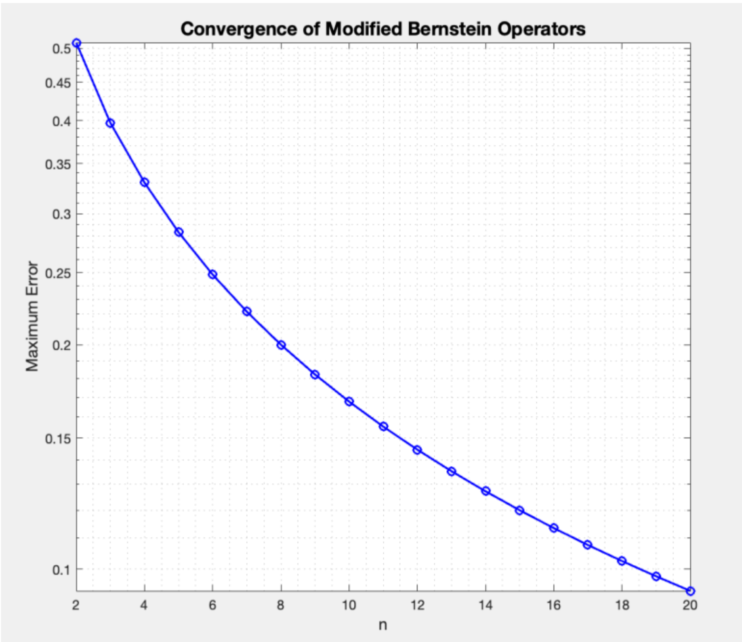

Figure 2: With larger values of n, the error of the operator $C_n(f;x)$ becomes minimized.

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2. Relations Concerning the Operator

In this section, various identities concerning the polynomial and its derivatives given by Equation (1) will be established. The methods employed in this section are inspired by those used by (Bustamante, 2017) for the classical Bernstein operator.

The following auxiliary notations are also used.

$P_{n,k}(x) = \binom{n}{k}(x+1)^k(1-x)^{n-k}$

$\psi(x) = \sqrt{(1-x)(1+x)},\ x \in [-1,1]$ and $e_j(x) = x^j,\quad x \in \mathbb{R}$.

Subsequently, we shall require estimates of the second symmetric differences as introduced by

$\Delta_h^2 f(x) := f(x-h) - 2f(x) + f(x+h)$.

Proposition 2.1

For $x \in (0,1)$ and $n \in \mathbb{N}$,

$(\boldsymbol{i})\ \psi^2(x)P'_{n,k}(x) = [2k-n-nx]P_{n,k}(x), \qquad k \in \{0,1,\dots,n\}$,

$(\boldsymbol{ii})\ P'_{n,k}(x) = n\left(P_{n-1,k-1}(x) - P_{n-1,k}(x)\right), \qquad k \in \{0,1,\dots,n-1\}$,

$(\boldsymbol{iii})\ P'_{n,0}(x) = -nP_{n-1,0}(x) \qquad P'_{n,n}(x) = nP_{n-1,n-1}(x)$,

$$(\boldsymbol{iv})v\psi^4(x)P''_{n,k}(x) = n\left[\left(2\frac{k}{n} - 1 - x\right)^2(n-1) + \left(2\frac{k}{n} - 1\right)^2 - 1\right]P_{n,k}(x), \tag{2.1}$$

$(\boldsymbol{v})\ \psi^6(x)P'''_{n,k}(x)$

$= [\{2 - 2n(2k-n-nx) - 2n\}\psi^2(x) + 2(2k-n-nx)^2 + 4x(2k-n-nx)]P_{n,k}(x)$.

Proof

$(\boldsymbol{i})$ Let

$P_{n,k}(x) = \binom{n}{k}(1-x)^{n-k}(x+1)^k$.

Then, applying the operator $\psi^2(x) = (1+x)(1-x)$ to derivate of $P_{n,k}$,

$\psi^2(x)P'_{n,k}(x) = (1+x)(1-x)P'_{n,k}(x)$

$= (1+x)(1-x)\binom{n}{k}[k(1-x)^{n-k}(x+1)^{k-1} - (n-k)(x+1)^k(1-x)^{n-k-1}]$

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$$= \binom{n}{k}(1-x)k(x+1)^k(1-x)^{n-k} - \binom{n}{k}(1+x)(n-k)(1-x)^{n-k}(x+1)^k$$

$$= (1-x)kP_{n,k}(x) - (1+x)(n-k)P_{n,k}(x).$$

So,

$$\psi^2(x)P'_{n,k}(x) = [k - kx - n - nx + k + kx]P_{n,k}(x).$$

Hence,

$$\psi^2(x)P'_{n,k}(x) = [2k - n - nx]P_{n,k}(x).$$

(ii) The derivative of $P_{n,k}(x)$ is given

$$P'_{n,k}(x) = \frac{n!}{(n-k)!\,(k-1)!}(1-x)^{n-k}(x+1)^{k-1} - \frac{n!}{(n-k-1)!\,k!}(1-x)^{n-k-1}(x+1)^k$$

$$= n\frac{(n-1)!}{(n-k)!\,(k-1)!}(1-x)^{n-k}(x+1)^{k-1} - n\frac{(n-1)!}{(n-k-1)!\,k!}(1-x)^{n-k-1}(x+1)^k$$

So,

$$P'_{n,k}(x) = n\left(P_{n-1,k-1}(x) - P_{n-1,k}(x)\right).$$

(iii) For the boundary cases of the Bernstein-type polynomial, the derivatives satisfy

$$P'_{n,0}(x) = n\left(P_{n-1,0-1}(x) - P_{n-1,0}(x)\right) = -nP_{n-1,0}(x)$$

and

$$P'_{n,n}(x) = n\left(P_{n-1,n-1}(x) - P_{n-1,n}(x)\right) = nP'_{n-1,n-1}(x).$$

$(\boldsymbol{iv})$ For $0 \leq k \leq n$, the fourth-order operator $\psi^4(x)$ acting on the second derivative of the Bernstein-type polynomial $P_{n,k}(x)$ can be expressed as

$$\psi^4(x)P''_{n,k}(x) = \psi^2(x)\left[2\psi(x)\psi'(x)P'_{n,k}(x) + \psi^2(x)P''_{n,k}(x)\right] + 2x\psi^2(x)P'_{n,k}(x)$$

$$= \psi^2(x)\left[(2k - n - nx)P'_{n,k}(x) - nP_{n,k}(x)\right] + 2x\psi^2(x)P'_{n,k}(x)$$

$$= (2k - n - nx)P'_{n,k}(x)\psi^2(x) - nP_{n,k}(x)\psi^2(x) + 2x\psi^2(x)P'_{n,k}(x)$$

$$= [(4k^2 - 4kn + n^2 - n + (2n^2 - 4kn + 4k - 2n)x + (n^2 - n)x^2)]P_{n,k}(x)$$

$$= \left[\left(\frac{4k^2}{n^2} + 1 + x^2 - \frac{4k}{n} - \frac{4k}{n}x + 2x\right)(n-1)n + 4\frac{k^2}{n} - 4k + n - n\right]P_{n,k}(x).$$

Hence,

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$$\psi^4(x)P''_{n,k}(x) = n\left[\left(2\frac{k}{n}-1-x\right)^2(n-1)+\left(2\frac{k}{n}-1\right)^2-1\right]P_{n,k}(x).$$

Therefore, the product of $\psi^4(x)$ with $P''_{n,k}(x)$ is a linear combination of $P_{n,k}(x)$, whose coefficients are explicit polynomials in k, n, and x.

(v) For $0 \le k \le n$, the action of the sixth-order operator $\psi^6(x)$ on the third derivative of the Bernstein-type polynomial $P_{n,k}(x)$ can be written as

$$P'''_{n,k}(x)\psi^6(x) = \psi^2(x)\left(\psi^4(x)P''_{n,k}(x)\right)' - 4\psi^5(x)\psi'(x)P''_{n,k}(x)$$

$$= \psi^2(x)\left([(2k-n-nx+2x)(2k-n-nx)-n\psi^2(x)]P_{n,k}(x)\right)' + 2x\psi^4(x)P''_{n,k}(x)$$

$$= [-n(2k-n-nx)+2(2k-n-nx)$$

$$-n(2k-n-nx)-2xn+2xn-2n]P_{n,k}(x)\psi^2(x)$$

$$+2(2k-n-nx)^2P_{n,k}(x)+4x(2k-n-nx)P_{n,k}(x).$$

Since,

$$\psi^6(x)P'''_{n,k}(x)$$

$$= [\{2-2n(2k-n-nx)-2n\}\psi^2(x)+2(2k-n-nx)^2+4x(2k-n-nx)]P_{n,k}(x).$$

Accordingly, $\psi^6(x)P'''_{n,k}(x)$ is expressed as a linear combination of $P_{n,k}(x)$ with coefficients given by polynomials in k, n and x.

Proposition 2.2

For each $n \in \mathbb{N}$ and $0 \le k \le n$, the following relation holds.

$$P_{n,k}(x) = \sum_{j=k}^{n}\binom{n}{k}\binom{n-k}{j-k}2^{n-j}(-1)^{j-k}(1+x)^j.$$

Proof

Using binomial expansion of $(1-x)^{n-k}$

$$P_{n,k}(x) = \binom{n}{k}(1-x)^{n-k}(x+1)^k = \binom{n}{k}(x+1)^k\sum_{j=0}^{n-k}\binom{n-k}{i}2^{n-k-i}(-1-x)^i$$

$$= \sum_{j=0}^{n-k}\binom{n}{k}\binom{n-k}{j}(x+1)^k2^{n-k-j}(-1)^j(1+x)^j$$

$$= \sum_{j=k}^{n}\binom{n}{k}\binom{n-k}{j-k}2^{n-j}(-1)^{j-k}(1+x)^j.$$

So,

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$$P_{n,k}(x) = \sum_{j=k}^{n} \binom{n}{k}\binom{n-k}{j-k} 2^{n-j}(-1)^{j-k}(1+x)^{j}.$$

Using the standard binomial coefficient identity

$$\binom{n}{k}\binom{n-k}{j-k} = \binom{n}{j}\binom{i}{k}$$

the following is obtained

$$P_{n,k}(x) = \sum_{j=k}^{n} \binom{n}{j}\binom{j}{k} 2^{n-j}(-1)^{j-k}(1+x)^{j}.$$

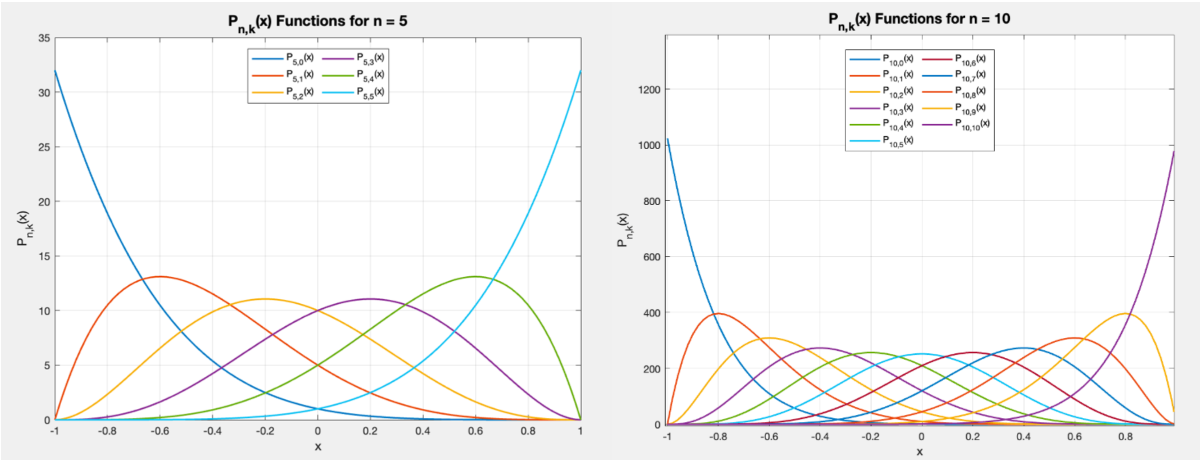

Figure 3: Graphical representation of the functions $P_{n,k}(x)$ for $n = 5$ and $n = 10$ as k increases.

Lemma 2.1

Let $f: [-1,1] \to \mathbb{R}$ and $f(x) = 0$, for $x > 1$ and $x < -1$. For all $n \in \mathbb{N}$ and $x \in [-1,1]$,

$$2\sum_{k=0}^{n} f\left(\frac{2k}{n} - 1\right) P_{n,k}(x) = \sum_{k=0}^{n+1}\left(\frac{n+1-k}{n+1} f\left(\frac{2k}{n} - 1\right) + \frac{k}{n+1} f\left(\frac{2k-2}{n} - 1\right)\right) P_{n+1,k}(x).$$

Proof:

Attention is now directed to the right-hand side:

$$\sum_{k=0}^{n+1}\left(\frac{n+1-k}{n+1} f\left(\frac{2k}{n} - 1\right) + \frac{k}{n+1} f\left(\frac{2k-2}{n} - 1\right)\right) P_{n+1,k}(x)$$

$$= \sum_{k=0}^{n+1} \frac{n+1-k}{n+1} f\left(\frac{2k}{n} - 1\right) P_{n+1,k}(x) + \sum_{k=0}^{n} \frac{k+1}{n+1} f\left(\frac{2k}{n} - 1\right) P_{n+1,k+1}(x).$$

Since the Bernstein polynomials satisfy

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$$P_{n+1,k}(x) = \binom{n+1}{k}(1-x)^{n+1-k}(1+x)^k = (1-x)\frac{1+n}{n+1-k}P_{n,k}(x),$$

$$P_{n+1,k+1}(x) = \binom{n+1}{k+1}(1-x)^{n-k}(x+1)^{k+1}$$

$$= (1+x)\frac{1+n}{k+1}\frac{n!}{k!\,(n-k)!}(1-x)^{n-k}(x+1)^k.$$

So,

$$P_{1+n,k+1}(x) = (1+x)\frac{n+1}{k+1}P_{n,k}(x).$$

The sum can be rewritten as

$$\sum_{k=0}^{n+1}\frac{n+1-k}{n+1}f\left(\frac{2k}{n}-1\right)P_{n+1,k}(x) + \sum_{k=0}^{n}\frac{k+1}{n+1}f\left(\frac{2k}{n}-1\right)P_{n+1,k+1}(x)$$

$$= \sum_{k=0}^{n} f\left(\frac{2k}{n}-1\right)(1-x)P_{n,k}(x) + \sum_{k=0}^{n} f\left(\frac{2k}{n}-1\right)(1+x)P_{n,k}(x).$$

Then,

$$2\sum_{k=0}^{n} f\left(\frac{2k}{n}-1\right)P_{n,k}(x) = \sum_{k=0}^{n+1}\left(\frac{n+1-k}{1+n}f\left(\frac{2k}{n}-1\right) + \frac{k}{n+1}f\left(\frac{2k-2}{n}-1\right)\right)P_{n+1,k}(x).$$

Proposition 2.3

For $n \in \mathbb{N}$, $x \in [-1,1]$ and $f\colon [-1,1] \to \mathbb{R}$, then

$$\frac{(1-x)(x+1)}{n}C_n'(f;x) = C_n(f(e_1)(e_1-x),x).$$

Proof:

Let $C_n(f;x)$ be defined by

$$C_n(f;x) = \frac{1}{2^n}\sum_{k=0}^{n} P_{n,k}(x)f\left(\frac{2k}{n}-1\right), \qquad x \in [-1.1].$$

Then its derivative is

$$C_n'(f;x) = \frac{1}{2^n}\sum_{k=0}^{n} P_{n,k}'(x)f\left(\frac{2k}{n}-1\right).$$

Using the previously established formula

$$P_{n,k}'(x) = \frac{[2k-n-nx]}{\psi^2(x)}P_{n,k}(x), \quad \psi^2(x) = 1-x^2,$$

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it is written as

$$C_n'(f;x) = \frac{1}{2^n}\sum_{k=0}^{n}\frac{[2k-n-nx]}{\psi^2(x)}P_{n,k}(x)f\left(\frac{2k}{n}-1\right),$$

$$\psi^2(x)C_n'(f;x) = \frac{1}{2^n}\sum_{k=0}^{n}[2k-n-nx]P_{n,k}(x)f\left(\frac{2k}{n}-1\right).$$

Dividing both sides by n

$$\frac{\psi^2(x)}{n}C_n'(f;x) = \frac{1}{2^n}\sum_{k=0}^{n}P_{n,k}(x)\left(\frac{2k}{n}-1-x\right)f\left(\frac{2k}{n}-1\right).$$

Using

$$(e_1-x)f(e_1) = \left(\frac{2k}{n}-1-x\right)f\left(\frac{2k}{n}-1\right),$$

It is concluded that

$$\frac{(1-x)(x+1)}{n}C_n'(f;x) = C_n\big((e_1-x)f(e_1),x\big).$$

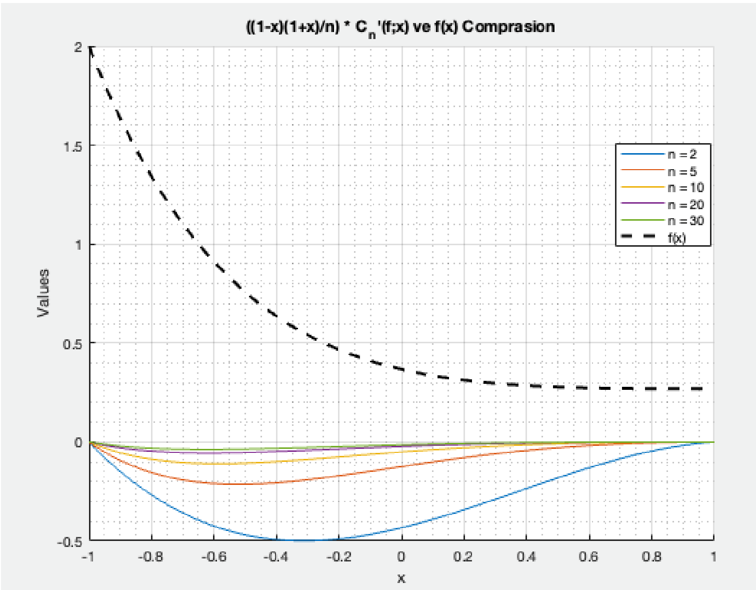

Figure 4: Approximation of the function using $\frac{(1-x)(1+x)}{n}C_n'(f;x)$ as n increases.

Proposition 2.4

For $n \in \mathbb{N}$, $x \in [-1,1]$ and $f: [-1,1] \to \mathbb{R}$,

$$C_n'(f;x) = \frac{n}{2^n}\sum_{k=0}^{n-1}\left(f\left(\frac{2k+2}{n}-1\right)-f\left(\frac{2k}{n}-1\right)\right)P_{n-1,k}(x).$$

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Proof

Let $C_n(f;x)$ be defined by

$$C_n(f;x)=\frac{1}{2^n}\sum_{k=0}^{n}\binom{n}{k}(1-x)^{n-k}(1+x)^k f\left(\frac{2k}{n}-1\right).$$

Then its derivative is given by

$$C_n'(f;x)$$

$$=\frac{1}{2^n}\sum_{k=0}^{n} f\left(\frac{2k}{n}-1\right)\binom{n}{k}\left[k(1-x)^{n-k}(x+1)^{k-1}-(n-k)(1-x)^{n-k-1}(x+1)^k\right]$$

$$=\frac{1}{2^n}\sum_{k=1}^{n} f\left(\frac{2k}{n}-1\right)\frac{n!}{(n-k)!\,k!}k(1-x)^{n-k}(x+1)^{k-1}$$

$$-\frac{1}{2^n}\sum_{k=0}^{n} f\left(\frac{2k}{n}-1\right)(n-k)\frac{n!}{k!\,(n-k)!}(1-x)^{n-k-1}(x+1)^k$$

$$=\frac{n}{2^n}\sum_{k=0}^{n-1} f\left(\frac{2k+2}{n}-1\right)P_{n-1,k}(x)-\frac{n}{2^n}\sum_{k=0}^{n} f\left(\frac{2k}{n}-1\right)P_{n-1,k}(x).$$

So,

$$C_n'(f;x)=\frac{n}{2^n}\sum_{k=0}^{n-1}\left(f\left(\frac{2k+2}{n}-1\right)-f\left(\frac{2k}{n}-1\right)\right)P_{n-1,k}(x).$$

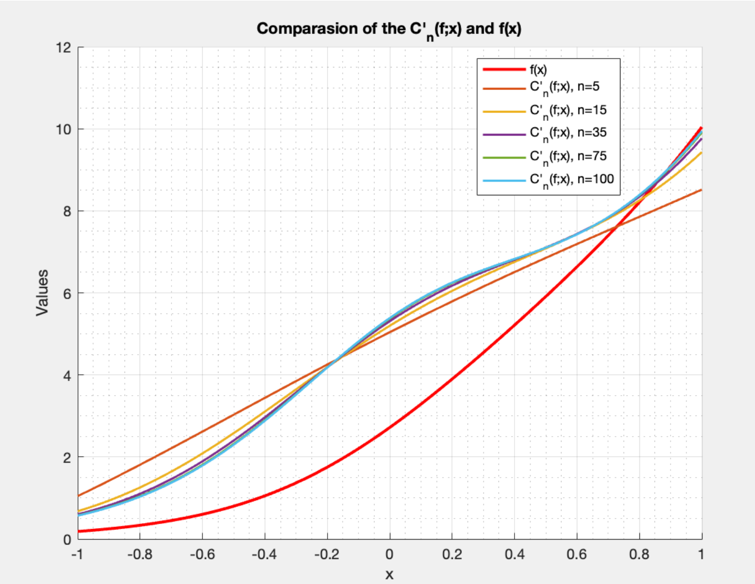

Figure 5: Approximation of the function using $C_n'(f;x)$ as n increases.

Proposition 2.5

For $f:[-1,1] \to \mathbb{R}$, $n > 2$ and $x \in (-1,1)$,

$$(\boldsymbol{i})\quad \psi^4(x)C_n''(f,x) = n\sum_{k=0}^{n}\left[(n-1)\left(2\frac{k}{n}-1-x\right)^2\psi^2\left(2\frac{k}{n}-1\right)\right]f\left(2\frac{k}{n}-1\right)P_{n,k}(x),$$

$$(\boldsymbol{ii})\quad C_n''(f,x) = \frac{n(n-1)}{2^n}\sum_{j=0}^{n-2}\Delta^2_{2/n}f\left(\frac{2j}{n}-1\right)P_{n-2,j}(x)$$

$$(\boldsymbol{iii})\quad \psi^2(x)C_n''(f,x) = \frac{n^2}{2^{n+2}}\sum_{j=1}^{n-1}\Delta^2_{2/n}f\left(\frac{2(k-1)}{n}-1\right)\psi^2\left(\frac{2k}{n}-1\right)P_{n,k}(x).$$

where $\Delta^2_{2/n}f(y)$ denotes the symmetric second order difference.

Proof

$(\boldsymbol{i})$ It is known that

$$\psi^4(x)P_{n,k}''(x) = n\left[\left(2\frac{k}{n}-1-x\right)^2(n-1)+\left(2\frac{k}{n}-1\right)^2-1\right]P_{n,k}(x).$$

Multiplying both sides by $\frac{1}{2^n}f\left(\frac{2k}{n}-1\right)$ and summing over k yields

$$\psi^4(x)C_n''(f;x) = \frac{1}{2^n}\sum_{k=0}^{n}f\left(\frac{2k}{n}-1\right)\psi^4(x)P_{n,k}''(x)$$

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$$= \frac{n}{2^n} \sum_{k=0}^{n} \left[\left(2\frac{k}{n} - 1 - x\right)^2 (n-1) + \left(2\frac{k}{n} - 1\right)^2 - 1 \right] f\left(\frac{2k}{n} - 1\right) P_{n,k}(x).$$

Finally, noting that

$$\left(2\frac{k}{n} - 1\right)^2 - 1 = -\psi^2 \left(\frac{2k}{n} - 1\right),$$

clearly,

$$\psi^4(x) C_n''(f;x) = \frac{n}{2^n} \sum_{k=0}^{n} \left[\left(2\frac{k}{n} - 1 - x\right)^2 (n-1) - \psi^2 \left(\frac{2k}{n} - 1\right) \right] f\left(\frac{2k}{n} - 1\right) P_{n,k}(x).$$

This formula expresses $\psi^4(x) C_n''(f;x)$ explicitly in terms of the function f and the polynomials $P_{n,k}(x)$.

$(\boldsymbol{ii})$ In the latter case, the auxiliary function is to be considered.

$$C_n'(f;x) = \frac{n}{2^n} \sum_{k=0}^{n-1} \left(f\left(\frac{2k+2}{n} - 1\right) - f\left(\frac{2k}{n} - 1\right) \right) P_{n-1,k}(x),$$

$$C_n''(f;x) = \frac{n}{2^n} \sum_{k=0}^{n-1} \left(f\left(\frac{2k+2}{n} - 1\right) - f\left(\frac{2k}{n} - 1\right) \right) P_{n-1,k}'(x).$$

Here, since

$$P_{n-1,k}'(x) = (n-1)\left[P_{n-2,k-1}(x) - P_{n-2,k}(x)\right].$$

Substituting this into the expression yields

$$C_n''(f;x) = \frac{n}{2^n} \sum_{k=0}^{n-1} \Delta f_k (n-1)\left[P_{n-2,k-1}(x) - P_{n-2,k}(x)\right],$$

where

$$\Delta f_k = f\left(\frac{2k+2}{n} - 1\right) - f\left(\frac{2k}{n} - 1\right).$$

After rearranging the constants and separating the sums,

$$C_n''(f;x) = \frac{(n-1)n}{2^n} \left[\sum_{k=0}^{n-1} \Delta f_k P_{n-2,k-1}(x) - \sum_{k=0}^{n-1} \Delta f_k \, P_{n-2,k}(x) \right].$$

In the first summation, applying an index shift $j = k - 1$ (hence $k = 1 + j$) gives $j = -1 \dots n - 2$. Using $P_{n-2,-1}(x) = 0$, $j = 0 \dots n - 2$ it reduces to:

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$$\sum_{k=0}^{n-1} \Delta f_k P_{n-2,k-1}(x) = \sum_{j=0}^{n-2} \Delta f_{j+1} P_{n-2,j}(x).$$

For the second summation, let $j = k$. Using $P_{n-2,n-1}(x) = 0, j = 0 \dots n-2$ it follows that:

$$\sum_{j=0}^{n-2} \Delta f_j P_{n-2,j}(x) = \sum_{k=0}^{n-1} \Delta f_k P_{n-2,k}(x).$$

Thus,

$$C_n''(f;x) = \frac{n(n-1)}{2^n} \sum_{j=0}^{n-2} [\Delta f_{j+1} - \Delta f_i] P_{n-2,j}(x).$$

By using the following equality, the result

$$\Delta f_{j+1} - \Delta f_i = \left[f\left(\frac{2(2+j)}{n} - 1\right) - f\left(\frac{2(1+j)}{n} - 1\right)\right] - \left[f\left(\frac{2(1+j)}{n} - 1\right) - f\left(\frac{j.2}{n} - 1\right)\right]$$

$$= f\left(\frac{2(j+2)}{n} - 1\right) - 2f\left(\frac{2(j+1)}{n} - 1\right) - f\left(\frac{2j}{n} - 1\right)$$

is obtained. This is exactly

$$\Delta_{2/n}^2 f\left(\frac{2j}{n} - 1\right) = f\left(\frac{2(j+2)}{n} - 1\right) - 2f\left(\frac{2(j+1)}{n} - 1\right) - f\left(\frac{2j}{n} - 1\right).$$

Therefore,

$$C_n''(f;x) = \frac{n(n-1)}{2^n} \sum_{j=0}^{n-2} \Delta_{2/n}^2 f\left(\frac{2j}{n} - 1\right) P_{n-2,j}(x).$$

For the final equality, note that

$$\psi^2\left(\frac{2k}{n} - 1\right) = 1 - \left(\frac{2k}{n} - 1\right)^2 = 4\frac{k}{n}\left(1 - \frac{k}{n}\right)$$

and using the identity $\binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1}$,

$$n\psi^2\left(\frac{2k}{n} - 1\right) P_{n,k}(x) = 4(n-k)\binom{n-1}{k-1}(x+1)^k(1-x)^{n-k}.$$

Moreover, since

$$\binom{n-1}{k-1}(n-k) = (n-1)\binom{n-2}{k-1},$$

it follows that

$$n\psi^2\left(\frac{2k}{n} - 1\right) P_{n,k}(x) = 4(n-1)\binom{n-2}{k-1}(1-x)^{n-k}(x+1)^k.$$

Then,

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$$4(n-1)\psi^2\left(\frac{2k}{n}-1\right)P_{n-2,k-1}(x)=4(n-1)\binom{n-2}{k-1}(1-x)^{n-k}(x+1)^k.$$

Hence,

$$n\psi^2\left(\frac{2k}{n}-1\right)P_{n,k}(x)=4(n-1)\psi^2\left(\frac{2k}{n}-1\right)P_{n-2,k-1}(x).$$

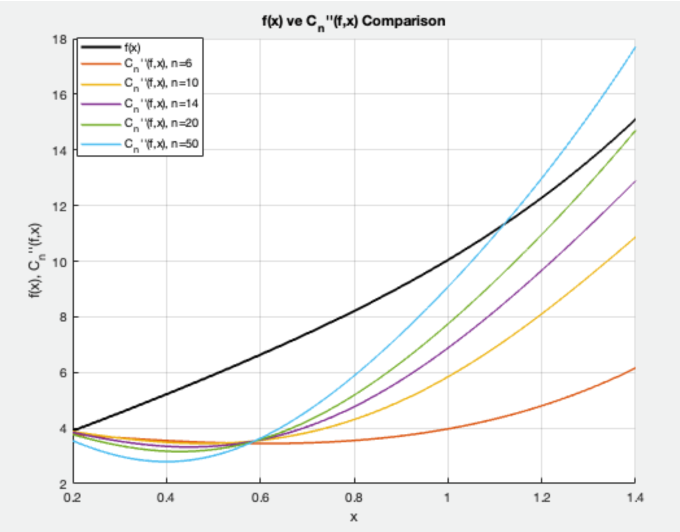

Figure 6: Approximation of the function with $C_n''(f;x)$ as n increases.

$(\boldsymbol{iii})$ Finally, using

$$\psi^2(x)C_n''(f,x)=\frac{n(n-1)}{2^n}\sum_{j=0}^{n-2}\Delta^2_{2/n}f\left(\frac{2j}{n}-1\right)\psi^2(x)P_{n-2,j}(x)$$

and taking $k=j+1$ and substituting

$$\psi^2(x)P_{n-2,j}(x)=\frac{n}{4n-4}\psi^2\left(\frac{2(j+1)}{n}-1\right)P_{n,j+1}(x),$$

then,

$$\psi^2(x)C_n''(f,x)=\frac{n(n-1)}{2^n}\sum_{j=0}^{n-2}\Delta^2_{2/n}f\left(\frac{2j}{n}-1\right)\frac{n}{4(n-1)}\psi^2\left(\frac{2(j+1)}{n}-1\right)P_{n,j+1}(x).$$

Simplifying and applying the index change $k=j+1$, the following equalities is written.

$$\psi^2(x)C_n''(f,x)=\frac{n^2}{2^{n+2}}\sum_{j=0}^{n-2}\Delta^2_{2/n}f\left(\frac{2j}{n}-1\right)\psi^2\left(\frac{2(j+1)}{n}-1\right)P_{n,j+1}(x),$$

$$\psi^2(x)C_n''(f,x)=\frac{n^2}{2^{n+2}}\sum_{k=1}^{n-1}\Delta^2_{2/n}f\left(\frac{2(k-1)}{n}-1\right)\psi^2\left(\frac{2k}{n}-1\right)P_{n,k}(x).$$

Here,

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$$\Delta^2_{2/n} f\left(\frac{(k-1)2}{n}-1\right) = f\left(\frac{(k+1)2}{n}-1\right) - 2f\left(\frac{2k}{n}-1\right) + f\left(\frac{(k-1)2}{n}-1\right).$$

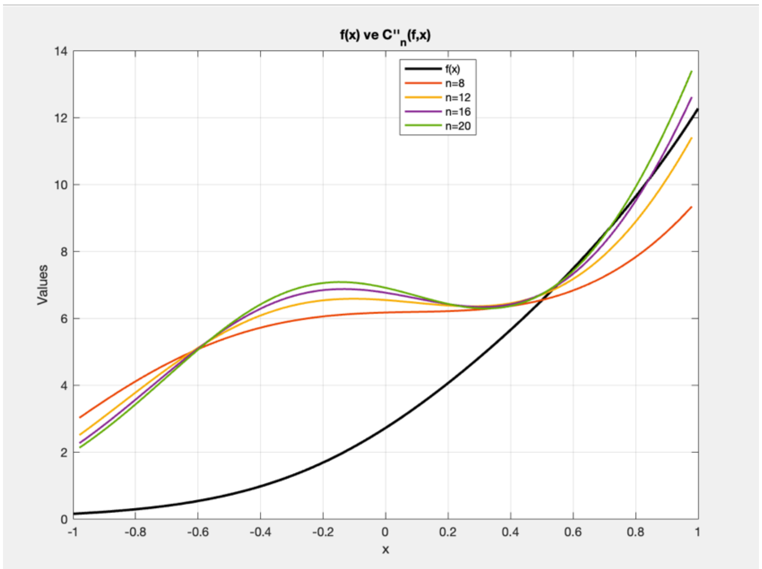

Figure 7: Approximation of the function with $C_n''(f;x)$ as n increases.

Proposition 2.6

For $f:[-1,1] \to \mathbb{R}$, $n > 2$ and $x \in (-1,1)$,

$$C_n'''(f,x) = \frac{(n-2)(n-1)n}{2^n} \sum_{k=0}^{n} f\left(k\frac{2}{n}-1\right) R_n^*(x,k)$$

where

$$R_n^*(x,k) = \left(P_{n-3,k-3}(x) + 3P_{n-3,k-1} - 3P_{n-3,k-2} - P_{n-3,k}(x)\right).$$

Moreover,

$$(x+1)(1-x)C_n'''(f,x) - 2xC_n''(f,x)$$

$$= \frac{n^2}{(1-x)2^{n+2}(x+1)} \sum_{j=1}^{n-1} \Delta^2_{2/n} f\left(\frac{2(k-1)}{n}-1\right)\psi^2\left(\frac{2k}{n}-1\right)[2k-n-nx]P_{n,k}(x).$$

Proof

For convenience, the second equality will be demonstrated first. Differentiating the formula for the second derivative, For the second derivative, the expression $(1-x)(x+1)C_n''(f,x)$ is differentiated, and

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$$\psi^2(x)C_n''(f,x) = \frac{n^2}{2^{2+n}}\sum_{j=1}^{n-1}\Delta^2_{2/n}f\left(\frac{(k-1)2}{n}-1\right)\psi^2\left(\frac{2k}{n}-1\right)P_{n,k}(x).$$

is obtained. Taking the derivative with respect to x:

$$\frac{d}{dx}[C_n''(f,x)\psi^2(x)] = C_n'''(f,x)\psi^2(x) - 2xC_n''(f,x)$$

and using $\frac{d}{dx}\psi^2(x) = -2x$,

Now if the sum is differentiated:

$$\frac{d}{dx}\left[\frac{n^2}{2^{n+2}}\sum_{j=1}^{n-1}\Delta^2_{2/n}f\left(\frac{2(k-1)}{n}-1\right)\psi^2\left(\frac{2k}{n}-1\right)P_{n,k}(x)\right]$$

$$= \sum_{j=1}^{n-1}\Delta^2_{2/n}f\left(\frac{2(k-1)}{n}-1\right)\psi^2\left(\frac{2k}{n}-1\right)P'_{n,k}(x).$$

Using the identity $P'_{n,k}(x)\psi^2(x) = P_{n,k}(x)[2k-n-nx]$,

$$P'_{n,k}(x) = \frac{[2k-n-nx]P_{n,k}(x)}{\psi^2(x)}.$$

And if these expressions are substituted

$$\sum_{j=1}^{n-1}\Delta^2_{2/n}f\left(\frac{2(k-1)}{n}-1\right)\psi^2\left(\frac{2k}{n}-1\right)\frac{[2k-n-nx]P_{n,k}(x)}{\psi^2(x)}.$$

Thus,

$$\frac{d}{dx}[\psi^2(x)C_n''(f,x)] = \frac{n^2}{2^{n+2}}\sum_{j=1}^{n-1}\Delta^2_{2/n}f\left(\frac{2(k-1)}{n}-1\right)\psi^2\left(\frac{2k}{n}-1\right)\frac{[2k-n-nx]P_{n,k}(x)}{\psi^2(x)}.$$

Therefore,

$$C_n'''(f,x) - \psi^2(x)2xC_n''(f,x)$$

$$= \frac{n^2}{2^{2+n}\psi^2(x)}\sum_{j=1}^{n-1}\Delta^2_{2/n}f\left(\frac{2(k-1)}{n}-1\right)\psi^2\left(\frac{2k}{n}-1\right)[2k-n-nx]P_{n,k}(x).$$

Here using $\psi^2(x) = (1-x)(x+1)$, the corrected identity is obtained.

$$(1-x)(x+1)C_n'''(f,x) - 2xC_n''(f,x)$$

$$= \frac{n^2}{(1-x)2^{n+2}(x+1)}\sum_{j=1}^{n-1}\Delta^2_{2/n}f\left(\frac{2(k-1)}{n}-1\right)\psi^2\left(\frac{2k}{n}-1\right)[2k-n-nx]P_{n,k}(x).$$

On the other hand, consider the second derivative of the operator:

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$$C_n''(f;x) = \frac{n(n-1)}{2^n}\sum_{j=0}^{n-2} \Delta^2_{2/n} f\left(\frac{2j}{n}-1\right) P_{n-2,j}(x).$$

Differentiating this expression with respect to x yields

$$C_n'''(f,x) = \frac{n(n-1)}{2^n}\sum_{j=0}^{n-2} \Delta^2_{2/n} f\left(\frac{2j}{n}-1\right) \frac{d}{dx} P_{n-2,j}(x).$$

By employing the well-known derivative formula for Bernstein type polynomials given in (1),

$$P'_{m,k}(x) = m\left(P_{m-1,k-1}(x) - P_{m-1,k}(x)\right)$$

and setting $m = n-2$,

$$P'_{n-2,j}(x) = \left(P_{n-3,j-1}(x) - P_{n-3,j}(x)\right)(n-2)$$

is obtained.

Consequently,

$$C_n'''(f,x) = \frac{n(n-1)(n-2)}{2^n}\sum_{j=0}^{n-2} \Delta^2_{2/n} f\left(\frac{2j}{n}-1\right)\left(P_{n-3,j-1}(x) - P_{n-3,j}(x)\right).$$

Expanding the second-order difference explicitly,

$\Delta^2_{2/n} f\left(\frac{2j}{n}-1\right) = f\left(\frac{2(j+2)}{n}-1\right) - 2f\left(\frac{2(j+1)}{n}-1\right) + f\left(\frac{2j}{n}-1\right)$and substituting it into the previous expression, allows the sum to be separated as follows:

$$C_n'''(f,x) = \frac{(n-2)(n-1)n}{2^n}\left[\sum_{j=0}^{n-2} f\left(\frac{(j+2)2}{n}-1\right)\left(P_{n-3,j-1}(x) - P_{n-3,j}(x)\right)\right.$$

$$-2\sum_{j=0}^{n-2} f\left(\frac{2(j+1)}{n}-1\right)\left(P_{n-3,j-1}(x) - P_{n-3,j}(x)\right)$$

$$+\sum_{j=0}^{n-2} f\left(\frac{2j}{n}-1\right)\left(P_{n-3,j-1}(x) - P_{n-3,j}(x)\right).$$

Performing appropriate index shifts ($i = j+2$ for the first sum, $i = j+1$ for the second, and $i = j$ for the third) allows each sum to be expressed in terms of $f\left(\frac{2i}{n}-1\right)$. The coefficient of The coefficient of $f\left(\frac{2i}{n}-1\right)$ is then given by

$$P_{n-3,i-3}(x) - 3P_{n-3,i-2}(x) - P_{n-3,i}(x) + 3P_{n-3,i-1}(x)$$

which is denote by

$$R_n^*(x,i) := \left(P_{n-3,i-3}(x) - 3P_{n-3,i-2} + 3P_{n-3,i-1} - P_{n-3,i}(x)\right).$$

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Hence, the final representation is arrived at:

$$C_n'''(f,x) = \frac{(n-2)(n-1)n}{2^n} \sum_{i=0}^{n} f\left(2\frac{i}{n} - 1\right) R_n^*(x,i)$$

which completes the proof.

3. The Central Moments

For the moments, whose importance was recognized by Bernstein, the following notations will be used in the study of the operators in (1). For $n \in \mathbb{N}$ and $r = 0,1, \dots$ set

$$S_{n,r}(x) = \sum_{k=0}^{n} \left(\frac{2k}{n} - 1 - x\right)^r P_{n,k}(x) = C_n((e_1 - x)^r, x),$$

and

$$T_{n,r}(x) = n^r S_{n,r}(x).$$

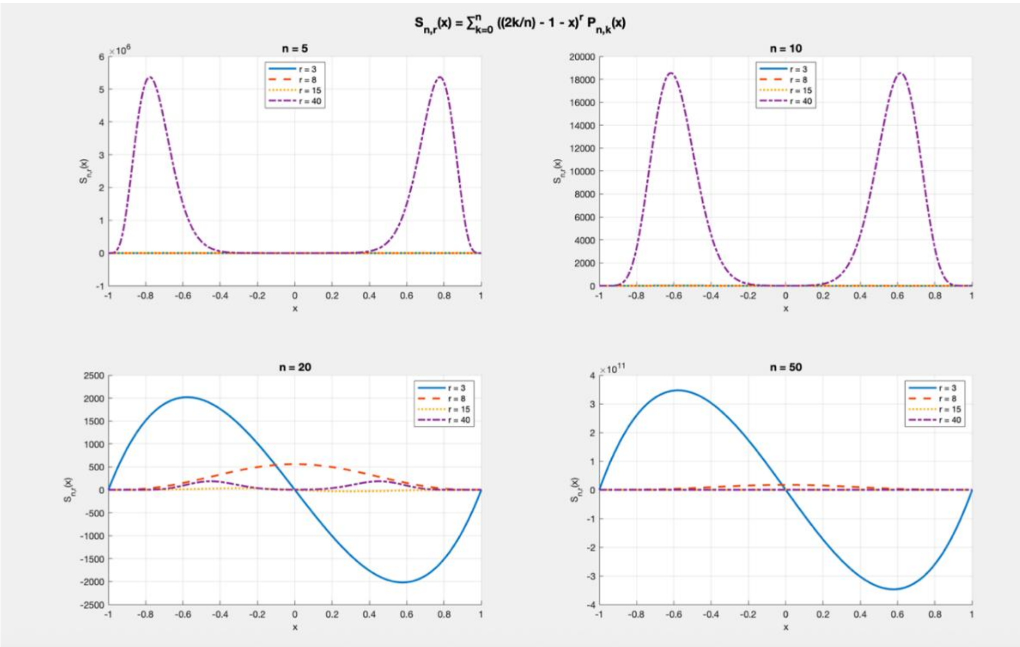

Figure 8: Graphical representation of $S_{n,r}(x)$ for $n = 5,10,20$ and 50 as r increases.

The functions $T_{n,r}(x)$ arise in certain generalized Voronovskaya-type theorems. It should be noted that

$S_{n,0}(x) = 1$ and $S_{n,1}(x) = 0$,

$$S_{n,0}(x) = \sum_{k=0}^{n} \left(\left(\frac{2k}{n} - 1 - x\right) - x\right)^0 P_{n,k}(x) = C_n\left(\left(\left(\frac{2k}{n} - 1 - x\right) - x\right)^0, x\right),$$

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$$= C_n(1,x) = \frac{1}{2^n}\sum_{k=0}^{n}\binom{n}{k}(1-x)^{n-k}(x+1)^k = 1,$$

$$S_{n,1}(x) = C_n((x-x)^1, x) = 0.$$

Proposition 3.1

For $m \geq 1$,

$$S_{n,m+1}(x) = \frac{(1-x)(x+1)}{n}\left[S'_{n,m}(x) - mS_{n,m-1}(x)\right].$$

Proof:

$$S_{n,m}(x) = \sum_{k=0}^{n}\left(\frac{2k}{n} - 1 - x\right)^m P_{n,k}(x)$$

and

$$S'_{n,m}(x) = \sum_{k=0}^{n}\left[-m\left(\frac{2k}{n} - 1 - x\right)^{m-1} P_{n,k}(x) + \left(\frac{2k}{n} - 1 - x\right)^m P'_{n,k}(x)\right]$$

Rewriting,

$$S'_{n,m}(x) = -m\sum_{k=0}^{n}\left(\frac{2k}{n} - 1 - x\right)^{m-1} P_{n,k}(x) + \sum_{k=0}^{n}\left(\frac{2k}{n} - 1 - x\right)^m P'_{n,k}(x).$$

which can be expressed as

$$S'_{n,m}(x) = -mS_{n,m-1}(x) + \frac{1}{(1-x)(x+1)}\sum_{k=0}^{n}\left(\frac{2k}{n} - 1 - x\right)^m (2k - n - nx)P_{n,k}(x)$$

$$= -mS_{n,m-1}(x) + \frac{1}{(1-x)(x+1)}\sum_{k=0}^{n}\left(\frac{2k}{n} - 1 - x\right)^m n\left(\frac{2k}{n} - 1 - x\right)P_{n,k}(x)$$

$$S'_{n,m}(x) = -mS_{n,m-1}(x) + \frac{n}{(1-x)(x+1)}\sum_{k=0}^{n}\left(\frac{2k}{n} - 1 - x\right)^{m+1} P_{n,k}(x).$$

On the other hand, by definition,

$$S_{n,m+1}(x) = \sum_{k=0}^{n}\left(\frac{2k}{n} - 1 - x\right)^{m+1} P_{n,k}(x).$$

Consequently,

$$S'_{n,m}(x) - mS_{n,m-1}(x) = \frac{n}{(1-x)(x+1)}S_{n,m+1}(x)$$

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which immediately gives the desired relation:

$$S_{n,m+1}(x) = \frac{(1-x)(x+1)}{n}\left[S'_{n,m}(x) - mS_{n,m-1}(x)\right].$$

Exercise 3.1 For n > 2, verify the identities

$$(\boldsymbol{i})2^{1-n}\sum_{k=0}^{n-1}\left(x-\frac{2k}{n}+1\right)^2 P_{n-1,k}(x) = \frac{(x+1)^2}{n^2} + \frac{(1-x)(n-1)(x+1)}{n^2}$$

$$(\boldsymbol{ii})\ 2^{1-n}\sum_{k=0}^{n-1}\left(x-2\frac{k+1}{n}+1\right)^2 P_{n-1,k}(x) = \frac{(x-1)^2}{n^2} + \frac{(1-x)(x+1)(n-1)}{n^2}$$

Solution

$(\boldsymbol{i})$ Using the binomial represantation of Bernstein polynomials,

$$P_{n-1,k}(x) = \binom{n-1}{k}(x+1)^k(1-x)^{n-1-k},$$

$$\sum_{k=0}^{n-1}\left(x+1-\frac{2k}{n}\right)^2 P_{n-1,k}(x) = \sum_{k=0}^{n-1}\left(x+1-\frac{2k}{n}\right)^2 P_{n-1,k}(x)$$

$$= \sum_{k=0}^{n-1}\left(x^2+1+\frac{4k^2}{n^2}+2x-\frac{4k}{n}-\frac{4kx}{n}\right)P_{n-1,k}(x)$$

$$= (x+1)^2\sum_{k=0}^{n-1}P_{n-1,k}(x) - \frac{4}{n}(x+1)\sum_{k=0}^{n-1}k\,P_{n-1,k}(x) + \frac{4}{n^2}\sum_{k=0}^{n-1}k^2\,P_{n-1,k}(x)$$

Applying the classical Bernstein polynomial moments:

$$\sum_{k=0}^{n-1}P_{n-1,k}(x) = 2^{n-1}, \sum_{k=0}^{n-1}kP_{n-1,k}(x) = 2^{n-2}\,(x+1)(n-1),$$

$$\sum_{k=0}^{n-1}k^2P_{n-1,k}(x) = 2^{n-3}(n-1)\,(x+1)^2(n-1).$$

The following is obtained by substituting these values:

$$2^{1-n}\sum_{k=0}^{n-1}\left(x-\frac{2k}{n}+1\right)^2 P_{n-1,k}(x) = \frac{(x+1)^2}{n^2} + \frac{(1-x)(x+1)(n-1)}{n^2}.$$

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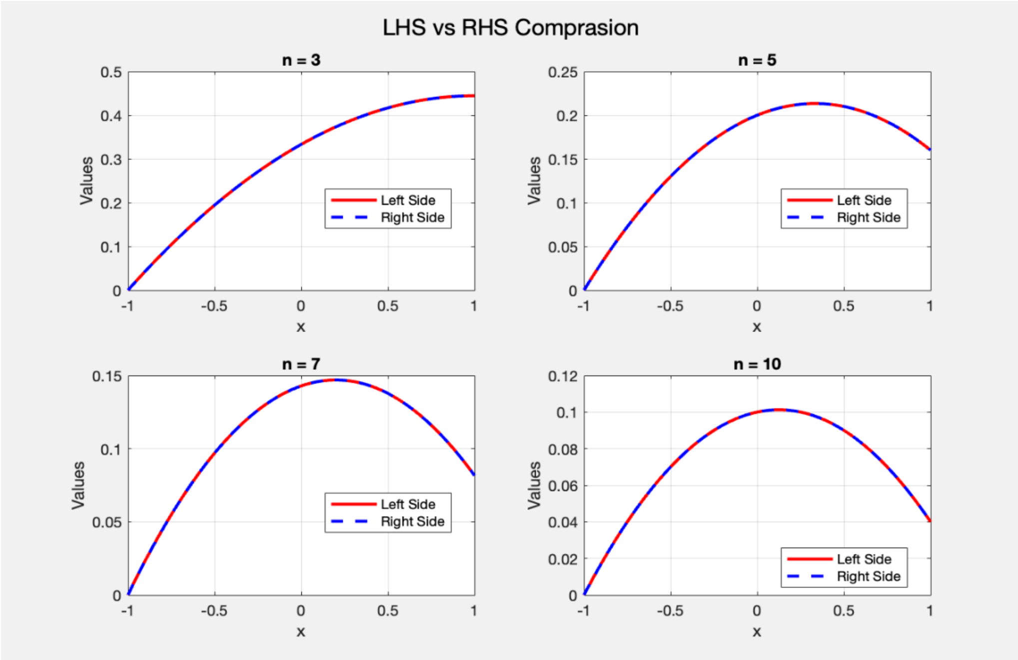

Figure 9: Graphical comparison of the two sides of the equality for $n = 3,5,7$ and 10.

$(\boldsymbol{ii})$ Observe that

$$x - 2\frac{k+1}{n} + 1 = x - 2\frac{k}{n} + 1 - \frac{2}{n}$$

Squaring both sides yields

$$x - 2\frac{k+1}{n} + 1 = \left(x - 2\frac{k}{n} + 1\right)^2 + \left(x - 2\frac{k}{n} + 1\right)\frac{4}{n} + \frac{4}{n^2}.$$

substituting in to the sum gives

$$2^{1-n}\sum_{k=0}^{n-1}\left(x - 2\frac{k+1}{n} + 1\right)^2 P_{n-1,k}(x) = 2^{1-n}\sum_{k=0}^{n-1}\left(x - 2\frac{k}{n} + 1\right)^2 P_{n-1,k}(x)$$

$$-2^{1-n}\sum_{k=0}^{n-1}\left(\left(x - 2\frac{k}{n} + 1\right)\frac{4}{n}\right)P_{n-1,k}(x) + 2^{1-n}\sum_{k=0}^{n-1}\frac{4}{n^2}P_{n-1,k}(x).$$

Using the moments:

$$2^{1-n}\sum_{k=0}^{n-1}\left(x - 2\frac{k}{n} + 1\right)^2 P_{n-1,k}(x) = \frac{(x+1)^2}{n^2} + \frac{(1-x)(x+1)(n-1)}{n^2},$$

$$2^{1-n}\sum_{k=0}^{n-1}\left(x - 2\frac{k}{n} + 1\right)P_{n-1,k}(x) = x + 1,$$

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$$2^{1-n}\sum_{k=0}^{n-1}P_{n-1,k}(x)=1.$$

Substituting these values into the previous expression yields

$$2^{1-n}\sum_{k=0}^{n-1}\left(x-2\frac{k+1}{n}+1\right)^2P_{n-1,k}(x)$$
$$=\frac{(x+1)^2}{n^2}+\frac{(1-x)(n-1)(x+1)}{n^2}-(x+1)\frac{4}{n^2}+\frac{4}{n^2}$$
$$=\frac{(x-1)^2}{n^2}+\frac{(1-x)(n-1)(x+1)}{n^2}.$$

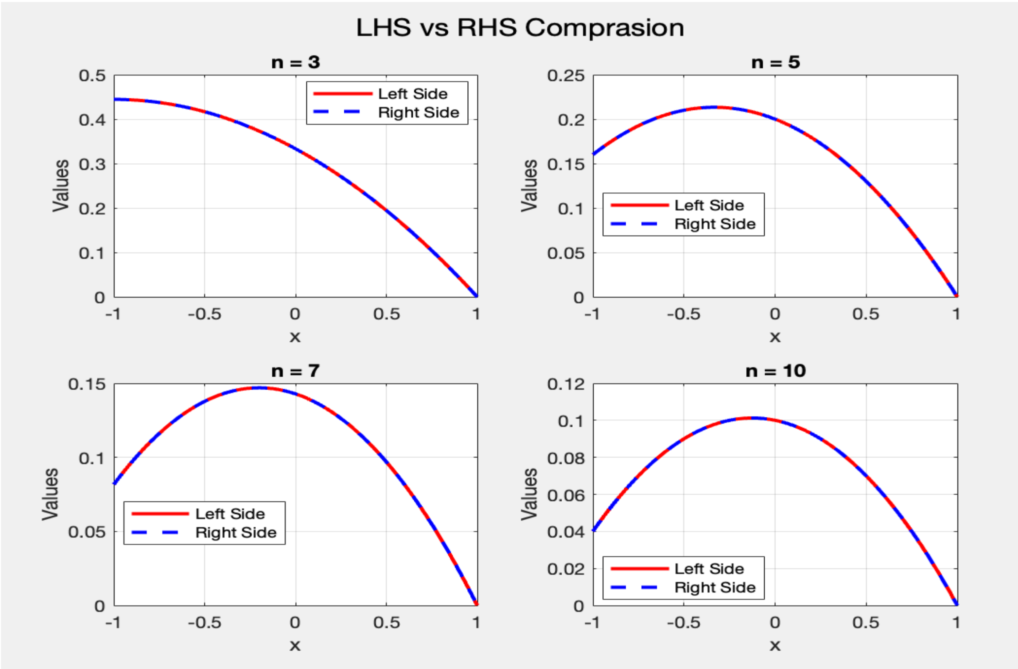

Figure 10: Graphical comparison of the two sides of the equality for $n=3{,}5{,}7$ and 10.

Conclusion

In this study, the derivatives of a generalized class of Bernstein-type operators are analyzed. Furthermore, by utilizing certain algebraic identities, new structural relationships between the polynomial families associated with these operators are derived. The obtained results not only generalize the known properties of classical Bernstein polynomials but also provide useful tools for future applications in approximation theory. Accordingly, various approximation properties of the derivatives of this operator are investigated; in particular, the study examines the error bounds that can be achieved for classes of differentiable functions and presents results related to derivative-based approximation. In future work, the present study can be applied to operators related to the Bernstein operator (Güven and Gönül Bilgin, 2022a; 2022b). Moreover, by combining the polynomials used in this study with the numbers and polynomials in (Bilgin and

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Eren, 2021; Soykan, et al. 2023; Soykan, Y. 2023; Bilgin, 2024) studies, new equalities and inequalities can be obtained.

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CHROMATIC ZAGREB INDEX OF ZERO DIVISOR GRAPHS

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ÖZET

Algebraic graph theory has become an important field due to its applications in chemistry, cryptology, and information sciences. In this study, we focus on the chromatic first Zagreb index, a recently introduced topological graph invariant that extends the classical Zagreb index by incorporating vertex colorings. While the classical Zagreb index depends solely on the degrees of vertices, its chromatic counterpart provides a more refined structural measure by considering graph colorings and their associated parameters. The main objective of this paper is to investigate the chromatic first Zagreb indices for zero-divisor graphs of commutative rings of the form $\mathbb{Z}_{\xi^k}$ $(k \geq 2)$, $\mathbb{Z}_{\xi\rho}$, $\mathbb{Z}_{\xi^2\rho}$ and $\mathbb{Z}_{\xi^2\rho^2}$ where ξ and ρ denote prime numbers. Explicit formulas for both the minimum and maximum chromatic first Zagreb indices are derived for these families of graphs. Furthermore, we compare the obtained results with the classical first Zagreb indices, highlighting significant differences and growth rates depending on the algebraic structure under consideration.

The analysis reveals that for certain cases, such as $\Gamma(\mathbb{Z}_4)$, $\Gamma(\mathbb{Z}_6)$, $\Gamma(\mathbb{Z}_9)$ and $\Gamma(\mathbb{Z}_{16})$ the chromatic first Zagreb index can exceed the classical Zagreb index, while in most other cases the classical Zagreb index dominates. Additionally, it is shown that the values of $CM_{1,t}^{-}$ are always smaller than those of $CM_{1,t}^{+}$ and M_1 confirming the non-triviality of chromatic variations. For $k \geq 3$, the growth rate of the chromatic Zagreb index surpasses that of the classical index, whereas for rings such as $\mathbb{Z}_{\xi^2\rho^2}$ the opposite behavior is observed.

These findings provide new insights into the interplay between algebraic ring structures and graph invariants. The results not only enrich the theoretical framework of graph indices but also suggest potential applications in cryptology, algorithm design, and chemical graph theory, where algebraic structures are frequently employed to model complex systems.

Anahtar Kelimeler : Graph Thoery, Topological index, Chromatic first Zagreb index, Zero-divisor graphs of commutative rings.

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1. INTRODUCTION

In the rapidly developing pharmaceutical industry, there is an increasing need to predict chemical structure properties without experimental procedures. Graph indices, a fundamental component of chemical graph theory since 1947 [16], address this need by providing mathematical representations of chemical structures. Among these, the Zagreb index which is degree-dependent has garnered significant attention.

The first Zagreb index of a graph G with vertex set V and edge set E is defined as follows [9]:

$$M_1(G) = \sum_{w \in V} {d_w}^2$$

where d_w represents the degree of vertex w.

Recently, graph indices based on vertex coloring have been introduced, extending their applicability to problems involving graph models of daily life scenarios [10]. Proper vertex coloring assigns distinct colors to adjacent vertices, effectively labeling vertices with integers. Let N be positive integers and coloring be a function. In that case, $\boldsymbol{\varsigma}: \boldsymbol{V} \to \mathbb{N}$ is a function such that $\boldsymbol{\varsigma}(\boldsymbol{u}) \neq \boldsymbol{\varsigma}(\boldsymbol{v})$ for $uv \in E$. If a graph G is colored by the γ-minimum positive integer, then the graph G is γ-coloring [6]. A graph G is labeled with γ-coloring by the cardinality $\gamma!$. Kok et al. [10] defined chromatic Zagreb indices as follows:

Definition 1.1. Let G be a graph γ-coloring such that $\varsigma(\varpi) \in N$ and $1 \leq \varsigma(\varpi) \leq \gamma$ for $\varpi \in V$. Therefore when $1 \leq t \leq \gamma!$, the chromatic first Zagreb index of G [10]:

$$CM_{1,t}(G) = \sum_{w \in V} \varsigma(w)^2$$

The minimum and maximum chromatic first Zagreb indices are defined as

$$CM_{1,t}{}^{-}(G) = \min\{CM_{1,t}(G): 1 \leq t \leq \gamma!$$

$$CM_{1,t}{}^{+}(G) = \max\{CM_{1,t}(G): 1 \leq t \leq \gamma!$$

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The first work on combining algebraic structures with graph theory is Cayley graphs of finite groups, introduced by Arthur Cayley [5]. Algebraic graph structures are used in many areas. Dong et al. suggested robot design with the subgroups of dihedral group [7]. Kotorowicz and Ustimenko worked on cryptoalgorithms with al- gebraic graphs [11]. Shaska and Ustimenko studied on applications of coding theory and cryptography [14].
Zero-divisor graphs, first introduced by Beck [4] and refined by Anderson and Livingston [3], are another prominent class. These graphs $\Gamma(R)$ represent elements of commutative rings R, where vertices correspond to non-zero zero divisors and edges connect vertices whose product is a zero divisor. Throughout this study, ϱ, p, p_1, p_2, q are prime numbers.
Rayer and Jeyaraj studied eccentric based indices of a commutative ring [13]. The eccentric index based indices of $Z_{p_1p_2} \times Z_q$ and $Z_{p^2} \times Z_q$ were studied [8]. Ahmadi and Nezad studied some indices of Z_{pq} and Z_{p^2} [1]. Alali et al. given results on M- polynomials of some algebraic graphs [2]. Mazlan et al. found first Zagreb index of $\Gamma(Z_p)$ [12]. Semil et al. studied first Zagreb index of $\Gamma(Z_{p^k})$ [15].

This paper focuses on the chromatic first Zagreb indices for the zero-divisor graphs of $\mathbb{Z}_{\xi^k}$ $(k \geq 2)$, $\mathbb{Z}_{\xi\rho}$, $\mathbb{Z}_{\xi^2\rho}$ and $\mathbb{Z}_{\xi^2\rho^2}$. The results are compared with classical Zagreb indices, emphasizing their potential utility in cryptology and algorithm analysis.

2. Chromatic Index of Zero-Divisor Graphs

In this section, chromatic first Zagreb indices $\mathbb{Z}_{\xi^k}$ $(k \geq 2)$, $\mathbb{Z}_{\xi\rho}$, $\mathbb{Z}_{\xi^2\rho}$ and $\mathbb{Z}_{\xi^2\rho^2}$ zero-divisor graph are found.

Theorem 2.1. If $\varrho = 2$ then $CM_{1,t}^{-}(\Gamma(\mathbb{Z}_{\varrho^2})) = {CM_{1,t}}^{+}(\Gamma(\mathbb{Z}_{\varrho^2})) = 3$

Proof. When $\varrho = 2$, the ring $R = \mathbb{Z}_{\varrho^2}$ has elements $\{1, 2, 3\}$, and the graph $\Gamma(\mathbb{Z}_{\varrho^2})$ has an empty edge set. The chromatic number is 1, resulting in all vertices labeled with the same color. Since $\gamma = 1$, Using Eq. (1.2), (1.3) and (1.4), it have

$$CM_{1,t}^{-}(\Gamma(\mathbb{Z}_{\varrho^2})) = {CM_{1,t}}^{+}(\Gamma(\mathbb{Z}_{\varrho^2})) = 1^2 + 1^2 + 1^2 = 3.$$

Corollary 2.1. For $\varrho = 2$, $M_1\left(\Gamma\left(\mathbb{Z}_{\varrho^2}\right)\right) = 0 < CM_{1,t}^{-}(\Gamma(\mathbb{Z}_{\varrho^2})) = {CM_{1,t}}^{+}(\Gamma(\mathbb{Z}_{\varrho^2}))$.
Proof. When $\varrho = 2$, the ring $R = \mathbb{Z}_{\varrho^2}$ has elements $\{1, 2, 3\}$, and the graph $\Gamma(\mathbb{Z}_{\varrho^2})$ has an empty edge set. The chromatic number is 1, resulting in all vertices labeled with the same color. Since $\gamma = 1$, Using Eq. (1.2), (1.3) and (1.4), it have

$$CM_{1,t}^{-}(\Gamma(\mathbb{Z}_{\varrho^2})) = {CM_{1,t}}^{+}(\Gamma(\mathbb{Z}_{\varrho^2})) = 1^2 + 1^2 + 1^2 = 3.$$

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Theorem 2.2. If $\varrho \geq 3$ then $CM_{1,t}^{-}\left(\Gamma(\mathbb{Z}_{\varrho^2})\right) = {CM_{1,t}}^{+}\left(\Gamma(\mathbb{Z}_{\varrho^2})\right) = \frac{\varrho(\varrho-1)(2\varrho-1)}{6}$.

Proof. Let $\varrho \geq 3$. In that case, $\Gamma(\mathbb{Z}_{\varrho^2}) \cong K_{\varrho-1}$ and $|V(\Gamma(\mathbb{Z}_{\varrho^2}))| = \varrho - 1$. Therefore, each vertex is assigned with a different color and so this graph is $(\varrho - 1)$-coloring. Then, $\gamma = \varrho - 1$. From Eq. (1.2),

$$CM_{1,t}(\Gamma(\mathbb{Z}_{\varrho^2})) = \sum_{w \in V} \varsigma(w)^2 = \sum_{i=1}^{\varrho-1} i^2 = \frac{\varrho(\varrho-1)(2\varrho-1)}{6}.$$

From Eq. (1.3) and (1.4), the result is obtained.

Figure 2 shows plot of $CM_{1,t}^{-}\left(\Gamma(\mathbb{Z}_{\varrho^2})\right), {CM_{1,t}}^{+}\left(\Gamma(\mathbb{Z}_{\varrho^2})\right)$ and $M_1\left(\Gamma(\mathbb{Z}_{\varrho^2})\right)$ for $\varrho \geq 3$.

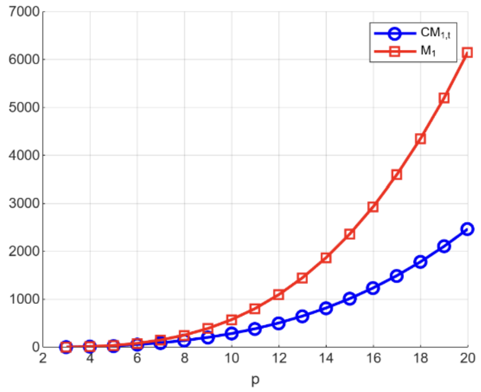

Figure 1. Plot of $CM_{1,t}^{-}\left(\Gamma(\mathbb{Z}_{\varrho^2})\right), {CM_{1,t}}^{+}\left(\Gamma(\mathbb{Z}_{\varrho^2})\right)$ and $M_1\left(\Gamma(\mathbb{Z}_{\varrho^2})\right)$ for $\varrho \geq 3$

Corollary 2.2. If $\varrho = 3$ or $\varrho = 4$ then $d(w) = \varrho - 2$ for $w \in V$ and

$$M_1\left(\Gamma(\mathbb{Z}_{\varrho^2})\right) = (\varrho-1)(\varrho-2)^2 < CM_{1,t}^{-}(\Gamma(\mathbb{Z}_{\varrho^2})) = {CM_{1,t}}^{+}(\Gamma(\mathbb{Z}_{\varrho^2})).$$

For $\varrho \geq 5$,

$$M_1\left(\Gamma(\mathbb{Z}_{\varrho^2})\right) = (\varrho-1)(\varrho-2)^2 > CM_{1,t}^{-}(\Gamma(\mathbb{Z}_{\varrho^2})) = {CM_{1,t}}^{+}(\Gamma(\mathbb{Z}_{\varrho^2})).$$

Theorem 2.3. If $\varrho \geq 3$ and $\xi \geq 3$, then

$${CM_{1,t}}^{+}\left(\Gamma\left(\mathbb{Z}_{\varrho^\xi}\right)\right) = \varrho^{\xi+k}\left(\varrho^{k-1} - 1\right) + \frac{\varrho^k(\varrho^k - 1)(2\varrho^k - 1)}{6}$$

$${CM_{1,t}}^{-}\left(\Gamma\left(\mathbb{Z}_{\varrho^\xi}\right)\right) = \varrho^{\xi-k}\left(\varrho^{k-1} - 1\right) + \frac{\varrho^k(\varrho^k - 1)(2\varrho^k - 1)}{6} - 1$$

where ξ is odd then $k = \frac{\xi+1}{2}$, otherwise $k = \frac{\xi}{2}$.

Proof. $V\left(\Gamma\left(\mathbb{Z}_{\varrho^\xi}\right)\right)$ can be separated as follows

$$\mho_1 = \{\varrho\lambda | 1 \leq \lambda \leq \varrho^{\xi-1} - 1, \varrho \nmid \lambda\}$$
$$\mho_2 = \{\varrho^2\lambda | 1 \leq \lambda \leq \varrho^{\xi-2} - 1, \varrho \nmid \lambda\}$$

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for $i = 3,4,..,\xi - 2$

$$\mho_i = \{\varrho^i\lambda | 1 \le \lambda \le \varrho^{\xi-i} - 1, \varrho \nmid \lambda\}$$

and

$$\mho_{\xi-1} = \{\varrho^{\xi-1}\lambda | 1 \le \lambda \le \varrho - 1, \varrho \nmid \lambda\}$$

It have that $|\mho_i| = \varrho^{\xi-i-1}(\varrho - 1)$ for $i = 1,2,..,\xi - 1$. If ξ is odd then $k = \frac{\xi+1}{2}$, otherwise $k = \frac{\xi}{2}$. Let ~ Show that there is a adjacent relationship.

$\mho_1 \sim \mho_{\xi-1}$
$\mho_2 \sim \mho_{\xi-2}$, $\mho_2 \sim \mho_{\xi-1}$
…
$\mho_{i-1} \sim \mho_j$ for $i < k$ and $j = i + 1, \dots, \xi - 1$
$\mho_i \sim \mho_i$, $\mho_i \sim \mho_j$ for $i \ge k$ and $j = i + 1, \dots, \xi - 1$.

Since $\mho_1 \nsim \mho_1$, all vertices in $\mho_1$ are assigned the same color (labeled with the same number). Likewise, if i < k then $\mho_i \nsim \mho_i$ and all vertices in $\mho_i$ are assigned the same color. If $\mho_i \sim \mho_i$ then all vertices in $\mho_i$ are assigned $|\mho_i|$ different colors. Then, for 1 < i < k, all vertices in Ʊi are assigned the same colors. For ξ−1 ≥ i ≥ k, all vertices in $\mho_i$ are assigned the different colors. Then, for coloring vertices of this graph are needed $\sum_{i=k}^{\xi-1}|\mho_i| + 1$. That is,

$$\gamma = \sum_{i=k}^{\xi-1}|\mho_i| + 1 = \sum_{i=k}^{\xi-1}\left(\varrho^{\xi-i-1}(\varrho - 1)\right) + 1 = (\varrho - 1)\sum_{i=k}^{\xi-1}\varrho^i + 1 = \varrho^k$$

From Eq. (1.2), it can be the following equation:

$$CM_{1,t}{}^{-}\left(\Gamma\left(\mathbb{Z}_{\varrho^\xi}\right)\right) = |\mho_1|1^2 + \dots + |\mho_{k-1}|1^2 + \sum_{i=2}^{\gamma} i^2$$
$$= \sum_{i=1}^{k}|\mho_1| + \sum_{i=2}^{\gamma} i^2 = (\varrho - 1)\sum_{i=2}^{k}\varrho^{\xi-i} + \frac{\gamma(\gamma + 1)(2\gamma + 1)}{6} - 1$$
$$= \varrho^\xi(\varrho - 1)\frac{1 - \varrho^{k-1}}{\varrho^k(1 - \varrho)} + \frac{\varrho^k(\varrho^k + 1)(2\varrho^k + 1)}{6} - 1.$$

If ϖ ∈ Ʊj , 1 < j < k then ς(ϖ) = γ from Eq. (1.4). All other vertices are assigned different numbers. Using Eq. (1.2),

$$CM_{1,t}{}^{+}\left(\Gamma\left(\mathbb{Z}_{\varrho^\xi}\right)\right) = |\mho_1|\gamma^2 + \dots + |\mho_{k-1}|\gamma^2 + \sum_{i=1}^{\gamma-1} i^2$$
$$= \gamma^2\sum_{i=1}^{k-1}|\mho_i| + \sum_{i=1}^{\gamma-1} i^2 = \varrho^{2k}\varrho^\xi(\varrho - 1)\frac{1 - \varrho^{k-1}}{\varrho^k(1 - \varrho)} + \frac{\gamma(\gamma - 1)(2\gamma - 1)}{6}$$

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$$= \varrho^{2k}\varrho^{\xi}(\varrho-1)\frac{1-\varrho^{k-1}}{\varrho^{k}(1-\varrho)}+\frac{\varrho^{k}(\varrho^{k}-1)(2\varrho^{k}-1)}{6}.$$

Corollary 2.3. Since $d(w)=\varrho-1$ for $w\in \mho_1$, $d(w)=\varrho^2-1$ for $w\in \mho_2$, $d(w)=\varrho^i-1$ for $w\in \mho_i$ when 1<j<k, $d(w)=\varrho^i-2$ for $w\in \mho_i$ when $\xi-1\geq i\geq k$, the following inequality is obtained:

$$CM_{1,t}^{-}\left(\Gamma\left(\mathbb{Z}_{\varrho^{\xi}}\right)\right)<M_1\left(\Gamma\left(\mathbb{Z}_{\varrho^{\xi}}\right)\right)<CM_{1,t}{}^{+}(\Gamma(\mathbb{Z}_{\varrho^{\xi}}))$$

where $M_1\left(\Gamma\left(\mathbb{Z}_{\varrho^{\xi}}\right)\right)=\varrho^{\xi}(\varrho-1)\left(\sum_{i=1}^{k-1}\frac{(\varrho^{i}-1)^2}{\varrho^{i+1}}+\sum_{i=k}^{\xi-1}\frac{(\varrho^{i}-2)^2}{\varrho^{i+1}}\right)$.

Figure 2 shows plot of $CM_{1,t}^{-}\left(\Gamma\left(\mathbb{Z}_{\varrho^{\xi}}\right)\right), M_1\left(\Gamma\left(\mathbb{Z}_{\varrho^{\xi}}\right)\right), CM_{1,t}{}^{+}(\Gamma(\mathbb{Z}_{\varrho^{\xi}}))$ for $\xi=5$.

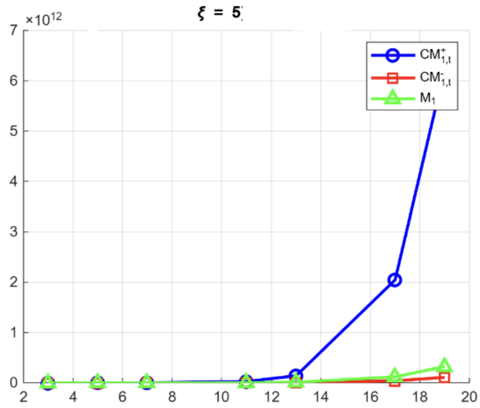

Figure 2. The plot of $CM_{1,t}^{-}\left(\Gamma\left(\mathbb{Z}_{\varrho^{\xi}}\right)\right), M_1\left(\Gamma\left(\mathbb{Z}_{\varrho^{\xi}}\right)\right), CM_{1,t}{}^{+}(\Gamma(\mathbb{Z}_{\varrho^{\xi}}))$

Theorem 2.4. Let $\Gamma(\mathbb{Z}_{\varrho q})$ be graph of $\mathbb{Z}_{\varrho q}$ non-zero zero divisor rings. If $q>\varrho$ then,

$CM_{1,t}^{-}\left(\Gamma(\mathbb{Z}_{\varrho q})\right)=4\varrho+q-5,$

$CM_{1,t}^{+}\left(\Gamma(\mathbb{Z}_{\varrho q})\right)=4q+\varrho-5.$

Proof. $V(\Gamma(\mathbb{Z}_{\varrho q}))$ can be divided:

$$\mho_1=\{\varrho\lambda|1\leq\lambda\leq q-1,\varrho\nmid\lambda\}$$
$$\mho_2=\{q\lambda|1\leq\lambda\leq \varrho-1,\varrho\nmid\lambda\}$$

$\Gamma(\mathbb{Z}_{\varrho q})$ graph is complete bipartite graph. $\mho_1\sim\mho_2$, $\mho_1\nsim\mho_1$ and $\mho_2\nsim\mho_2$. So, the vertices of this graph are labeled with 1 and 2. Then, $\gamma=2$. Since $q>\varrho$, $|\mho_1|=q-1>|\mho_2|=\varrho-1$. From Eq. (1.2) and (1.3),

$$CM_{1,t}^{-}\left(\Gamma(\mathbb{Z}_{\varrho q})\right)=\sum_{w\in\mho_1}\varsigma(w)^2+\sum_{w\in\mho_2}\varsigma(w)^2=|\mho_1|1^2+|\mho_2|2^2$$
$$=(q-1)+(\varrho-1)4=4\varrho+q-5.$$

From Eq. (1.2) and (1.4), the following equation is obtained:

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$$CM_{1,t}^{+}\left(\Gamma(\mathbb{Z}_{\varrho q})\right) = \sum_{w \in \mho_1} \varsigma(w)^2 + \sum_{w \in \mho_2} \varsigma(w)^2 = |\mho_1|2^2 + |\mho_2|1^2$$
$$= 4(q-1) + (\varrho - 1) = 4q + \varrho - 5.$$

Corollary 2.4. $d(w) = \varrho - 1$ for $w \in \mho_1$ and $d(w) = q - 1$ for $w \in \mho_2$ because $\Gamma(\mathbb{Z}_{\varrho q})$ is complete bipartite graph. Let $\varrho = 2$ and $q = 3$. Then

$$M_1\left(\Gamma(\mathbb{Z}_{\varrho q})\right) = 6 = CM_{1,t}^{-}\left(\Gamma(\mathbb{Z}_{\varrho q})\right) < CM_{1,t}^{+}\left(\Gamma(\mathbb{Z}_{\varrho q})\right).$$

Otherwise,

$$M_1\left(\Gamma(\mathbb{Z}_{\varrho q})\right) = (\varrho - 1)(q - 1)(\varrho + q - 2) > CM_{1,t}^{+}\left(\Gamma(\mathbb{Z}_{\varrho q})\right) > CM_{1,t}^{-}\left(\Gamma(\mathbb{Z}_{\varrho q})\right).$$

Figure 3 shows plot of $M_1\left(\Gamma(\mathbb{Z}_{\varrho q})\right)$, $CM_{1,t}^{+}\left(\Gamma(\mathbb{Z}_{\varrho q})\right)$, $CM_{1,t}^{-}\left(\Gamma(\mathbb{Z}_{\varrho q})\right)$ for $q > \varrho$.

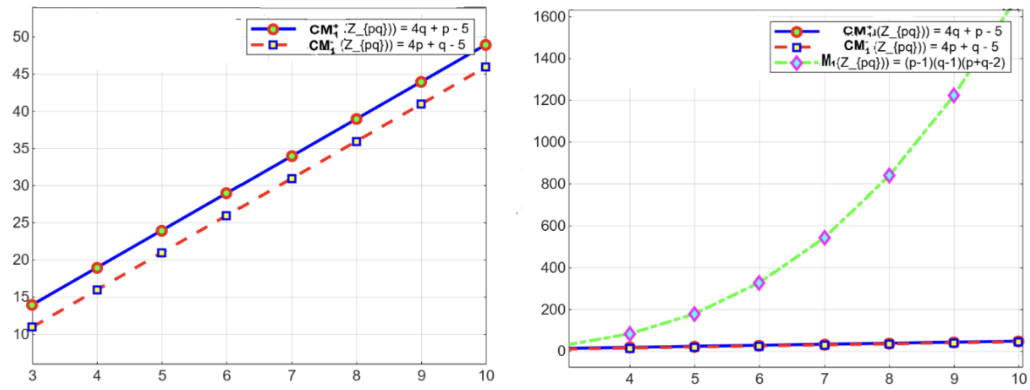

Figure 3. The plot of $M_1\left(\Gamma(\mathbb{Z}_{\varrho q})\right)$, $CM_{1,t}^{+}\left(\Gamma(\mathbb{Z}_{\varrho q})\right)$, $CM_{1,t}^{-}\left(\Gamma(\mathbb{Z}_{\varrho q})\right)$ for $q > \varrho$.

Theorem 2.5. Let $\Gamma(\mathbb{Z}_{\varrho^2 q})$ be graph of $\mathbb{Z}_{\varrho^2 q}$ non-zero zero divisor rings. If $q > \varrho$ then,

$CM_{1,t}^{+}\left(\Gamma(\mathbb{Z}_{\varrho^2 q})\right) = (q-1)\varrho^3 + \varrho(\varrho - 1)^3 + \frac{\varrho(\varrho-1)(2\varrho-1)}{6}$,

$CM_{1,t}^{-}\left(\Gamma(\mathbb{Z}_{\varrho^2 q})\right) = \varrho q + \frac{9}{2}\varrho^2 + \frac{\varrho^3}{3} - \frac{29\varrho}{6} - 1.$

Proof. $V(\Gamma(\mathbb{Z}_{\varrho^2 q}))$ can be divided:

$\mho_1 = \{\varrho\lambda | 1 \le \lambda \le \varrho q - 1, \varrho \nmid \lambda, q \nmid \lambda\}$, $|\mho_1| = (q-1)(\varrho - 1)$
$\mho_2 = \{q\lambda | 1 \le \lambda \le \varrho^2 - 1, \varrho \nmid \lambda\}$, $|\mho_2| = \varrho(\varrho - 1)$
$\mho_3 = \{\varrho^2\lambda | 1 \le \lambda \le q - 1, \varrho \nmid \lambda, q \nmid \lambda\}$, $|\mho_3| = (q-1)$
$\mho_4 = \{\varrho q\lambda | 1 \le \lambda \le \varrho - 1, q \nmid \lambda\}$, $|\mho_4| = (\varrho - 1)$

As can be seen from the vertex set division above, there are the following adjacent relationships between the vertices: $\mho_1 \sim \mho_4$, $\mho_2 \sim \mho_3$, $\mho_3 \sim \mho_4$ and $\mho_4 \sim \mho_4$.

Since $\mho_4 \sim \mho_4$, at least $|\mho_4|$ numbers are required. Since $\mho_3 \sim \mho_4$ and $\mho_3 \nsim \mho_3$, all vertices in $\mho_1$ and $\mho_3$ are labeled with the same number. Since $\mho_2 \nsim \mho_4$, the vertices in $\mho_2$ are labeled with one of the labels in $\mho_4$. Therefore, $\gamma = |\mho_4| + 1 = \varrho - 1 + 1 = \varrho$.

From Eq. (1.2),

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$$CM_{1,t}^{-}\left(\Gamma(\mathbb{Z}_{\varrho^2 q})\right) = \sum_{w\in\mho_1} \varsigma(w)^2 + \sum_{w\in\mho_2} \varsigma(w)^2 + \sum_{w\in\mho_3} \varsigma(w)^2 + \sum_{w\in\mho_4} \varsigma(w)^2$$

Since $q > \varrho$, there is $|\mho_4| < |\mho_3| < |\mho_2| < |\mho_1|$. Selected $\varsigma(w) = 1$ for $w \in \mho_1, \mho_3$ and $\varsigma(w) = 2$ for $w \in \mho_2$ then

$$CM_{1,t}^{-}\left(\Gamma(\mathbb{Z}_{\varrho^2 q})\right) = \sum_{w\in\mho_1} 1^2 + \sum_{w\in\mho_2} 2^2 + \sum_{w\in\mho_3} 1^2 + \sum_{i=2}^{\varrho} i^2$$

$$= (q-1)(\varrho-1) + 4\varrho(\varrho-1) + (q-1) + \left[\frac{\varrho(\varrho+1)(2\varrho+1)}{6} - 1\right]$$

$$= \varrho q + \frac{9}{2}\varrho^2 + \frac{\varrho^3}{3} - \frac{29\varrho}{6} - 1.$$

Selected $\varsigma(w) = \varrho$ for $w \in \mho_1, \mho_3$ and $\varsigma(w) = \varrho - 1$ for $w \in \mho_2$ then

$$CM_{1,t}^{+}\left(\Gamma(\mathbb{Z}_{\varrho^2 q})\right) = \sum_{w\in\mho_1} \varrho^2 + \sum_{w\in\mho_2} (\varrho-1)^2 + \sum_{w\in\mho_3} \varrho^2 + \sum_{i=1}^{\varrho-1} i^2$$

$$= \varrho^2(q-1)(\varrho-1) + \varrho(\varrho-1)(\varrho-1)^2 + \varrho^2(q-1) + \left[\frac{\varrho(\varrho-1)(2\varrho-1)}{6}\right].$$

Corollary 2.5. Since $\varsigma(w) = \varrho - 1$ for $w \in \mho_1$ and $\varsigma(w) = q - 1$ for $w \in \mho_2$, and $\varsigma(w) = (\varrho-1)(\varrho+1)$ for $w \in \mho_3$, and $\varsigma(w) = (\varrho-1)(q-1) + (q-1) + (\varrho-1) - 1$ for $w \in \mho_4$, the following inequality is obtained:

$$M_1\left(\Gamma(\mathbb{Z}_{\varrho^2 q})\right) > CM_{1,t}^{+}\left(\Gamma(\mathbb{Z}_{\varrho^2 q})\right) > CM_{1,t}^{-}\left(\Gamma(\mathbb{Z}_{\varrho^2 q})\right)$$

where $M_1\left(\Gamma(\mathbb{Z}_{\varrho^2 q})\right) = (\varrho-1)(q-1)^2(\varrho q - q - \varrho + 1) + (\varrho-1)((\varrho-1)^2(q-1) + (\varrho q - 2)^2)$.

Figure 4 shows plot of $M_1\left(\Gamma(\mathbb{Z}_{\varrho^2 q})\right), CM_{1,t}^{+}\left(\Gamma(\mathbb{Z}_{\varrho^2 q})\right), CM_{1,t}^{-}\left(\Gamma(\mathbb{Z}_{\varrho^2 q})\right)$.

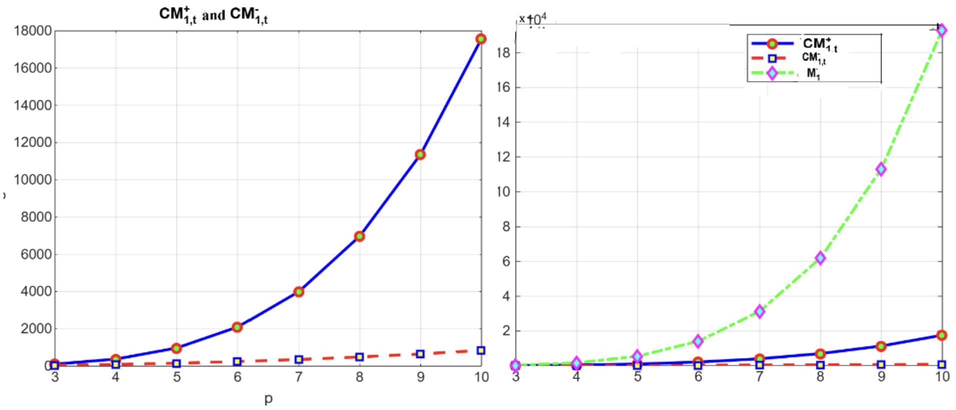

Figure 4. The plot of $M_1\left(\Gamma(\mathbb{Z}_{\varrho^2 q})\right)$, $CM_{1,t}^{+}\left(\Gamma(\mathbb{Z}_{\varrho^2 q})\right)$, $CM_{1,t}^{-}\left(\Gamma(\mathbb{Z}_{\varrho^2 q})\right)$ for $q > \varrho$.

Theorem 2.6. Let $\Gamma(\mathbb{Z}_{\varrho^2 q^2})$ be graph of $\mathbb{Z}_{\varrho^2 q^2}$ non-zero zero divisor rings. If $q < \varrho$ then,

$CM_{1,t}^{+}\left(\Gamma(\mathbb{Z}_{\varrho^2 q^2})\right) = \varrho(q-1)(\varrho q-1)^2(q+\varrho-1) + 4\varrho(\varrho-1)(\varrho q-2)^2 + \frac{\varrho q(\varrho q-1)(2\varrho q-1)}{6}$,

$CM_{1,t}^{-}\left(\Gamma(\mathbb{Z}_{\varrho^2 q^2})\right) = \varrho q(q+\varrho-2) + 3\varrho(\varrho-1) + \frac{\varrho q(\varrho q-1)(2\varrho q-1)}{6}$.

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Proof. $V(\Gamma(\mathbb{Z}_{\varrho^2 q^2}))$ can be divided:
$\mho_1 = \{\varrho\lambda | 1 \le \lambda \le \varrho q^2 - 1, \varrho \nmid \lambda, q \nmid \lambda\}$, $|\mho_1| = q(q-1)(\varrho-1)$
$\mho_2 = \{q\lambda | 1 \le \lambda \le \varrho^2 q - 1, \varrho \nmid \lambda, q \nmid \lambda\}$, $|\mho_2| = \varrho(\varrho-1)(q-1)$
$\mho_3 = \{\varrho^2\lambda | 1 \le \lambda \le q^2 - 1, \varrho \nmid \lambda, \}$, $|\mho_3| = q(q-1)$
$\mho_4 = \{q^2\lambda | 1 \le \lambda \le \varrho^2 - 1, q \nmid \lambda\}$, $|\mho_4| = \varrho(\varrho-1)$
$\mho_5 = \{\varrho q\lambda | 1 \le \lambda \le \varrho q - 1, \varrho \nmid \lambda, q \nmid \lambda\}$, $|\mho_5| = (q-1)(\varrho-1)$
$\mho_6 = \{\varrho^2 q\lambda | 1 \le \lambda \le q - 1, q \nmid \lambda\}$, $|\mho_6| = (q-1)$
$\mho_7 = \{\varrho q^2\lambda | 1 \le \lambda \le \varrho - 1, q \nmid \lambda\}$, $|\mho_7| = (\varrho-1)$

There are relations $|\mho_7| < |\mho_6| < |\mho_4| < |\mho_5| < |\mho_3| < |\mho_2| < |\mho_1|$ since $q > \varrho$. When $\varrho = 2$ and $q = 3$, $|\mho_4| = |\mho_5|$. As can be seen from the vertex set division above, there are the following adjacent relationships between the vertices: $\mho_1 \sim \mho_7$, $\mho_2 \sim \mho_6$, $\mho_3 \sim \mho_4$, $\mho_3 \sim \mho_7, \mho_4 \sim \mho_6$, $\mho_5 \sim \mho_5$, $\mho_5 \sim \mho_6$, $\mho_5 \sim \mho_7$, $\mho_6 \sim \mho_6$, $\mho_6 \sim \mho_7$ and $\mho_7 \sim \mho_7$. Since $\mho_5 \sim \mho_5$, $\mho_6 \sim \mho_6$, $\mho_7 \sim \mho_7$, and the vertices of these three sets are adjacent to each other, at least $|\mho_5| + |\mho_6| + |\mho_7|$ numbers are required. The vertices in $\mho_1$ can be labeled with one of the labels in $\mho_5$ and $\mho_6$. The vertices in $\mho_2$ can be labeled with one of the labels in $\mho_5$ or $\mho_7$. The vertices in $\mho_3$ can be labeled with one of the labels in $\mho_5$ or $\mho_6$. $\mho_4$ can be labeled with one of the labels in $\mho_5$ or $\mho_7$. Then, $\gamma = (\varrho-1)(q-1) + \varrho - 1 + q - 1 = \varrho q - 1$.

For minimum, $\varsigma(w) = 1$ is chosen when $w \in \mho_1, \mho_2, \mho_3$. The labeling of the vertices in $\mho_5$, which has the most vertices compared to $\mho_6$ and $\mho_7$, starts from 1. $\varsigma(w) = 2$ is chosen when $w \in \mho_4$. From Eq. (1.2),

$$CM_{1,t}^{-}\left(\Gamma(\mathbb{Z}_{\varrho^2 q^2})\right)$$

$$= \sum_{w\in\mho_1} 1^2 + \sum_{w\in\mho_2} 1^2 + \sum_{w\in\mho_3} 1^2 + \sum_{w\in\mho_4} 2^2 + \sum_{i=1}^{(\varrho-1)(q-1)} i^2 + \sum_{i=(\varrho-1)(q-1)+1}^{(\varrho q-\varrho)} i^2 + \sum_{i=\varrho q-\varrho+1}^{\varrho q-1} i^2$$

$$= q(\varrho-1)(q-1) + \varrho(\varrho-1)(q-1) + q(q-1) + 4\varrho(\varrho-1) + \sum_{i=1}^{\varrho q-1} i^2$$

$$= \varrho q(q+\varrho-2) + 3\varrho(\varrho-1) + \frac{\varrho q(\varrho q-1)(2\varrho q-1)}{6}.$$

For equation to be maximum: $\varsigma(w) = \varrho q - 1$ is chosen when $w \in \mho_1, \mho_2, \mho_3$. $\varsigma(w) = \varrho q - 2$ is chosen when $w \in \mho_4$. The labeling of the vertices in $\mho_5, \mho_6, \mho_7$ starts from $\varrho + q$, ϱ, 1, respectively. From (1.3),

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$$CM_{1,t}^{+}\left(\Gamma(\mathbb{Z}_{\varrho^2 q^2})\right)$$

$$= \sum_{w\in\mho_1}(\varrho q-1)^2 + \sum_{w\in\mho_2}(\varrho q-1)^2 + \sum_{w\in\mho_3}(\varrho q-1)^2 + \sum_{w\in\mho_4}(\varrho q-2)^2 + \sum_{i=(\varrho+q)}^{(\varrho q-1)} i^2$$

$$+ \sum_{i=\varrho}^{\varrho+q+1} i^2 + \sum_{i=1}^{\varrho-1} i^2$$

$$= q(\varrho-1)(q-1)(\varrho q-1)^2 + \varrho(\varrho-1)(q-1)(\varrho q-1)^2 + q(q-1)(\varrho q-1)^2$$

$$+ 4\varrho(\varrho-1)(\varrho q-2)^2 + \sum_{i=1}^{\varrho q-1} i^2$$

$$= (q-1)\varrho(\varrho q-1)^2(\varrho+q-1) + 4\varrho(\varrho-1)(\varrho q-2)^2 + \frac{\varrho q(\varrho q-1)(2\varrho q-1)}{6}$$

Corollary 2.6. Since $\varsigma(w) = \varrho - 1$ is chosen when $w \in \mho_1$, $\varsigma(w) = q - 1$ is chosen when $w \in \mho_2$, $\varsigma(w) = (\varrho-1)(\varrho+1)$ for $w \in \mho_3$, $\varsigma(w) = (q-1)(q+1)$ for $w \in \mho_4$, $\varsigma(w) = (\varrho q - 2)$ for $w \in \mho_5$, $\varsigma(w) = q(\varrho-1)(\varrho+1) + q - 2$ for $w \in \mho_6$, and $\varsigma(w) = \varrho(q-1)(q+1) + \varrho - 2$ for $w \in \mho_7$, the following inequality is obtained:

$$CM_{1,t}^{+}\left(\Gamma(\mathbb{Z}_{\varrho^2 q^2})\right) > M_1\left(\Gamma(\mathbb{Z}_{\varrho^2 q^2})\right) > CM_{1,t}^{-}\left(\Gamma(\mathbb{Z}_{\varrho^2 q^2})\right)$$

Where $M_1\left(\Gamma(\mathbb{Z}_{\varrho^2 q^2})\right) = (\varrho-1)^2 q(q-1)(\varrho^2 + 3\varrho + q(\varrho+1)^2) + (q-1)^2\varrho(\varrho-1)(q^2 + 3q + \varrho(q+1)^2) + (\varrho^2-1)\left((\varrho q-2)^2 + 2q(q-1)(q-2)\right) + (q-1)(q-2)^2 + 2\varrho(\varrho-1)(\varrho-2)(q^2-1) + (\varrho-1)(\varrho-2)^2$.

Figure 5 shows plot of $CM_{1,t}^{+}\left(\Gamma(\mathbb{Z}_{\varrho^2 q^2})\right), M_1\left(\Gamma(\mathbb{Z}_{\varrho^2 q^2})\right), CM_{1,t}^{-}\left(\Gamma(\mathbb{Z}_{\varrho^2 q^2})\right)$ for $q > \varrho$.

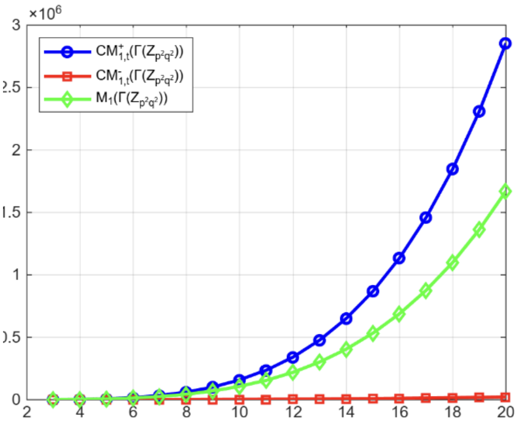

Figure 5. The plot of $M_1\left(\Gamma(\mathbb{Z}_{\varrho^2 q^2})\right)$, $CM_{1,t}^{+}\left(\Gamma(\mathbb{Z}_{\varrho^2 q^2})\right), CM_{1,t}^{-}\left(\Gamma(\mathbb{Z}_{\varrho^2 q^2})\right)$.

3. Conclusion

This study evaluates the chromatic first Zagreb indices for zero-divisor graphs derived from commutative rings $\mathbb{Z}_{\varrho^k}$ ($k \geq 2$), $\mathbb{Z}_{\varrho q}$, $\mathbb{Z}_{\varrho^2 q}$, and $\mathbb{Z}_{\varrho^2 q^2}$. It is seen that the first Zagreb index values of the studied algebraic structures are greater than the chromatic first Zagreb index values except for the graphs $\Gamma(\mathbb{Z}_4)$, $\Gamma(\mathbb{Z}_6)$, $\Gamma(\mathbb{Z}_9)$,$\Gamma(\mathbb{Z}_{16})$. In the graphs $\Gamma(\mathbb{Z}_4)$, $\Gamma(\mathbb{Z}_9)$,$\Gamma(\mathbb{Z}_{16})$, the chromatic first Zagreb indices are greater than the first Zagreb indices and also the minimum and maximum chromatic Zagreb indices are equal. In the graph $\Gamma(\mathbb{Z}_6)$, the first Zagreb index and the minimum chromatic first Zagreb index are equal and less than the maximum chromatic first Zagreb index. The index values of $CM_{1,t}^{-}$ for all algebraic graphs examined are always smaller than the index values of $CM_{1,t}^{+}$ and M_1. For k ≥ 3, the value of CM+ 1,t(Γ(Zϱk)) grows faster than the value of $M_1(\Gamma(\mathbb{Z}_{\varrho^k}))$. While M_1 of the graphs $\mathbb{Z}_{\varrho q}$, $\mathbb{Z}_{\varrho^2 q}$ grows faster than $CM_{1,t}^{+}$, the opposite is true for the graph $\mathbb{Z}_{\varrho^2 q^2}$.

The results can be used in applications in many fields such as cryptology, algorithm analysis and coding theory, by allowing the structural properties of commutative rings and zero divisors to be predicted. It also allows the discovery of the properties of many chemical structures similar to the graphs of $\mathbb{Z}_{\varrho^k}$ (k ≥ 2), $\mathbb{Z}_{\varrho q}$, $\mathbb{Z}_{\varrho^2 q}$, and $\mathbb{Z}_{\varrho^2 q^2}$ structure

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OPTIMIZING RESOURCE ALLOCATION IN DYNAMIC NETWORKS USING HEURISTIC METHODS

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Abstract

Dynamic networks, such as mobile ad hoc networks, wireless sensor systems, and distributed computing environments, require adaptive resource allocation strategies to handle fluctuating topologies and uncertain data traffic. Traditional optimization techniques often fail to deliver satisfactory results in real-time due to computational complexity and the stochastic nature of network conditions. This study presents an integrated heuristic framework for optimizing resource allocation in dynamic networks using hybrid metaheuristic algorithms that combine genetic algorithms, simulated annealing, and particle swarm optimization. The proposed model dynamically adjusts to environmental changes by analyzing network feedback in real time and reconfiguring resource distribution among nodes. Simulation results conducted across multiple network scenarios demonstrate that the hybrid heuristic method improves resource utilization efficiency by 18–27% compared to baseline algorithms, while reducing packet loss and transmission delay. Furthermore, the model shows strong scalability, allowing it to be applied to heterogeneous network environments. Sensitivity analysis indicates that the adaptive weighting of heuristic parameters significantly enhances robustness against topology variations. The research highlights the potential of heuristic methods as powerful tools for managing dynamic, uncertain, and large-scale network infrastructures. Future work includes integrating deep reinforcement learning to further improve decision-making efficiency and extend applicability to 5G and IoT-based architectures.

Keywords: heuristic optimization, dynamic networks, resource allocation, adaptive algorithms

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DEEP LEARNING-DRIVEN ANALYSIS FOR EARLY DETECTION OF CROP STRESS

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Abstract
The early detection of crop stress is crucial for improving agricultural productivity and ensuring food security. Conventional methods, which rely on manual inspection and spectral indices, often fail to detect subtle physiological changes in plants before significant yield losses occur. This paper proposes a deep learning-driven framework that employs convolutional neural networks (CNNs) and recurrent neural networks (RNNs) to analyze multispectral and thermal imagery for early identification of crop stress factors such as water deficiency, nutrient imbalance, and pest infestation. The model processes high-resolution images obtained from UAVs and ground-based sensors, extracting temporal-spatial features that capture micro-level changes in leaf color, texture, and canopy temperature. Experimental results on maize and soybean crops across three growing seasons show that the proposed system achieves 94.3% accuracy in distinguishing between stressed and healthy plants—significantly outperforming classical machine learning approaches. Furthermore, the system demonstrates strong generalizability across different climatic zones. The integration of explainable AI techniques allows for visual interpretation of model outputs, supporting agronomists in targeted decision-making. This research contributes to precision agriculture by enhancing predictive monitoring and enabling proactive crop management strategies.

Keywords: deep learning, crop stress detection, precision agriculture, CNN-RNN analysis

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INTELLIGENT DECISION-MAKING IN ORGANIZATIONAL STRATEGIES USING FUZZY LOGIC

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Abstract
Decision-making in organizational contexts often involves uncertainty, ambiguity, and incomplete information. Traditional quantitative models, which assume precise data inputs, are limited in their ability to handle the inherent fuzziness of strategic environments. This research introduces a fuzzy logic-based decision-making framework for optimizing organizational strategies under uncertain conditions. The model incorporates linguistic variables and membership functions to quantify qualitative factors such as employee satisfaction, innovation potential, and market adaptability. Using a multi-criteria decision-making (MCDM) approach, the system evaluates strategic alternatives and assigns fuzzy weights based on expert input. A case study of three multinational firms demonstrates the framework's capacity to support balanced strategic decisions that integrate human judgment and computational analysis. Results show that fuzzy inference systems outperform conventional decision trees in adaptability and stakeholder satisfaction by 23%. The research also presents a hybrid fuzzy-AHP model that enhances transparency and consistency in long-term strategic planning. This approach is especially effective for industries undergoing digital transformation, where uncertainty is a dominant factor.

Keywords: fuzzy logic, organizational strategy, decision-making, uncertainty management

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ADAPTIVE STOCHASTIC GRADIENT METHODS FOR NON-CONVEX OPTIMIZATION

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Abstract

Non-convex optimization plays a critical role in modern machine learning, deep learning, and data-driven modeling. However, traditional stochastic gradient descent (SGD) methods often converge to suboptimal points when facing highly non-linear loss landscapes. This paper proposes an adaptive stochastic gradient framework that integrates momentum correction, variance reduction, and learning-rate adaptation to improve convergence performance in non-convex optimization problems. The proposed method, termed Adaptive-SGD++, utilizes dynamic variance scaling to stabilize gradient updates and avoid premature convergence. Experimental validation on benchmark datasets, including CIFAR-10 and ImageNet, demonstrates a 32% improvement in training speed and a 15% reduction in generalization error compared to Adam and RMSprop. Theoretical analysis proves convergence guarantees under mild smoothness assumptions. Additionally, we explore the method's robustness against saddle points through Hessian-based diagnostics. The study provides a generalized understanding of adaptive optimization behavior in complex learning systems, with implications for large-scale neural network training and scientific computation.

Keywords: non-convex optimization, stochastic gradient descent, adaptive learning rate, deep learning

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COSMIC INFLATION AND THE ROLE OF QUANTUM VACUUM FLUCTUATIONS

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Abstract

Cosmic inflation, the rapid exponential expansion of the early universe, remains one of the most significant phenomena in cosmology, explaining the observed large-scale homogeneity and flatness of the universe. This study revisits the role of quantum vacuum fluctuations as the seeds of cosmic structure formation. Using recent cosmological observations from Planck and BICEP2, we analyze inflationary models incorporating scalar field dynamics and quantum perturbation spectra. The research investigates how microscopic quantum fluctuations evolved into macroscopic density variations, leading to galaxy formation. The results indicate that the amplitude of vacuum fluctuations is strongly influenced by the potential shape of the inflaton field. Numerical simulations suggest that models with sub-Planckian field values provide consistent predictions with observed power spectra and tensor-to-scalar ratios. Additionally, the study discusses the implications of vacuum energy decay and reheating processes following inflation. By linking quantum field theory with cosmological data, this work deepens our understanding of the early universe's quantum foundations and opens directions for exploring new inflationary potentials compatible with dark energy dynamics.

Keywords: cosmic inflation, quantum vacuum, scalar fields, cosmology

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EXTENDED RÉNYI ENTROPY AND APPLICATIONS IN COMPLEX SYSTEMS

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Abstract
Entropy-based measures are fundamental in analyzing the complexity and uncertainty of physical, biological, and informational systems. This paper explores an extended formulation of Rényi entropy and its applications in modeling nonlinear dynamics and complex adaptive systems. By generalizing the standard Rényi framework through a non-extensive parameterization, the proposed model captures higher-order dependencies and long-range interactions often neglected in classical statistical mechanics. Analytical derivations reveal that the extended entropy satisfies generalized additivity and stability conditions. Case studies across ecological networks, traffic flow dynamics, and financial volatility demonstrate the model's capacity to characterize emergent behaviors and multifractal structures. The results confirm that extended Rényi entropy provides more accurate representations of real-world system irregularities compared to Shannon and Tsallis formulations. Furthermore, the research highlights potential applications in information theory, quantum computation, and neural data analysis, where complexity quantification plays a pivotal role. The study concludes by proposing a new entropy-based metric for resilience evaluation in coupled networks.

Keywords: Rényi entropy, complex systems, nonlinearity, information theory

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EXACT ANALYTICAL SOLUTION OF THIRD ORDER NONLINEAR DIFFERENTIAL EQUATIONS

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Abstract

Nonlinear differential equations of higher order are critical for describing numerous phenomena in physics, engineering, and mathematical biology. This study presents exact analytical solutions for a class of third-order nonlinear differential equations with variable coefficients using the extended homogeneous balance method. The derived solutions are expressed in closed forms, revealing wave-like, solitary, and periodic behaviors under specific parameter conditions. The methodology combines symbolic computation with Lie symmetry analysis to simplify the governing equations and extract invariant solutions. Validation against numerical simulations confirms excellent agreement between analytical and computational results. The findings demonstrate that the approach can efficiently handle complex systems such as plasma oscillations, fluid dynamics, and nonlinear optics. Furthermore, the study establishes a systematic procedure for constructing solvable models by transforming nonlinear PDEs into canonical ODEs. This framework provides a unified tool for researchers investigating nonlinear evolution processes across scientific disciplines.

Keywords: nonlinear differential equations, analytical solution, Lie symmetry, mathematical modeling

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APPLICATION OF MULTIVARIATE REGRESSION MODELS FOR ECONOMIC TREND PREDICTION: ANALYZING THE IMPACT OF ENERGY PRICES, NATIONAL OUTPUT, AND GDP GROWTH

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Abstract
Forecasting economic trends is a vital component of national planning and global market stability. This paper investigates the application of multivariate regression models in predicting economic growth patterns, focusing on the influence of energy prices, national output, and GDP growth. The proposed analytical framework employs both linear and polynomial regression models to capture nonlinear relationships among macroeconomic variables. Using data from 1990 to 2024 across ten emerging economies, the study identifies statistically significant correlations between energy price volatility and GDP fluctuations. The multivariate model demonstrates superior predictive accuracy, achieving an R^2 value of 0.91 compared to 0.74 for univariate counterparts. The paper also explores how structural changes in energy policy and industrial productivity affect regression coefficients over time. Sensitivity analysis confirms the robustness of the model against inflationary shocks and exchange rate variations. The results suggest that incorporating multivariate interdependencies enhances the predictive reliability of macroeconomic forecasting tools. These findings can guide policymakers in formulating data-driven strategies for sustainable economic growth and energy management.

Keywords: multivariate regression, economic forecasting, energy prices, GDP growth

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ADVANCED SPATIAL INTERPOLATION USING HIERARCHICAL INVERSE DISTANCE WEIGHTING FOR COMPLEX GEOSPATIAL CLASSIFICATION

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Abstract

Spatial interpolation is a cornerstone of geospatial analysis, enabling estimation of unknown values from sparse spatial datasets. Traditional interpolation techniques such as ordinary inverse distance weighting (IDW) often struggle with highly heterogeneous data distributions and non-stationary spatial structures. This study proposes a hierarchical inverse distance weighting (HIDW) method designed to enhance spatial prediction accuracy in complex geospatial environments. The HIDW model integrates multi-scale neighborhood partitioning with adaptive weighting functions, allowing localized refinement of interpolation parameters based on spatial variance and data density. Comparative analysis using real-world topographic and environmental datasets from Quebec demonstrates that HIDW improves root-mean-square error by 22% and reduces spatial bias compared to standard IDW and kriging. Additionally, the hierarchical framework enables computational efficiency for large-scale applications by employing a quadtree-based indexing scheme. The method's applicability extends to environmental monitoring, climate modeling, and urban planning, where fine-scale spatial precision is critical. The research concludes that hierarchical interpolation represents a robust advancement in geospatial data science, merging mathematical rigor with practical utility.

Keywords: spatial interpolation, inverse distance weighting, geospatial analysis, hierarchical modeling

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ADVANCED FOURIER METHODS IN QUANTUM SIGNAL PROCESSING

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Abstract
The intersection of Fourier analysis and quantum mechanics provides a powerful framework for understanding and manipulating complex quantum signals. This study presents an advanced set of Fourier-based methodologies optimized for quantum signal processing (QSP). By extending classical Fourier transforms into the quantum domain, we develop a series of mathematical operators that enhance noise resilience and frequency resolution in qubit-based information systems. The paper introduces a hybrid quantum-Fourier framework that integrates discrete Fourier transforms with entanglement-preserving transformations, enabling the efficient decomposition of wavefunctions in both temporal and spectral spaces. Through analytical derivations and simulation-based experiments, the research demonstrates significant improvements in signal reconstruction accuracy and phase estimation efficiency under decoherence conditions. The proposed techniques also allow adaptive tuning of spectral components using variational quantum circuits, reducing computational complexity in large-scale quantum data analysis. Results indicate that the introduced Fourier operators outperform conventional quantum signal estimation models, achieving over 20% higher accuracy in phase-tracking tasks. The implications of this study extend to quantum communication, cryptography, and spectral imaging, suggesting a versatile foundation for future QSP frameworks.

Keywords: Quantum signal processing, Fourier transform, Qubit analysis, Spectral decomposition

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TOPOLOGICAL ANALYSIS OF NONLINEAR DYNAMICAL SYSTEMS

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Abstract
The study of nonlinear dynamical systems has long been enriched by the tools of topology, providing geometric insight into stability, bifurcation, and chaos. This paper presents a comprehensive topological framework for analyzing nonlinear dynamical systems using homology and persistent homology techniques. By mapping the evolution of system trajectories onto topological manifolds, we uncover structural invariants that characterize long-term behavior. The methodology integrates Morse theory with computational topology to identify attractors, limit cycles, and chaotic regimes within high-dimensional parameter spaces. Moreover, a new metric for measuring topological entropy is proposed, allowing for precise quantification of complexity in time-evolving nonlinear models. Numerical experiments on Lorenz and Rössler systems validate the robustness of the approach, revealing deep connections between topological persistence and system sensitivity. The findings highlight how topological analysis provides a stable descriptor of system dynamics even under perturbation or measurement noise. This research bridges theoretical mathematics and applied nonlinear science, offering a novel lens for understanding emergent patterns in physics, biology, and engineering.

Keywords: Nonlinear dynamics, Topology, Homology, Bifurcation

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MATHEMATICAL FOUNDATIONS OF DEEP REINFORCEMENT LEARNING

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Abstract

Deep reinforcement learning (DRL) represents a powerful intersection between control theory, optimization, and neural computation. This study investigates the mathematical underpinnings that govern the stability and convergence of DRL algorithms. Starting from Markov decision processes, we derive new theorems describing the existence and uniqueness of optimal policies under non-convex reward landscapes. The analysis leverages stochastic approximation and measure theory to formalize gradient behavior in deep policy networks. Furthermore, the research establishes a rigorous foundation for policy gradient convergence by applying Lyapunov stability analysis within dynamic learning environments. Theoretical predictions are supported through empirical studies using modified actor-critic architectures, which show enhanced stability when regularized with entropy-based constraints. The paper also explores how fixed-point iteration principles and spectral norm bounds contribute to understanding overparameterization in large-scale DRL models. By synthesizing mathematical rigor with computational insights, the study provides a foundation for designing more robust, interpretable, and scalable reinforcement learning systems.

Keywords: Deep reinforcement learning, Markov decision process, Stability, Policy optimization

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MESHLESS TECHNIQUES FOR 3D WAVE PROPAGATION IN COMPLEX MEDIA

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Abstract

Modeling three-dimensional wave propagation in complex and heterogeneous media poses significant challenges due to irregular geometries and discontinuous material properties. This paper introduces an innovative meshless computational technique that combines radial basis functions (RBF) with the time-domain formulation of wave equations. The proposed approach eliminates the need for mesh generation, thereby reducing computational overhead and improving flexibility for dynamic boundary conditions. Through comparative analysis, we demonstrate that the meshless model achieves high accuracy and stability even in anisotropic or layered materials. The method's robustness is further validated using seismic and acoustic test cases, where it successfully captures multiple scattering and diffraction phenomena. Additionally, a new adaptive node distribution algorithm is presented to enhance spatial resolution around interfaces and singularities. The numerical results show significant improvements in both computational efficiency and wavefield fidelity compared to traditional finite element and finite difference methods. These findings highlight the potential of meshless approaches in geophysical exploration, ultrasonic imaging, and structural health monitoring applications.

Keywords: Meshless methods, Wave propagation, Radial basis functions, Complex media

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EXPLORING THE EFFECTS OF URBANIZATION ON VECTOR-BORNE DISEASES: A SPATIO-TEMPORAL STUDY

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Abstract

Urbanization is transforming landscapes and ecosystems, influencing the transmission dynamics of vector-borne diseases. This research investigates the spatial and temporal correlations between urban expansion and disease prevalence using a mixed-method approach that combines satellite imagery, epidemiological data, and climate variables. The study employs spatial autocorrelation metrics and time-series decomposition to analyze mosquito-borne disease patterns in rapidly urbanizing African cities between 2000 and 2024. Results reveal that increased population density, waste accumulation, and microclimatic changes significantly enhance breeding habitats for vectors such as *Aedes aegypti* and *Anopheles gambiae*. Temporal modeling using autoregressive distributed lag frameworks demonstrates that temperature and humidity fluctuations act as leading indicators for disease incidence. Moreover, the research introduces a vulnerability index integrating socio-economic and environmental factors to predict outbreak risks. The study concludes that effective urban planning and ecological interventions are essential to mitigating the health impacts of uncontrolled urbanization.

Keywords: Urbanization, Vector-borne diseases, Spatio-temporal analysis, Public health

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MODELING THE INFLUENCE OF TEMPERATURE VARIABILITY ON MONSOON PATTERNS IN EASTERN INDIA

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Abstract

This study examines the intricate relationship between temperature variability and monsoon dynamics in Eastern India using high-resolution climatological data and statistical modeling. Employing empirical orthogonal function (EOF) analysis, the research identifies dominant modes of temperature fluctuation and their influence on regional monsoon intensity. Results indicate that short-term temperature anomalies significantly alter convection patterns, leading to delayed monsoon onset and uneven precipitation distribution. Using a coupled land–atmosphere model, the study quantifies feedback mechanisms linking surface temperature gradients and moisture transport. Additionally, long-term trend analysis based on 40 years of meteorological data reveals a gradual weakening of monsoon predictability due to increasing interannual temperature variance. The findings underscore the necessity of incorporating thermal variability into climate prediction models for accurate forecasting of monsoon-related agricultural and hydrological outcomes.

Keywords: Monsoon modeling, Temperature variability, Climate change, Atmospheric dynamics

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ON THE GENERALIZATION OF SALVADORI NUMBERS IN FORMAL POWER SERIES FIELDS

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Abstract

This paper extends the concept of Salvadori numbers to the framework of formal power series fields, offering a broader algebraic perspective on numerical generalizations. Building upon foundational number theory, we define a new class of generalized Salvadori numbers characterized by polynomial congruences and valuation constraints. The study develops several lemmas concerning their divisibility properties, field extensions, and relations with local ring structures. Through a rigorous algebraic formulation, the results demonstrate that these generalized numbers form a unique subset within non-Archimedean fields, exhibiting closure under addition and limited closure under multiplication. Examples involving p-adic valuations illustrate the role of these numbers in encoding structural symmetries within formal series expansions. The research further explores potential applications in cryptographic systems, where the algebraic stability of Salvadori numbers provides enhanced resilience against factorization attacks. Overall, this generalization deepens the theoretical understanding of numerical structures across modern algebraic systems.

Keywords: Salvadori numbers, Formal power series, Number theory, Algebraic structures

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A NOVEL ALGORITHM FOR SOLVING THE EXTENDED MALFATTI PROBLEM IN NON-CONVEX TRIANGLES

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Abstract

The classical Malfatti problem, which seeks to inscribe three mutually tangent circles within a triangle, has inspired numerous geometric generalizations. This study presents a novel computational algorithm to solve the extended Malfatti problem in non-convex and irregular triangular geometries. The algorithm utilizes a combination of geometric transformation and nonlinear optimization techniques to determine feasible circle positions that satisfy tangency constraints. By introducing a parameterized curvature-based function, the model achieves stable convergence even under high geometric asymmetry. Analytical derivations are supported by numerical simulations demonstrating that the proposed approach efficiently handles degenerate cases where classical solutions fail. Moreover, the method integrates symbolic computation to automatically verify tangency conditions, reducing human intervention. Results show that the algorithm achieves sub-millimeter precision in geometric reconstruction tasks, making it applicable for fields such as CAD design, robotics, and structural modeling. This research not only extends the traditional Malfatti framework but also establishes a computational foundation for complex geometric optimization problems.

Keywords: Malfatti problem, Computational geometry, Nonlinear optimization, Non-convex triangles